Robust Guaranteed Cost Filtering for Uncertain Systems with Time-Varying Delay Via LMI Approach

Jong Hae Kim

Abstract: In this paper, we consider the guaranteed cost filtering design method for time-varying delay systems with parameter uncertainties by LMI(Linear Matrix Inequality) approach. The objective is to design a stable guaranteed cost filter which minimizes the guaranteed cost of the closed loop system in filtering error dynamics. The sufficient conditions for the existence of filter, the guaranteed cost filter design method, and the guaranteed cost upper bound are proposed by LMI technique in terms of all finding variables. Finally, we give an example to check the validity of the proposed method.

Keywords: guaranteed cost filtering, time delay systems, parameter uncertainty, LMI

I. Introduction

Since the filtering design that can handle model uncertainties has been one of the interesting problems, much effort[1]-[4] has been devoted to the development of filtering design method. Petersen and McFarlane[1] presented the results on the design of robust state feedback controllers and steady-state robust state estimator for a class of uncertain linear systems with norm bounded uncertainty. Wang and Unbehauen[2] considered the observer design problem for uncertain linear systems with D-stability constraints. Xie and Soh[3] studied the problem of Kalman filter design for uncertain systems using Riccati equation approach. Also, Xie et al.[4] treated the problem of H_∞ estimation for discrete time linear uncertain systems. However, they just considered parameter uncertain systems without time delay using Riccati equation technique. Also, the extensive use of optimization criteria like the H_2 and/or H_∞ norm has consolidated the importance of estimation and filtering in linear systems theory[5]-[8]. Geromel et al.[5][6] dealt with H_2 and H_∞ robust filtering for discrete time systems and convex bounded uncertain systems by LMI techniques. Also, Palhares et al.[7][8] considered the problem of mixed L_2-L_∞/H_∞ filtering for uncertain systems and robust H_∞ filtering design with pole constraints for discrete time systems using LMI approach.

Recently, many works treated time delay systems in control part because the time delay is frequently a source of instability and encountered in various engineering systems. However, most of filtering works did not consider time delay in parameter uncertain systems. Recently, Yu et al.[9] proposed guaranteed cost control methods for uncertain systems with time delay. More recently, Kim[10] treated the problem of designing guaranteed cost state feedback controller for the generalized time-varying delay systems with delayed state and control input by LMI approach. However, there are no papers considering both time delay and parameter uncertainty to guarantee the upper bound of cost function in filtering design problems. And, most of works considering guaranteed cost filtering problem without time delay were somewhat conservative because some variables should be determined before finding solutions from Algebraic Riccati equations. Also, we want to develop the guaranteed cost filtering design algorithms as a dual part of guaranteed cost control design method[10]. However, the proposed result is not derived directly from the work[10].

Therefore, we propose the guaranteed cost filtering design method for parameter uncertain time delay systems without any pre-selections of variables by LMI technique. Moreover, we present an optimization problem to get the optimal guaranteed cost filter and the upper bound of guaranteed cost.

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II. Guaranteed cost filtering design

Consider a linear time-varying delay system

$$\begin{aligned} \dot{x}(t) &= (A+\Delta A(t))x(t)+(A_d+\Delta A_d(t))x(t-d(t)) \\ y(t) &= (C+\Delta C(t))x(t) \\ x(t) &= \phi_1(t), \ -d(0) \le t \le 0 \end{aligned} \tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state vector, $y(t)\in\mathbf{R}^r$ is the measurement output vector, $\phi_1(t)$ is an initial value function, and all matrices have proper dimensions. Here, time-varying delay is satisfied with

$$0 \le d(t) \le \infty, \ \dot{d}(t) \le \gamma < 1, \tag{2}$$

and time-varying delay is the known state delay. The parameter uncertainties are defined as

$$\begin{aligned} \begin{bmatrix} \Delta A(t) \\ \Delta C(t) \end{bmatrix} &= \begin{bmatrix} H_1 \\ H_2 \end{bmatrix} F_1(t)E, \\ \Delta A_d(t) &= H_d F_2(t) E_d. \end{aligned} \tag{3}$$

Here, unknown matrix is defined as

$$F_i(t) \in \Omega := \{F_i(t): F_i(t)^T F_i(t) \le I(i=1,2), \text{ the elements of } F_i(t) \text{ are Lebesgue measurable}\}. \tag{4}$$

We assume that the system (1) is asymptotically stable. This assumption guarantees that the boundedness of the filtering error holds, since the asymptotic stability of the filtering error dynamics depends on the states of the system (1). Our aim is to design a stable linear guaranteed cost filter described by

$$\dot{\hat{x}}(t) = G\hat{x}(t) + A_d\hat{x}(t-d(t)) + Ky(t) \tag{5}$$

where, G and K are filter variables. If we take the error state vector as follows:

$$e(t) := x(t) - \hat{x}(t), \tag{6}$$

then the error dynamics is obtained

$$\begin{aligned} \dot{e}(t) &= Ge(t) + (A - KC - G)x(t) + A_d e(t-d(t)) \\ &+ (\Delta A(t) - K\Delta C(t))x(t) + \Delta A_d(t)x(t-d(t)) \\ z(t) &= Le(t) \end{aligned} \tag{7}$$

by defining the error state output as $z(t) = Le(t)$. If we define the following augmented state vector

$$x_f(t) := \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \tag{8}$$

then the filtering error dynamics is given by

$$\begin{aligned} \dot{x}_f(t) &= (A_f + H_f F_1(t) E_f) x_f(t) \\ &\quad + (A_{df} + H_{df} F_2(t) E_{df}) x_f(t-d(t)) \\ &:= \overline{A}_f x_f(t) + \overline{A}_{df} x_f(t-d(t)) \\ z(t) &= C_f x_f(t) \\ x_f(t) &= \phi_f(t) = \begin{bmatrix} \phi_1(t) \\ \phi_2(t) \end{bmatrix}, \quad -d(0) \le t \le 0, \end{aligned} \tag{9}$$

where some notations are denoted by

$$\begin{aligned} &A_f = \begin{bmatrix} A & 0 \\ A - KC - G & G \end{bmatrix}, A_{df} = \begin{bmatrix} A_d & 0 \\ 0 & A_d \end{bmatrix}, C_f = [0 \ \ L], \\ &H_f = \begin{bmatrix} H_1 \\ H_1 - KH_2 \end{bmatrix}, H_{df} = \begin{bmatrix} H_d \\ H_d \end{bmatrix}, E_f = [E \ \ 0], E_{df} = [E_d \ \ 0], \end{aligned} \tag{10}$$

and $\phi_2(t)$ is an initial error value function. Here, we introduce guaranteed cost

$$J = \int_0^\infty z(t)^T z(t) dt. \tag{11}$$

Therefore, our objective is to develop the stable guaranteed cost filtering design method satisfying the minimization of guaranteed cost (11). In the following, we present an LMI optimization problem to get the optimal guaranteed cost filter and the upper bound of guaranteed cost bound.

Theorem 1: If the following optimization problem

$$\text{minimize } \{\alpha + tr(Q)\} \text{ subject to} \tag{12}$$

$$\begin{bmatrix} \Psi_1 & \Psi_2 & P_1 A_d & 0 & P_1 H_1 & P_1 H_d \\ * & \Psi_3 & 0 & P_2 A_d & P_2 H_1 - M_2 H_2 & P_2 H_d \\ * & * & -(1-\gamma)S_1 + \beta_2 E_d^T E_d & -(1-\gamma)S_2 & 0 & 0 \\ * & * & * & -(1-\gamma)S_3 & 0 & 0 \\ * & * & * & * & -\beta_1 I & 0 \\ * & * & * & * & * & -\beta_2 I \end{bmatrix} < 0, \tag{13}$$

$$-\alpha + \phi_1(0)^T P_1 \phi_1(0) + \phi_2(0)^T P_2 \phi_2(0) < 0, \tag{14}$$

$$-Q + N_1^T S_1 N_1 + N_2^T S_2 N_1 + N_1^T S_2 N_2 + N_2^T S_3 N_2 < 0 \tag{15}$$

has a solution positive definite matrices(or scalar) P_1, P_2, S_1, S_2, S_3, α, Q, β_1, β_2 and matrices M_1, M_2, then (5) is a guaranteed cost filter and $J^* = \alpha + tr(Q)$ is an upper bound of guaranteed cost. Here, $*$ represents the elements below the main diagonal of a symmetric matrix, $tr(\cdot)$ denotes the trace of a matrix, and some notations are defined as

$$\begin{aligned} &\Psi_1 = A^T P_1 + P_1 A + S_1 + \beta_1 E^T E \\ &\Psi_2 = A^T P_2 - C^T M_2^T - M_1^T + S_2 \\ &\Psi_3 = M_1^T + M_1 + L^T L + S_3 \\ &M_1 = P_2 G \\ &M_2 = P_2 K \\ &\beta_i = \frac{1}{\varepsilon_i}, \quad i = 1, 2 \\ &\int_{-d(0)}^0 \phi_f(\tau)\phi_f(\tau)^T d\tau = NN^T = \begin{bmatrix} N_1 \\ N_2 \end{bmatrix} [N_1^T \ \ N_2^T]. \end{aligned} \tag{16}$$

Proof: If we take a Lyapunov functional

$$V(x_f(t)) = x_f(t)^T P x_f(t) + \int_{t-d(t)}^t x_f(\tau)^T S x_f(\tau) d\tau, \tag{17}$$

then the derivative of (17) is given by

$$\begin{aligned} \dot{V}(x_f(t)) &= \dot{x}_f(t)^T P x_f(t) + x_f(t)^T P \dot{x}_f(t) \\ &\quad + x_f(t)^T S x_f(t) \\ &\quad - (1 - \dot{d}(t)) x_f(t-d(t))^T S x_f(t-d(t)) \\ &\le \dot{x}_f(t)^T P x_f(t) + x_f(t)^T P \dot{x}_f(t) \\ &\quad + x_f(t)^T S x_f(t) \\ &\quad - (1-\gamma) x_f(t-d(t))^T S x_f(t-d(t)) \\ &:= V_a(x_f(t)). \end{aligned} \tag{18}$$

The linear matrix inequality (13) implies that

$$\dot{V}(x_f(t)) \le V_a(x_f(t)) < -z(t)^T z(t) < 0. \tag{19}$$

Therefore we have

$$\begin{bmatrix} x_f(t) \\ x_f(t-d(t)) \end{bmatrix}^T \begin{bmatrix} \Sigma & P\overline{A}_{df} \\ * & -(1-\gamma)S \end{bmatrix} \begin{bmatrix} x_f(t) \\ x_f(t-d(t)) \end{bmatrix} < 0, \tag{20}$$

where, $\Sigma = \overline{A}_f^T P + P\overline{A}_f + C_f^T C_f + S$. And (20) is changed to

$$\begin{bmatrix} x_f(t) \\ x_f(t-d(t)) \end{bmatrix}^T \begin{bmatrix} \Phi_1 & PA_{df} \\ * & \Phi_2 \end{bmatrix} \begin{bmatrix} x_f(t) \\ x_f(t-d(t)) \end{bmatrix} < 0 \tag{21}$$

by the following lemma

$$\begin{aligned} &2x_f(t)^T P H_f F_1(t) E_f x_f(t) \\ &\le \varepsilon_1 x_f(t)^T P H_f H_f^T P x_f(t) + \frac{1}{\varepsilon_1} x_f(t)^T E_f^T E_f x_f(t), \\ &2x_f(t)^T P H_{df} F_2(t) E_{df} x_f(t-d(t)) \\ &\le \varepsilon_2 x_f(t)^T P H_{df} H_{df}^T P x_f(t) \\ &+ \frac{1}{\varepsilon_2} x_f(t-d(t))^T E_{df}^T E_{df} x_f(t-d(t)). \end{aligned} \tag{22}$$

Here,

$$\begin{aligned} \Phi_1 &= A_f^T P + PA_f + C_f^T C_f + S \\ &\quad + \varepsilon_1 P H_f H_f^T P + \frac{1}{\varepsilon_1} E_f^T E_f + \varepsilon_2 P H_{df} H_{df}^T P, \\ \Phi_2 &= -(1-\gamma)S + (1/\varepsilon_2) E_{df}^T E_{df}, \end{aligned}$$

and $\varepsilon_i, i = 1, 2,$ is a positive constant. By Schur complements, the matrix inequality

$$\begin{bmatrix} \Phi & PA_{df} \\ * & -(1-\gamma)S+\frac{1}{\varepsilon_2}E_{df}^TE_{df} \end{bmatrix} < 0 \tag{23}$$

is equivalent to

$$\begin{bmatrix} \Theta & PA_{df} & PH_f & PH_{df} \\ * & -(1-\gamma)S+\frac{1}{\varepsilon_2}E_{df}^TE_{df} & 0 & 0 \\ * & * & -\frac{1}{\varepsilon_1}I & 0 \\ * & * & * & -\frac{1}{\varepsilon_2}I \end{bmatrix} < 0, \tag{24}$$

where, $\Theta = A_f^TP + PA_f + C_f^TC_f + S + \frac{1}{\varepsilon_1}E_f^TE_f$. And if we set

$$P = \begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}, \quad S = \begin{bmatrix} S_1 & S_2 \\ S_2 & S_3 \end{bmatrix}, \tag{25}$$

then (24) is transformed into

$$\begin{bmatrix} \Lambda_1 & \Lambda_2 & P_1A_d & 0 & P_1H_1 & P_1H_d \\ * & \Lambda_3 & 0 & P_2A_d & P_2H_1-P_2KH_2 & P_2H_d \\ * & * & -(1-\gamma)S_1+\frac{1}{\varepsilon_2}E_d^TE_d & -(1-\gamma)S_2 & 0 & 0 \\ * & * & * & -(1-\gamma)S_3 & 0 & 0 \\ * & * & * & * & -\frac{1}{\varepsilon_1}I & 0 \\ * & * & * & * & * & -\frac{1}{\varepsilon_2}I \end{bmatrix} < 0, \tag{26}$$

where,

$$\Lambda_1 = A^TP_1 + P_1A + S_1 + \frac{1}{\varepsilon_1}E^TE,$$
$$\Lambda_2 = A^TP_2 - C^TK^TP_2 - G^TP_2 + S_2,$$
$$\Lambda_3 = G^TP_2 + P_2G + L^TL + S_3.$$

Using some changes of variables, $M_1 = P_2G$, $M_2 = P_2K$, and $\frac{1}{\varepsilon_i} = \beta_i$, $i = 1, 2$, (26) is transformed into (13). Furthermore, by the integrating both sides of the inequality (19) from 0 to T_f and using the initial condition, we obtain

$$-\int_0^{T_f} z(t)^Tz(t)dt > x_f(T_f)^TPx_f(T_f) - x_f(0)^TPx_f(0) + \int_{T_f-d(0)}^{T_f} x_f(\tau)^TSx_f(\tau)d\tau - \int_{-d(0)}^{0} x_f(\tau)^TSx_f(\tau)d\tau. \tag{27}$$

As the closed loop system is asymptotically stable, when $T_f \to \infty$, some terms go to zero. Hence we get

$$\int_0^{\infty} z(t)^Tz(t)dt \le \phi_f(0)^TP\phi_f(0) + \int_{-d(0)}^{0} \phi_f(\tau)^TS\phi_f(\tau)d\tau. \tag{28}$$

This is an upper bound of guaranteed cost. The first term of right hand side in (28) is changed to

$$-\alpha + \phi_f(0)^T P\phi_f(0) < 0.$$

This is equivalent to (14). The second term of right hand side in (28) has the following relations

$$\int_{-d(0)}^{0} \phi_f(\tau)^TS\phi_f(\tau)d\tau = \int_{-d(0)}^{0} tr(\phi_f(\tau)^TS\phi_f(\tau))d\tau = tr(NN^TS) = tr(N^TSN) < tr(Q). \tag{29}$$

Therefore, $-Q + N^TSN < 0$ is equal to (15). ■

Hence, we can get the optimal guaranteed cost filter. Also, all solutions including filter variables($G = P_2^{-1}M_1$, $K = P_2^{-1}M_2$) and the upper bound of guaranteed cost ($J^* = \alpha + tr(Q)$) can be calculated simultaneously because the proposed sufficient conditions are LMIs in terms of all finding variables.

Remark 1: The optimization problem in Theorem 1 can be solvable easily using the command of 'mincx' in LMI Toolbox[11], which is numerically efficient owing to recent advances in convex optimization. Therefore, all solutions (P_1, P_2, S_1, S_2, S_3, α, Q, β_1, β_2, M_1, and M_2) can be obtained at the same time.

Remark 2: The proposed guaranteed cost filter design algorithm can be extended into various guaranteed cost filter problems including continuous and discrete time systems such as multiple time delay systems, convex bounded uncertain systems, interconnected systems, and so on. Moreover, the presented guaranteed cost filter design algorithm includes the design method for the parameter uncertain systems without time delay.

III. Numerical example

In order to check the validity of the proposed filter design algorithm, we consider a parameter uncertain system with time-varying delay

$$\begin{aligned} \dot{x}(t) &= \left\{\begin{bmatrix} -1 & 0 \\ 1 & -2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} F_1(t)[1\ \ 1]\right\}x(t) \\ &\quad + \left\{\begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} F_2(t)[1\ \ 1]\right\}x(t-d(t)) \\ y(t) &= \{[1\ \ 0] + 0.1F_1(t)[1\ \ 1]\}x(t) \\ z(t) &= [1\ \ 1]e(t) \\ d(t) &= 2 + 0.2\sin t, \ \phi_f(t) = [e^{t+1}\ \ 0\ \ 0.1\ \ 1]^T. \end{aligned} \tag{30}$$

All solutions are obtained simultaneously using LMI control Toolbox[11] as follows:

$$\begin{aligned} &P_1 = \begin{bmatrix} 0.0193 & 0.0082 \\ 0.0082 & 0.0050 \end{bmatrix}, \ P_2 = \begin{bmatrix} 0.3417 & 0.4603 \\ 0.4603 & 0.6623 \end{bmatrix}, \\ &S_1 = \begin{bmatrix} 0.0139 & 0.0143 \\ 0.0143 & 0.0153 \end{bmatrix}, \ S_2 = 10^{-3} \times \begin{bmatrix} 0.0656 & -0.0366 \\ -0.0366 & 0.4245 \end{bmatrix}, \\ &S_3 = \begin{bmatrix} 0.0111 & -0.0472 \\ -0.0472 & 0.3014 \end{bmatrix}, \ \alpha = 0.1460, \\ &Q = \begin{bmatrix} 0.0001 & 0 & 0 & 0 \\ 0 & 0.0506 & -0.0000 & 0 \\ 0 & -0.0000 & 0.0001 & 0 \\ 0 & 0 & 0 & 0.0001 \end{bmatrix}, \\ &M_1 = \begin{bmatrix} -1.0422 & -0.9251 \\ -0.9055 & -1.3110 \end{bmatrix}, \ M_2 = \begin{bmatrix} 1.1520 \\ 1.1358 \end{bmatrix}, \\ &\beta_1 = 0.0032, \ \beta_2 = 0.0105. \end{aligned} \tag{31}$$

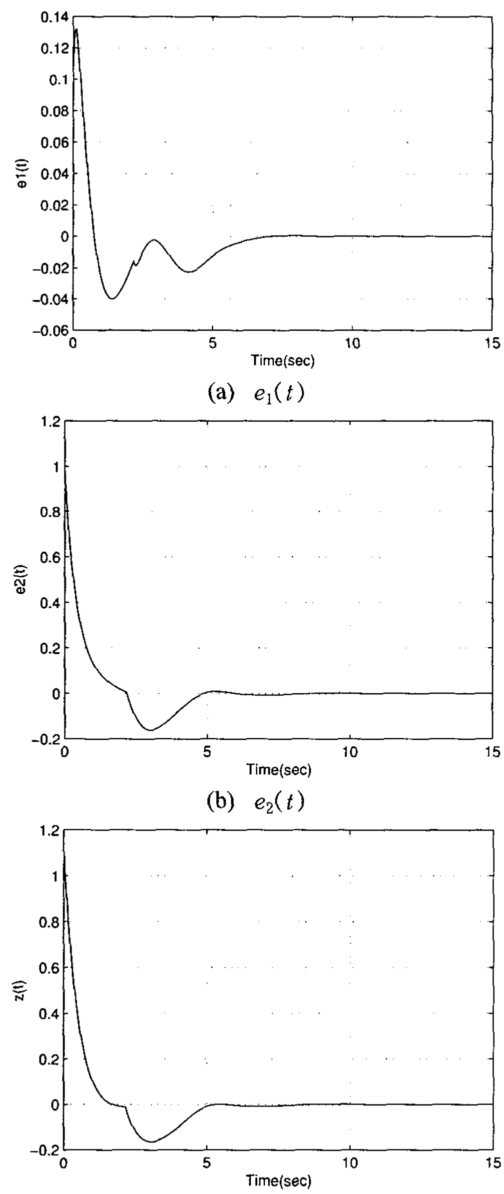
Therefore, the guaranteed cost filter and the upper bound of guaranteed cost are

$$\begin{aligned}\hat{x}(t) &= \begin{bmatrix} -19.0243 & -0.6358 \\ 11.8565 & -1.5376 \end{bmatrix}\hat{x}(t) \\ &+\begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.2 \end{bmatrix}\hat{x}(t-d(t))+\begin{bmatrix} 16.7099 \\ -9.9002 \end{bmatrix}y(t), \\ \mathcal{J}^* &= 0.1557. \end{aligned} \tag{32}$$

Moreover, the obtained filter guarantees an optimal guaranteed cost. For computer simulation, unknown matrices are defined by

$$\begin{aligned} F_1(t) &= \sin t, \\ F_2(t) &= \cos t. \end{aligned} \tag{33}$$

The trajectories of error states and error state output are shown in Fig. 1. Therefore, the obtained filter guarantees not only asymptotic stability of filtering error dynamics but also minimization of upper bound in guaranteed cost function against parameter uncertainties and time-varying delay.



(a) $e_1(t)$

(b) $e_2(t)$

(c) $z(t)$

Fig. 1. The trajectories of error state and error state output.

V. Conclusion

In this paper, we proposed the guaranteed cost filter design algorithm for time delay systems with parameter uncertainties. The sufficient conditions for the existence of filter and guaranteed cost filter design method were presented using LMI technique. Since the proposed conditions were LMI forms in terms of all finding variables, all solutions could be calculated at the same time. Also, we checked the validity of the proposed method by an example.

References

[1] I. R. Petersen and D. C. McFarlane, "Optimal guaranteed cost control and filtering for uncertain linear systems," *IEEE Transaction on Automatic Control*, vol. 39, no. 9, pp. 1971-1977, 1994.

[2] Z. Wang and H. Unbehauen, "Robust state observer design guaranteeing D-stability," *International Journal of Systems Science*, vol. 29, no. 12, pp. 1417-1426, 1998.

[3] L. Xie and Y. C. Soh, "Robust Kalman filtering for uncertain systems," *Systems and Control Letters*, vol. 22, pp. 123-129, 1994.

[4] L. Xie, C. E. de Souza, and M. Fu, "H_∞ estimation for discrete time linear uncertain systems," *International Journal of Robust and Nonlinear Control*, vol. 1, pp. 11-23, 1991.

[5] J. C. Geromel, J. Bernussou, G. Garcia, and M. C. Oliveira, "H_2 and H_∞ robust filtering for discrete-time linear systems," *Proc. of IEEE Conference on Decision and Control*, Tampa, Florida, USA, pp. 632-637, 1998.

[6] J. C. Geromel and M. C. Oliveira, "H_2 and H_∞ robust filtering for convex bounded uncertain systems," *Proc. of IEEE Conference on Decision and Control*, Tampa, Florida, USA, pp. 146-151, 1998.

[7] R. M. Palhares and P. L. D. Peres, "Mixed L_2-L_∞/H_∞ filtering for uncertain linear systems: An LMI approach," *ISIE '99-Bled.*, pp. 1070-1075, 1999.

[8] R. M. Palhares and P. L. D. Peres, "Robust H_∞ filtering design with pole constraints for discrete-time systems: An LMI approach," *Proc. of American control Conference*, San Diego, California, USA, pp. 4418-4422, 1999.

[9] L. Yu and J. Chu, "An LMI approach to guaranteed cost control of linear uncertain time delay systems," *Automatica*, vol. 35, pp. 1155-1159, 1999.

[10] J. H. Kim, "Guaranteed cost control of parameter uncertain systems with time delay," *Transactions on Control, Automation and Systems Engineering*, vol. 2, no. 1, pp. 19-23, 2000.

[11] P. Gahinet, A. Nemirovski, A. J. Laub, and M. Chilali, *LMI Control Toolbox*, The Math Works Inc., 1995.

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