Adaptive Approaches on the Sliding Mode Control of Robot Manipulators

Jae-Sam Park, Gueon-Sang Han, Hyun-Sik Ahn, and Do-Hyun Kim

Abstract: In this paper, adaptive algorithms on the sliding mode control for robust tracking control of robot manipulators are presented. The presented algorithms use adaptation laws for tuning both the sliding mode gain and the thickness of the boundary layer to reject a discontinuous control input, and to improve the tracking performance. It is shown that the robustness of the developed adaptive algorithms are guaranteed by the sliding mode control law and that the algorithms are globally convergent in the presence of disturbances and modeling uncertainties. Computer simulations are performed for a two-link manipulator, and the results show good properties of the proposed adaptive algorithms under large manipulator parameter uncertainties and disturbances.

Keywords: adaptive algorithms, sliding mode control, robust tracking, robot manipulators

I. Introduction

In recent years, methodology known as sliding mode control has been researched actively, and the sliding mode control has effectively used in the tracking control of robot manipulators by many researchers[2][3][5][6]. The concept of sliding mode control has been studied in detail in [1][7][8] where it has been used to stabilize a class of non-linear systems.

For faster manipulator dynamics in the presence of model uncertainties such as parameter perturbations, unknown joint frictions and inertias, and external disturbances, various types of adaptive sliding mode controllers have been developed to alter the control signal to account for changes in robot dynamics and disturbances in the environment, so as to improve the overall performance of the conventional sliding mode control algorithms, for example [3][6]. The algorithm in [3] uses the adaptation law for tuning the boundary layer of sliding mode controller. Another in [6], the adaptation law is applied to a sliding mode control algorithm to have the sliding mode gain adjusted continuously during operation.

In this paper, a new type of adaptive sliding mode controller, which tunes both the sliding mode gain and the boundary layer thickness, for robust tracking control of robot manipulators is presented. The algorithm uses adaptation laws for tuning both the sliding mode gain and the thickness of the boundary layer to reject a discontinuous control input, and to improve the tracking performance of the manipulator. It is shown that the robustness of the developed adaptive algorithms are guaranteed by the sliding mode control law and that the algorithms are globally convergent in the presence of disturbances and modeling uncertainties.

The proposed algorithm has advantages that i) it tunes both the thickness of a boundary layer and the sliding mode gain for sliding mode controller. Therefore, the tracking performance can be improved, it is good for rejection of control chattering phenomenon, and fairly large parameter variation and disturbances can be handled. ii) the adaptation laws of the proposed scheme are simple both for the gain adaptation and the boundary layer adaptation. Thus the computational load required is roughly same as that of a PID controller. Therefore the proposed scheme is easy to implement to a real-time control with no extra high cost.

The proposed adaptive algorithms are applied to a two-link robot manipulator and computer simulations are performed. The simulation results show the good properties of the developed schemes.

The organization of this paper is as follows: section 2 gives some mathematical formulations which will be useful to develop the adaptive sliding mode control algorithm for manipulators; section 3 and section 4 present adaptive sliding mode control algorithms; section 5 contains the computer simulation results for the proposed adaptive sliding mode control algorithms which show that the proposed algorithms possess good properties under large manipulator parameter uncertainties and disturbances; section 6 concludes the paper.

II. Problem formulation

Consider the rigid body dynamic n-link manipulator derived via the Euler-Lagrange equations [4]:

$$\begin{aligned} M(q)\ddot{q} + h(q,\dot{q}) + \tau_d(t) &= \tau(t) \\ h(q,\dot{q}) &= V(q,\dot{q}) + D\dot{q} + G(q) \end{aligned} \tag{1}$$

where $q \in \boldsymbol{R}^n$ and $\dot{q} \in \boldsymbol{R}^n$ are joint angle and angular velocity, respectively. $M(q) = M^T(q) \in \boldsymbol{R}^{n\times n}$ is the inertia matrix, which is symmetric, positive definite. $V(q,\dot{q}) \in \boldsymbol{R}^n$ contains centrifugal and Coriolis terms. $D\dot{q}$ and $G(q)$ describe viscous friction and gravity, respectively. For simplicity, these terms are combined and expressed as $h(q,\dot{q})$ in (1). $\tau_d \in \boldsymbol{R}^n$ represents the unknown disturbances, such as static friction or Coulomb friction. $\tau \in \boldsymbol{R}^n$ is the vector of input torques.

In joint space, the control problem for robot manipulators

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is to synthesize a control law for the torques such that the joint output, $q(t) \in \boldsymbol{R}^n$, traces the desired trajectory, $q_d(t) \in \boldsymbol{R}^n$, with a certain precision defined by

$$\tilde{\mathbf{q}} = [\tilde{q} \ \ \dot{\tilde{q}}]^{\top}, \quad \|\tilde{q}\| = \|q - q_d\| \le \gamma_1, \quad \|\dot{\tilde{q}}\| = \|\dot{q} - \dot{q}_d\| \le \gamma_2, \ \gamma_1 > 0, \ \gamma_2 > 0. \tag{2}$$

It is assumed that $q_d(t)$, $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are well defined and bounded for all operational time t.

Generally, the transient dynamics of SMC(Sliding Mode Control) consists of two conditions: a reaching condition and a sliding condition. Under the reaching condition, the desired response aims to reach the switching manifold in finite time. The switching manifold Z is written as

$$Z = \{ \tilde{\mathbf{q}} \mid z(\tilde{\mathbf{q}}) = 0 \} \tag{3}$$

where z denotes a switching function,

$$z = \dot{\tilde{q}} + \lambda \tilde{q} \tag{4}$$

with $\lambda = diag(\lambda_1, \lambda_2, \cdots, \lambda_n)$, $\lambda_i > 0$.

Parameters of the switching manifold dominate the dynamic behavior of the system during sliding mode control. The Lyapunov function approach is one of methods for specifying reaching condition or the sliding condition. The reaching condition or the sliding condition is obtained as [7]

$$\frac{1}{2}\frac{d}{dt} z^{\top} z \le -\check{\eta}\|z\| \tag{5}$$

where $\check{\eta}$ is strictly positive and constant.

From (1) and (4), we have $\dot{z} = M^{-1}(\tau - h - \tau_d) - \ddot{q}_d + \lambda \dot{\tilde{q}}$, or

$$M\dot{z} = M(\lambda \dot{\tilde{q}} - \ddot{q}_d) + \tau - h - \tau_d. \tag{6}$$

Lemma 1: Suppose that $M > 0$ is a bounded differentiable matrix function of q and τ_d is bounded on $\dot{q}$. Then, there exists constant $\eta > 0$ such that

$$z^{\top}[M(\lambda \dot{\tilde{q}} - \ddot{q}_d) - h - \tau_d] + \frac{1}{2} z^{\top} \dot{M} z \le \phi\eta\|z\|, \ \forall q, \dot{q} \tag{7}$$

with

$$\phi = 1 + \|z\| + \|z\|^2. \tag{8}$$

Proof: It is well known that M is positive definite and a bounded differentiable matrix function of q, $V(q, \dot{q})$ is a function at most quadratic in $\dot{q}$, and the gravitation $G(q)$ is bounded. Note that τ_d is a function of at most first order in $\dot{q}$. Noting that q_d and $\dot{q}_d$ are bounded, we have that q and $\dot{q}$ are bounded on $\|\tilde{q}\|$ and $\|\dot{\tilde{q}}\|$ respectively. From (4), $\dot{\tilde{q}} = z - \lambda\tilde{q}$ and thus $\tilde{q}(s) = [1 - T(s)]z(s)$ with $T(s) = \lambda(sI + \lambda)^{-1}$. Denote the H_∞ norm of a stable transfer function by $\|\cdot\|_\infty$. Then, it follows that $\|\dot{\tilde{q}}\| \le \|z\| + \|T\|_\infty\|z\|$. Since $T(s)$ is stable, $\|T\|_\infty$ is bounded and thus $\|\dot{\tilde{q}}\|$ is bounded on $\|z\|$. This in turn implies that $\dot{q}$ is bounded on z. Since M is a bounded differentiable function of q, $\dot{M}$ is bounded on $\dot{q}$. Therefore, it is clear that there exist bounded nonlinear functions $\theta(t), f_1(t), f_2(t) \in \boldsymbol{R}$ such that

$$z^{\top}[M(\lambda \dot{\tilde{q}} - \ddot{q}_d) - h - \tau_d] + \frac{1}{2} z^{\top} \dot{M} z \le [\theta(t) + f_1(t)\|z\| + f_2(t)\|z\|^2]\|z\|, \ \forall q, \dot{q}.$$

Let $\eta = \max\left(\sup_t \|\theta(t)\|, \ \sup_t \|f_1(t)\|, \ \sup_t \|f_2(t)\| \right)$. Then, clearly (7) holds.

III. Adaptive sliding mode control

In this section, we propose an adaptive sliding mode control law for the uncertain system (1). The control law is designed as

Algorithm 1: Adaptive sliding mode control law

$$\tau = -k_1 z - u_s, \ k_1 > 0$$

$$u_s = \begin{cases} \phi\bar{\eta}\dfrac{z}{\|z\|} & \text{if } \|z\| > \varepsilon \\ \phi\bar{\eta}\dfrac{z}{\varepsilon} & \text{otherwise} \end{cases} \quad ; 1 > \varepsilon > 0 \tag{9}$$

$$\dot{\bar{\eta}} = -\alpha\bar{\eta} + \beta\phi\|z\|, \ 1 > \alpha > 0, \ \beta > 0, \ \bar{\eta}(0) \ge 0$$

with ϕ defined by (8). In (9), we see that u_s is the sliding-mode torque vector with z as the sliding surface and gain $\bar{\eta}$ is adjusted adaptively such that $\bar{\eta} > \eta$, where η defined by Lemma 1. Then, we have the following result.

Theorem 1: Consider the system (1) with the control law (9). The closed-loop system is globally stable in the sense that $\bar{\eta}$ is bounded and

a) z is globally bounded by

$$\|z\| \le \varepsilon \operatorname{pos}(\eta - \bar{\eta} + 1) \le \varepsilon \tag{10}$$

where

$$\operatorname{pos}(\eta - \bar{\eta} + 1) = \begin{cases} \eta - \bar{\eta} + 1, & \text{if } \eta - \bar{\eta} + 1 > 0 \\ 0 & \text{otherwise} \end{cases}. \tag{11}$$

b) the tracking error $\tilde{q}$ is globally bounded by

$$\|\tilde{q}\| \le \frac{\varepsilon}{\lambda} \operatorname{pos}(\eta - \bar{\eta} + 1) \le \frac{\varepsilon}{\lambda}. \tag{12}$$

Proof: Choose the Lyapunov candidate function

$$v = \frac{1}{2}\left(z^{\top} M z + \frac{1}{\beta} \tilde{\eta}^{\top} \tilde{\eta}\right), \ \tilde{\eta} = \bar{\eta} - \eta. \tag{13}$$

From (6), the time derivative of the function (13) along the trajectories of (1) is

$$\begin{aligned} \dot{v} &= \frac{1}{2} z^{\top} \dot{M} z + z^{\top} M \dot{z} + \dot{\tilde{\eta}}^{\top} \tilde{\eta}/\beta \\ &= \frac{1}{2} z^{\top} \dot{M} z + z^{\top}[M(\lambda \dot{\tilde{q}} - \ddot{q}_d) - h - \tau_d + \tau] + \dot{\bar{\eta}}^{\top} \tilde{\eta}/\beta. \end{aligned} \tag{14}$$

When $\|z\| > \varepsilon$, $z^{\top} u_s = \phi\bar{\eta}\|z\|$ from (9), and from Lemma 1, $\dot{v}$ can be expressed as

$$\begin{aligned} \dot{v} &\le \phi\eta\|z\| - k_1\|z\|^2 - \phi\bar{\eta}\|z\| + \dot{\bar{\eta}}^T \tilde{\eta}/\beta \\ \Rightarrow \dot{v} &\le \phi\eta\|z\| - k_1\|z\|^2 - \phi\bar{\eta}\|z\| + (-\alpha\bar{\eta} + \beta\phi\|z\|)^T \tilde{\eta}/\beta \\ \Rightarrow \dot{v} &\le -k_1\|z\|^2 - \alpha\bar{\eta}^T(\bar{\eta}-\eta)/\beta. \end{aligned} \tag{15}$$

Note that $\alpha\bar{\eta}\ge 0$. Thus α and β are chosen so that $\bar{\eta}$ is adjusted (adapted) to satisfy $\bar{\eta}\ge\eta$ whenever $\|z\|>\varepsilon$. Then, for any $\|z\|$ with $\|z\|>\varepsilon$, $\dot{v}<0$. This implies that $\|z\|$ is bounded by $\|z\|\le\varepsilon<1$.

Thus, after the transient response, $\|z\|\le\varepsilon<1$. In this case, from (9), $z^T u_s = \phi\|z\|^2/\varepsilon$. Then, $\dot{v}$ can be expressed as

$$\begin{aligned} \dot{v} &\le \phi\eta\|z\| - k_1\|z\|^2 - \phi\bar{\eta}\|z\|^2/\varepsilon + \dot{\bar{\eta}}^T \tilde{\eta}/\beta \\ \Rightarrow \dot{v} &\le \phi\eta\|z\| - k_1\|z\|^2 - \phi\bar{\eta}\|z\|^2/\varepsilon + (-\alpha\bar{\eta} + \beta\phi\|z\|)^T \tilde{\eta}/\beta \\ \Rightarrow \dot{v} &\le -k_1\|z\|^2 + \phi\bar{\eta}\|z\|(I - \frac{\|z\|}{\varepsilon}) - \frac{\alpha}{\beta}\bar{\eta}^T(\bar{\eta}-\eta). \end{aligned} \tag{16}$$

From (16), we see that if α and β are chosen to satisfy $\frac{\alpha}{\beta}\ge\phi\|z\|$ (note that $\|z\|\le\varepsilon<1$), then $\dot{v}\le-(k_1\|z\|)\|z\|$, which satisfies the sliding condition of (5), for all

$$1-\frac{\|z\|}{\varepsilon} < \bar{\eta}-\eta \;\Rightarrow\; \|z\| > (\eta-\bar{\eta}+1)\varepsilon. \tag{17}$$

This implies that z is bounded by

$$\|z\| \le \varepsilon\,\mathrm{pos}(\eta-\bar{\eta}+1) \tag{18}$$

where pos($\cdot$) is defined by (11). Note that $\bar{\eta}\ge\eta$, thus $\mathrm{pos}(\eta-\bar{\eta}+1)\le 1$. From (9), we see that $\bar{\eta}$ is decreasing when $\beta\phi\|z\|<\alpha\bar{\eta}$ and vise versa. Thus, α and β are chosen to satisfy $\beta\phi\|z\|<\alpha\bar{\eta}$ whenever z is bounded by (18), then $\bar{\eta}$ is bounded. Therefore, we can conclude that $\|z\|$ is globally bounded by (10).

From the definition of the switching surface (4), the tracking error $\tilde{q}$ can be obtained from the first order filter relationship

$$\tilde{q} = \int_0^t e^{-\lambda(t-\tau)} z(\tau)d\tau. \tag{19}$$

From (18), (19) can be expressed as

$$\begin{aligned} \|\tilde{q}\| &\le \varepsilon\,\mathrm{pos}(\eta-\bar{\eta}+1)\int_0^t e^{-\lambda(t-\tau)}d\tau \\ \Rightarrow \|\tilde{q}\| &\le \frac{\varepsilon}{\lambda}\,\mathrm{pos}(\eta-\bar{\eta}+1)(I-e^{-\lambda t}) \le \frac{\varepsilon}{\lambda}\,\mathrm{pos}(\eta-\bar{\eta}+1). \end{aligned} \tag{20}$$

Therefore, $\|\tilde{q}\|$ is globally bounded by (12).

IV. Time varying thickness of the boundary layer

Note that (9) is an adaptive sliding mode control algorithm with an adaptively adjusted gain $\bar{\eta}$, and with z as the sliding surface.

It is known [7] that while sliding mode control has good robustness, the control law has to be discontinuous across $z(t)$. So, the thickness of the boundary layer $\varepsilon>0$ is chosen to eliminate the chattering of the control law. With the control algorithm of (9), small tracking errors can be achieved by choosing a small ε. However, small ε will usually result in undesirable vibration on the control signal when $\bar{\eta}$ is large. A reasonable way to choose ε is to set it large when $\|z\|$ is large and to set it small when $\|z\|$ is small. This motivates the use of a time-varying thickness(ε) of the boundary layer.

From this section, we denote the boundary layer thickness ε as $\hat{\varepsilon}$, which means the estimated boundary layer thickness. When $\|z\|\ge\hat{\varepsilon}$, we need to guarantee that the distance to the boundary layer always decrease. Thus, the condition (5) needs to be modified to satisfy[7]

$$\|z\|\ge\hat{\varepsilon} \;\Rightarrow\; \frac{1}{2}z^T z \le (\dot{\hat{\varepsilon}}-\check{\eta})\|z\|. \tag{21}$$

By comparing (21) and (5), we see that the sliding mode gain $\bar{\eta}$ of (9) should be replaced with

$$\phi\hat{\eta} = \phi\bar{\eta} - \dot{\hat{\varepsilon}} \;\Rightarrow\; \hat{\eta} = \bar{\eta} - \phi^{-1}\dot{\hat{\varepsilon}}. \tag{22}$$

We see from (22) that the varying pattern of the thickness $\hat{\varepsilon}$ is closely related with that of $\hat{\eta}$. Thus we can suggest following control law and adaptation law

Algorithm 2: Self-tuning of the boundary layer thickness

$$\begin{aligned} \tau &= -k_2 z - u_{as},\quad k_2>0 \\ u_{as} &= \begin{cases} \phi\hat{\eta}\dfrac{z}{\|z\|} & \text{if } \|z\|>\hat{\varepsilon} \\ \phi\hat{\eta}\dfrac{z}{\hat{\varepsilon}} & \text{otherwise} \end{cases} \\ \dot{\hat{\eta}} &= -\alpha_1\hat{\eta} + \beta_1\phi\|z\| \\ &\begin{cases} \hat{\varepsilon} = \hat{\varepsilon}(0),\; \dot{\hat{\varepsilon}} = -\alpha_2\hat{\varepsilon} + \beta_2\phi\hat{\eta}, & \text{if } \hat{\varepsilon} > \hat{\varepsilon}(0) \\ \dot{\hat{\varepsilon}} = -\alpha_2\hat{\varepsilon} + \beta_2\phi\hat{\eta}, & \text{otherwise} \end{cases} \end{aligned} \tag{23}$$

with $1>\alpha_1>0,\ \beta_1>0,\ \hat{\eta}(0)\ge 0,$
$1>\alpha_2>0,\ \beta_2>0,\ 1>\hat{\varepsilon}(0)>0$

where sat($\cdot$) is defined by (10).

Theorem 2: Consider the system (1) with the control law (23). The closed-loop system is globally stable in the sense that $\hat{\eta}$ and $\hat{\varepsilon}$ are bounded and

a) z is globally bounded by

$$\|z\| \le \hat{\varepsilon}\,\mathrm{pos}[\eta-(1+\beta_2)\hat{\eta}+1+\alpha_2\phi^{-1}\hat{\varepsilon}] \le \hat{\varepsilon}. \tag{24}$$

b) the tracking error $\tilde{q}$ is globally bounded by

$$\|\tilde{q}\| \le \frac{\hat{\varepsilon}}{\lambda}\,\mathrm{pos}[\eta-(1+\beta_2)\hat{\eta}+1+\alpha_2\phi^{-1}\hat{\varepsilon}] \le \frac{\hat{\varepsilon}}{\lambda} \tag{25}$$

where pos ($\cdot$) is defined by (14).

Proof: Choose the Lyapunov candidate function with $\tilde{\eta}$ is derived from (23) as

$$\begin{aligned} v &= \frac{1}{2}\left(z^T M z + \frac{1}{\beta_1}\tilde{\eta}\cdot\tilde{\eta}\right), \\ \tilde{\eta} &= \bar{\eta}-\eta = \hat{\eta}+\phi^{-1}\dot{\hat{\varepsilon}}-\eta = (1+\beta_2)\hat{\eta}-\alpha_2\phi^{-1}\hat{\varepsilon}-\eta. \end{aligned} \tag{26}$$

In (23), we see that $\hat{\varepsilon}$ is the output of 1'st order filter with $\hat{\eta}$ as an input. If α_2, β_2 are chosen carefully so that $\hat{\varepsilon}$ is not changed drastically, then we can let $\dot{\hat{\varepsilon}}\approx 0$ and thus

$\hat{\eta}=\hat{\eta}$. When $\|z\|\geq\hat{\varepsilon}$, from (9) we have that $z^T u_{as}=\phi\hat{\eta}\|z\|$, and from Lemma 1, $\dot{v}$ can be expressed as

$$\begin{aligned}\dot{v} &= \frac{1}{2}z^T\dot{M}z+z^T M\dot{z}+\dot{\hat{\eta}}^T\tilde{\eta}/\beta \quad (27)\\ &= \frac{1}{2}z^T\dot{M}z+z^T[M(\lambda\dot{\tilde{q}}-\ddot{q}_d)-h-\tau_d+\tau]+\dot{\hat{\eta}}^T\tilde{\eta}/\beta\\ \dot{v} &\leq \phi\eta\|z\|-k_2\|z\|^2-\phi\hat{\eta}\|z\|+\dot{\hat{\eta}}^T\tilde{\eta}/\beta_1\\ \Rightarrow \dot{v} &\leq (\hat{\varepsilon}-k_2\|z\|)\|z\|-\alpha_1\tilde{\eta}^T[(1+\beta_2)\hat{\eta}-\alpha_2\phi^{-1}\hat{\varepsilon}-\eta]/\beta_1\end{aligned}$$

In (23), α_i and β_i are chosen so that $\hat{\eta}$ is adjusted (adapted) to satisfy $\hat{\eta}\geq(\eta+\alpha_2\phi^{-1}\hat{\varepsilon})/(1+\beta_2)$ for any $\|z\|$ and $\hat{\eta}$ with $\hat{\eta}>\eta$, $\dot{v}\leq(\hat{\varepsilon}-k_2\|z\|)\|z\|$. This implies that $\|z\|$ is bounded by $\|z\|\leq\hat{\varepsilon}<1$.

Thus, after the transient response, $\|z\|\leq\hat{\varepsilon}<1$. In this case, $\dot{v}$ can be expressed as

$$\begin{aligned}\dot{v} &= \frac{1}{2}z^T\dot{M}z+z^T[M(\lambda\dot{\tilde{q}}-\ddot{q}_d)-h-\tau_d+\tau]+\dot{\hat{\eta}}^T\tilde{\eta}/\beta_1\\ &\leq \phi\eta\|z\|-k_2\|z\|^2-\phi\hat{\eta}\|z\|^2/\hat{\varepsilon}+\dot{\hat{\eta}}^T\tilde{\eta}/\beta_1 \quad (28)\\ \Rightarrow\dot{v} &\leq (\hat{\varepsilon}-k_2\|z\|)\|z\|+\phi\hat{\eta}\|z\|(I-\frac{\|z\|}{\hat{\varepsilon}})-\frac{\alpha_1}{\beta_1}\tilde{\eta}^T\\ &\quad [(1+\beta_2)\hat{\eta}-\alpha_2\phi^{-1}\hat{\varepsilon}-\eta].\end{aligned}$$

In (28), we see that if α_1 and β_1 are chosen to satisfy $\frac{\alpha_1}{\beta_1}\geq\phi\|z\|$ (note that $\|z\|\leq\varepsilon$), then $\dot{v}\leq(\hat{\varepsilon}-k_2\|z\|)\|z\|$, which satisfies the sliding condition of (21), for all

$$\begin{aligned}&1-\frac{\|z\|}{\hat{\varepsilon}}<(1+\beta_2)\hat{\eta}-\alpha_2\hat{\varepsilon}-\eta\\ &\Rightarrow \|z\|>[\eta-(1+\beta_2)\hat{\eta}+\alpha_2\phi^{-1}\hat{\varepsilon}+1]\hat{\varepsilon}. \quad (29)\end{aligned}$$

This implies that z is bounded by

$$\|z\| \leq \hat{\varepsilon}\cdot \text{pos}[\eta-(1+\beta_2)\hat{\eta}+1+\alpha_2\phi^{-1}\hat{\varepsilon}] \quad (30)$$

with pos($\cdot$) defined by (11). Note from (29) that $\text{pos}[\eta-(1+\beta_2)\hat{\eta}+1+\alpha_2\phi^{-1}\hat{\varepsilon}]\leq 1$. From (23), we see that $\hat{\eta}$ is decreasing when $\beta_1\phi\|z\|<\alpha_1\hat{\eta}$ and vise versa. Thus, α_1 and β_1 are chosen to satisfy $\beta_1\phi\|z\|<\alpha_1\hat{\eta}$ whenever z is bounded by (30), then $\hat{\eta}$ is bounded. Also from (23), we see that $\hat{\varepsilon}$ is reset as $\hat{\varepsilon}=\hat{\varepsilon}(0)$ whenever $\hat{\varepsilon}>\hat{\varepsilon}(0)$, which means $\hat{\varepsilon}$ is bounded. Therefore, we can conclude that $\|z\|$ is globally bounded by (24).

From the definition of the switching surface (4), the tracking error $\tilde{q}$ can be obtained from the first order filter relationship

$$\tilde{q}=\int_0^t e^{-\lambda(t-\tau)}z(\tau)d\tau. \quad (31)$$

From (30), $\|z\| \leq \hat{\varepsilon}\cdot\text{pos}[\eta-(1+\beta_2)\hat{\eta}+1+\alpha_2\phi^{-1}\hat{\varepsilon}]$, thus (30) can be expressed as

$$\begin{aligned}\|\tilde{q}\| &\leq \hat{\varepsilon}\cdot\text{pos}[\eta-(1+\beta_2)\hat{\eta}+1+\\ &\quad \alpha_2\phi^{-1}\hat{\varepsilon}]\int_0^t e^{-\lambda(t-\tau)}d\tau \quad (32)\\ \Rightarrow \|\tilde{q}\| &\leq \hat{\varepsilon}\cdot\text{pos}[\eta-(1+\beta_2)\hat{\eta}+1+\\ &\quad \alpha_2\phi^{-1}\hat{\varepsilon}](I-e^{-\lambda t})\leq\frac{\hat{\varepsilon}}{\lambda}.\end{aligned}$$

Therefore, $\|\tilde{q}\|$ is globally bounded by (25).

With the adaptation law in (23), the sliding mode gain and the thickness of the boundary layer are time-varying and nonlinear. $\hat{\varepsilon}$ is increasing or decreasing through a first order filter according to $\hat{\eta}$. This results in a large ε when the sliding mode gain is large and a small ε when the sliding mode gain is small. Note that $\hat{\eta}$ is increasing or decreasing according to $\|z\|$. Therefore, this control scheme can supply a fast and robust correction to the control law.

V. Simulation results

A simple two-link robot manipulator shown in Figure 1 has been simulated, controlled by the adaptive sliding mode control with self-tuning the boundary thickness algorithms developed in this paper. The manipulator was modelled as a set of nonlinear coupled differential equations as described in [4]

$$\begin{aligned}\tau_1 &= m_2 l_2^2(\ddot{q}_1+\ddot{q}_2)+m_2 l_1 l_2 c_2(2\ddot{q}_1+\ddot{q}_2)+(m_1+\\ &\quad m_2)l_2^1\ddot{q}_1-m_2 l_1 l_2 s_2\dot{q}_2^{\,2}-2m_2 l_1 l_2 s_2\dot{q}_1\dot{q}_2+ \quad (33)\\ &\quad m_2 l_2 q s_{12}+(m_1+m_2)l_1 g s_1\\ \tau_2 &= m_2 l_2^2(\ddot{q}_1+\ddot{q}_2)+m_2 l_1 l_2 c_2\ddot{q}_1+m_2 l_1 l_2 s_2\dot{q}_1^2+m_2 l_2 g s_{12}\end{aligned}$$

where $c_1:=\cos(q_1)$, $s_{12}:=\sin(q_1+q_2)$, *etc.*.

The desired trajectory had the form

$$\begin{aligned}q_{d1} &= 1+0.2\sin(\pi t)\\ q_{d2} &= 1-0.2\cos(\pi t)\end{aligned} \quad (34)$$

for $t\in[0,4]$, and the disturbance τ_d was added in the form of

$$\tau_d=5\begin{bmatrix}\sin(4\pi t)\\ \sin(4\pi t)\end{bmatrix}. \quad (35)$$

Parameters used in the simulation were

$$l_1=l_2=1\text{ m},\quad m_1=m_2=1\text{kg}. \quad (36)$$

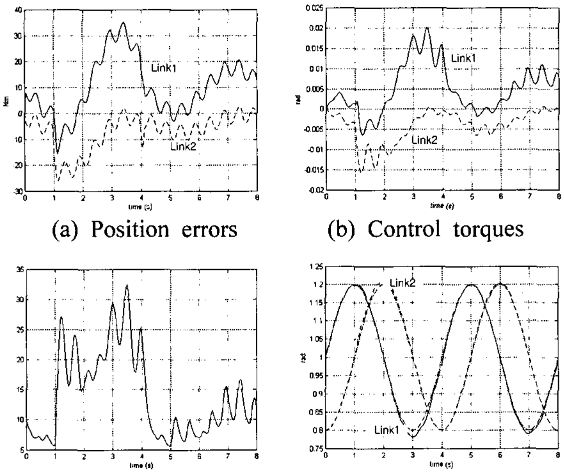
While the manipulator was being operated, m_2 and m_1 were changed from 1kg to 3kg and from 1kg to 1.5kg respectively at $t=1$ sec, and both link mass were changed back to 1kg at $t=4$ sec. The sampling time was set to be 10^{-3} sec and the plant initial states were set as

$$q_{d1}=1,\quad q_{d2}=0.8,\quad \dot{q}_{d1}=0.2\pi,\quad \dot{q}_{d2}=0. \quad (37)$$

First, we applied the control algorithm 1 of (9) to the system (33) with the controller parameters were chosen to be

$$\lambda=6I_2,\ \eta=2,\ \bar{\eta}=20,\ \alpha=0.003,\ \beta=400,\ \varepsilon=.05. \quad (38)$$

The simulation results are shown in Figure 1: a) position errors; b) control torques for each link of the manipulator; c) the adaptation results of $\bar{\eta}$; and d) link trajectories. It can be seen in Figure 2(a) and c) that the tracking errors $\tilde{q}$ were increasing due to the step change of m_1 and m_2 at $t=1$ sec, and that the sliding mode control gain $\bar{\eta}$ was adjusted to reduce the tracking error. As a result, the tracking errors were decreased by the adaptiveness, and the system eventually achieved the desired bounded tracking errors for the given parameter changes.



(a) Position errors (b) Control torques

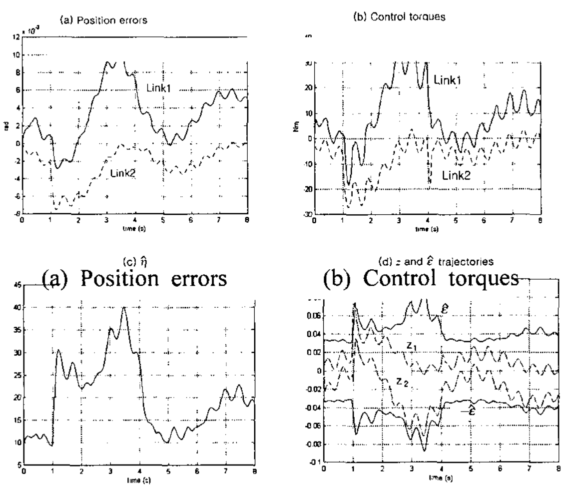
(c) $\bar{\eta}$ (d) Link positions

Fig. 1. Simulation results under adaptive sliding mode control Algorithm 1.

Next, we applied the control algorithm 2 of (23) to the system (33) with the controller parameters were chosen to be

$$\lambda=20,\ \alpha_1=0.021,\ \beta_1=45,$$
$$\hat{k}(0)=3,\ \alpha_2=0.005,\ \beta_2=0.05,\ \hat{\phi}(0)=0.004\ . \quad (39)$$

The simulation results are shown in Figure 2 : a) tracking errors; b) control torques for each link of the manipulator; c) the adaptation results of $\hat{\eta}$; and d) the boundary layer thickness $\hat{\varepsilon}$ variation and z-trajectories. We see that the tracking performance under algorithm 2 is much better than that under algorithm 1, because varying both the sliding mode control gain $\hat{\eta}$ and the thickness of the boundary layer allow us to make better use of the available bandwidth.



(a) Position errors (b) Control torques

(c) $\hat{\eta}$ (d) z and $\hat{\varepsilon}$trajectories

Fig. 2. Simulation results under adaptive sliding mode control Algorithm 2.

Note that the simulation results achieved in the presence of the disturbance of (34) over the interval $t\in[0,8]$, indicate that the proposed algorithm worked effectively for both the given parameter uncertainties and the disturbances.

VI. Conclusions

In this paper, adaptive algorithms which use adaptation laws for tuning both the sliding mode gain and the thickness of the boundary layer has been proposed to reject a discontinuous control input, and to improve the tracking performance. With this scheme, the tracking performance can be improved, good for rejection of control chattering phenomenon, and fairly large parameter variation and disturbances can be handled.

It is shown that the robustness of the developed adaptive algorithms are guaranteed by the sliding mode control law and that the algorithms are globally convergent in the presence of disturbances and modeling uncertainties.

The proposed adaptive algorithms are applied to a two-link robot manipulator and computer simulations are performed. The simulation results show the good properties of the developed schemes, because varying both the sliding mode control gain $\hat{\eta}$ and the thickness of the boundary layer $\hat{\varepsilon}$ allow us to make better use of the available bandwidth.

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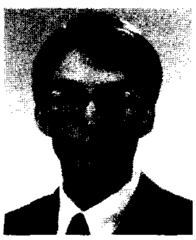

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