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SOME CURVATURE CONDITIONS OF n-DIMENSIONAL CR-SUBMANIFOLDS OF $(n-1)$ CR-DIMENSION IN A COMPLEX PROJECTIVE SPACE II

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ABSTRACT. In the previous paper we studied n-dimensional CR-submanifolds of $(n-1)$ CR-dimension immersed in a complex projective space $CP^{(n+p)/2}$, and especially determined such submanifolds under curvature conditions related to vertical direction. In the present article we determine such submanifolds under curvature conditions related to horizontal direction.

1. Introduction

Let M be an n-dimensional CR-submanifold of $(n-1)$ CR-dimension isometrically immersed in a complex space form $M^{(n+p)/2}(c)$. Denoting by $(J, \overline{g})$ the Kählerian structure of $M^{(n+p)/2}(c)$, it follows by definition (cf. [1, 3, 5, 6, 9, 12, 14, 16]) that the maximal J-invariant subspace

$$\mathcal{D}_x := T_xM \cap JT_xM$$

of the tangent space T_xM of M at each point x in M has constant dimension $(n-1)$. So there exists a unit vector field U_1 tangent to M such that

$$\mathcal{D}_x^{\perp} = \text{Span}\{U_1\}, \quad \forall x \in M,$$

where $\mathcal{D}_x^{\perp}$ denotes the subspace of T_xM complementary orthogonal to $\mathcal{D}_x$. Moreover, the vector field N_1 defined by

$$N_1 := JU_1 \tag{1.1}$$

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is normal to M and satisfies

$$JTM \subset TM \oplus \text{ Span}\{N_1\}.$$

Hence we have, for any tangent vector field X and for a local orthonormal basis $\{N_1, N_\alpha\}_{\alpha=2,\ldots,p}$ of normal vectors to M, the following decomposition in tangential and normal components :

$$JX = FX + u^1(X)N_1, \tag{1.2}$$

$$JN_\alpha = -U_\alpha + PN_\alpha, \quad \alpha = 1, \ldots, p. \tag{1.3}$$

Since the structure $(J, \overline{g})$ is Hermitian and $J^2 = -I$, we can easily see from (1.2) and (1.3) that F and P are skew-symmetric linear endomorphisms acting on T_xM and $T_xM^\perp$, respectively and that

$$g(FU_\alpha, X) = -u^1(X)\overline{g}(N_1, PN_\alpha), \tag{1.4}$$

$$g(U_\alpha, U_\beta) = \delta_{\alpha\beta} - \overline{g}(PN_\alpha, PN_\beta), \tag{1.5}$$

where $T_xM^\perp$ denotes the normal space of M at x and g the metric on M induced from $\overline{g}$. Furthermore we also have

$$g(U_\alpha, X) = u^1(X)\delta_{1\alpha} \tag{1.6}$$

and consequently

$$g(U_1, X) = u^1(X), \quad U_\alpha = 0, \quad \alpha = 2, \ldots, p. \tag{1.7}$$

Next, applying J to (1.2) and using (1.3) and (1.7), we have

$$F^2X = -X + u^1(X)U_1, \quad u^1(X)PN_1 = -u^1(FX)N_1, \tag{1.8}$$

from which, taking account of the skew-symmetry of P and (1.4),

$$u^1(FX) = 0, \quad FU_1 = 0, \quad PN_1 = 0. \tag{1.9}$$

Thus (1.3) may be written in the form

$$JN_1 = -U_1, \quad JN_\alpha = PN_\alpha, \quad \alpha = 2, \ldots, p. \tag{1.10}$$

Moreover we may put

$$PN_\alpha = \sum_{\beta=2}^{p} P_{\alpha\beta} N_\beta, \quad \alpha = 2, \ldots, p, \tag{1.11}$$

where $(P_{\alpha\beta})$ is a skew-symmetric matrix which satisfies

$$\sum_{\gamma=2}^{p} P_{\alpha\gamma} P_{\gamma\beta} = -\delta_{\alpha\beta}, \quad \alpha, \beta = 2, \ldots, p. \tag{1.12}$$

These results tell us that (F, g, U_1, u^1) defines an almost contact metric structure on M (cf. [3, 5, 6, 12]). Recently Okumura and Vanhecke [12] studied the normal almost contact metric case when $M^{(n+p)/2}(c)$ is a complex projective space $CP^{(n+p)/2}$ and proved

THEOREM O-V. *Let M be a CR-submanifold of $(n-1)$ CR-dimension isometrically immersed in $CP^{(n+p)/2}$ and let the normal field N_1 be parallel with respect to the normal connection. If the almost contact metric structure (F, g, U_1, u^1) induced in M is normal, which is equivalent to the condition $A_1 F = F A_1$ on M, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some odd-dimensional spheres and A_1 denotes the shape operator corresponding to N_1 (π is the Hopf fibration $S^{n+p+1}(1) \to CP^{(n+p)/2}$).*

On the other hand, when $\pi^{-1}(M)$ is (1) an Einstein space or (2) a locally symmetric space, it is well known (cf. [2, 7, 8, 10, 11]) that $\pi^{-1}(M)$ has parallel second fundamental form. Projecting the quantities on $\pi^{-1}(M)$ onto M in $CP^{(n+p)/2}$, we can consider CR-submanifolds of $(n-1)$ CR-dimension with the conditions corresponding to (1) or (2). In this paper we shall study such CR-submanifolds of $(n-1)$ CR-dimension isometrically immersed in $CP^{(n+p)/2}$ by using Theorem O-V.

2. Fundamental equations for CR-submanifolds of $(n-1)$ CR-dimension

We first let M be as in section 1 and use the same notations as shown in that section. We denote by $\overline{\nabla}$ and ∇ the Levi-Civita connection on $M^{(n+p)/2}(c)$ and M, respectively. Then the Gauss and Weingarten equations are given by

$$\overline{\nabla}_X Y = \nabla_X Y + h(X, Y), \tag{2.1}$$

$$\overline{\nabla}_X N_\alpha = -A_\alpha X + D_X N_\alpha, \quad \alpha = 1, \ldots, p \tag{2.2}$$

for any tangent vector fields X, Y to M. Here D denotes the normal connection induced from $\overline{\nabla}$ in the normal bundle $TM^{\perp}$ of M, and h and A_α the second fundamental form and the shape operator corresponding to N_α, respectively. It is clear that h and A_α are related by

$$h(X,Y) = \sum_{\alpha=1}^{p} g(A_\alpha X, Y) N_\alpha.$$

Especially we put

$$D_X N_\alpha = \sum_{\beta=1}^{p} s_{\alpha\beta}(X) N_\beta. \tag{2.3}$$

Then $(s_{\alpha\beta})$ is the skew-symmetric matrix of connection forms of D. Now, by using (2.1)-(2.3) and taking account of the Kähler condition $\overline{\nabla} J = 0$, we differentiate (1.2) and (1.3) covariantly and compare the tangential and normal parts. Then we can easily find that

$$(\nabla_X F)Y = u^1(Y)A_1 X - g(A_1 Y, X)U_1, \tag{2.4}$$

$$(\nabla_X u^1)(Y) = g(FA_1 X, Y), \tag{2.5}$$

$$\nabla_X U_1 = FA_1 X, \tag{2.6}$$

$$g(A_\alpha U_1, X) = -\sum_{\beta=2}^{p} s_{1\beta}(X) P_{\beta\alpha}, \quad \alpha = 2, \dots, p \tag{2.7}$$

for any X, Y tangent to M.

In the rest of this paper *we suppose that the normal field N_1 is parallel with respect to the normal connection D.* Hence (2.3) gives

$$s_{1\alpha} = 0, \quad \alpha = 2, \dots, p, \tag{2.8}$$

which together with (2.7) yields

$$A_\alpha U_1 = 0, \quad \alpha = 2, \dots, p. \tag{2.9}$$

On the other hand the ambient manifold $M^{(n+p)/2}(c)$ is of constant holomorphic sectional curvature c and consequently its Riemannian curvature tensor $\overline{R}$ satisfies

$$\begin{aligned}\overline{R}(\overline{X},\overline{Y})\overline{Z} = \frac{c}{4}\{&\overline{g}(\overline{Y},\overline{Z})\overline{X} - \overline{g}(\overline{X},\overline{Z})\overline{Y} + \overline{g}(J\overline{Y},\overline{Z})J\overline{X} \\ &- \overline{g}(J\overline{X},\overline{Z})J\overline{Y} - 2\overline{g}(J\overline{X},\overline{Y})J\overline{Z}\}\end{aligned}$$

for any $\overline{X}, \overline{Y}, \overline{Z}$ tangent to $M^{(n+p)/2}(c)$ (cf. [15,16]). So, the equations of Gauss, Codazzi and Ricci imply

$$(2.10)\quad \begin{aligned} R(X,Y)Z =& \frac{c}{4}\{g(Y,Z)X - g(X,Z)Y + g(FY,Z)FX \\ &- g(FX,Z)FY - 2g(FX,Y)FZ\} \\ &+ \sum_{\alpha}\{g(A_\alpha Y,Z)A_\alpha X - g(A_\alpha X,Z)A_\alpha Y\}, \end{aligned}$$

$$(2.11)\quad \begin{aligned} &(\nabla_X A_1)Y - (\nabla_Y A_1)X \\ &= \frac{c}{4}\{g(X,U_1)FY - g(Y,U_1)FX - 2g(FX,Y)U_1\}, \end{aligned}$$

$$(2.12)\quad [A_1, A_\alpha] = 0, \quad \alpha = 2, \dots, p$$

for any X, Y, Z tangent to M with the aid of (2.8), where R denotes the Riemannian curvature tensor of M.

3. Fibrations and immersions

In this section n-dimensional CR-submanifolds of $(p-1)$ CR-dimension isometrically immersed in $CP^{(n+p)/2}$ only will be considered. Moreover we shall use the assumption and the notations as in section 2.

Let $S^{n+p+1}(a)$ be the hypersphere of radius $a(>0)$ in $C^{(n+p+2)/2}$ the complex space of complex dimension $(n+p+2)/2$, which is identified with the Euclidean $(n+p+2)$-space $\mathbb{R}^{n+p+2}$. The unit sphere $S^{n+p+1}(1)$ will be briefly denoted by S^{n+p+1}. Let $\widetilde{\pi} : S^{n+p+1} \to CP^{(n+p)/2}$ be the natural projection of S^{n+p+1} onto $CP^{(n+p)/2}$ defined by the Hopf-fibration $S^1 \to S^{n+p+1} \to CP^{(n+p)/2}$. As is well known (cf. [3, 5, 12, 14, 15]), S^{n+p+1} admits a Sasakian structure $\widetilde{\xi}$ and each fibre $\widetilde{\pi}^{-1}(x)$ of x in $CP^{(n+p)/2}$ is a maximal integral submanifold of the distribution spanned by $\widetilde{\xi}$. Thus the base space $CP^{(n+p)/2}$ admits the induced Kähler structure of constant holomorphic sectional curvature 4 (cf. [3, 5, 12, 14, 15]). Moreover we have a fibration $\pi : \pi^{-1}(M) \to M$ which is compatible with the Hopf-fibration π. More precisely speaking $\pi : \pi^{-1}(M) \to M$ is a fibration with totally geodesic fibers such that the following diagram is commutative :

$$\begin{array}{ccc} \pi^{-1}(M) & \xrightarrow{\ i'\ } & S^{n+p+1} \\ {\scriptstyle \pi}\downarrow & & \downarrow{\scriptstyle \widetilde{\pi}} \\ M & \xrightarrow{\ i\ } & CP^{(n+p)/4} \end{array}$$

where $i' : \pi^{-1}(M) \to S^{n+p+1}$ and $i : M \to CP^{(n+p)/2}$ are isometric immersions.

Now, let ξ be the unit vector fields tangent to the fibers of $\pi^{-1}(M)$ such that $i'_*\xi = \widetilde{\xi}$. (In what follows we shall delete the i' and i'_* in our notation.) Furthermore we denote by X^* the horizontal lift of a vector field X tangent to M. Then the horizontal lifts N^*_α $(\alpha = 1, \dots, p)$ of the normal vectors N_α to M form an orthonormal basis of normal vectors to $\pi^{-1}(M)$ in S^{n+p+1}. Let A'_α and $s'_{\alpha\beta}$ be the corresponding shape operators and normal connection forms, respectively. Then, as shown in [3, 4, 5, 12, 13, 14, 15], the fundamental equations for the submersion π are given by

$$'\nabla_{X^*}Y^* = (\nabla_X Y)^* + g'((FX)^*, Y^*)\xi, \tag{3.1}$$

$$'\nabla_{X^*}\xi = {}'\nabla_\xi X^* = -(FX)^*, \tag{3.2}$$

where g' denotes the Riemannian metric of $\pi^{-1}(M)$ induced from $\widetilde{g}$ that of S^{n+p+1} and $'\nabla$ the Levi-Civita connection with respect to g'. The similar equations are valid for the submersion $\widetilde{\pi}$ by replacing F (resp. ξ) with J (resp. $\widetilde{\xi}$) respectively. We denote by $\widetilde{\nabla}$ and $'\nabla^\perp$ the Levi-Civita connection for $\widetilde{g}$ and the normal connection of $\pi^{-1}(M)$ induced from $\widetilde{\nabla}$, respectively. Since the diagram is commutative, $\widetilde{\nabla}_{X^*}N^*_\alpha$ implies

$$\begin{aligned} '\nabla^\perp_{X^*}N^*_\alpha - A'_\alpha X^* &= (\overline{\nabla}_X N_\alpha)^* + \widetilde{g}((FX)^*, N^*_\alpha)\widetilde{\xi} \\ &= -(A_\alpha X)^* + g(U_\alpha, X)^*\xi + (\nabla^\perp_X N_\alpha)^* \end{aligned}$$

because of (2.2), (2.5) and (3.1), from which, comparing the tangental and normal parts, we have

$$A'_\alpha X^* = (A_\alpha X)^* - g(U_\alpha, X)^*\xi, \tag{3.3}$$

$$'\nabla^\perp_{X^*}N^*_\alpha = (\nabla^\perp_X N_\alpha)^*. \tag{3.4}$$

Next, calculating $\widetilde{\nabla}_\xi N^*_\alpha$ and using (2.2), (2.5) and (3.2), we have

$$'\nabla^\perp_\xi N^*_\alpha - A'_\alpha\xi = -(FN_\alpha)^* = U^*_\alpha - (PN_\alpha)^*,$$

which yields

$$A'_\alpha\xi = -U^*_\alpha, \tag{3.5}$$

$$'\nabla^\perp_\xi N^*_\alpha = -(PN_\alpha)^*. \tag{3.6}$$

Hence (3.3) and (3.5) with $\alpha = 1$ and (3.6) imply

$$A'_1 X^* = (A_1 X)^* - g(U, X)^* \xi, \tag{3.7}$$

$$A'_1 \xi = -U^*, \tag{3.8}$$

$$s'_{\alpha\beta}(X^*) = s_{\alpha\beta}(X)^*, \quad s'_{\alpha\beta}(\xi) = -P_{\alpha\beta}. \tag{3.9}$$

First of all we recall the co-Gauss equations for the submersion π (cf. [4, 13, 14, 15]). Taking account of (3.1) and (3.2), we have

$$\begin{aligned} {}'\nabla_{X^*}{}'\nabla_{Y^*} Z^* =& (\nabla_X \nabla_Y Z)^* + \{g'((FX)^*, (\nabla_Y Z)^*) \\ &+ g'((F\nabla_X Y)^*, Z^*) + g'(((\nabla_X F)Y)^*, Z^*) \\ &+ g'((FY)^*, (\nabla_X Z)^*)\}\xi - g'((FY)^*, Z^*)(FX)^*, \end{aligned} \tag{3.10}$$

$$[X^*, Y^*] = [X, Y]^* + 2g'((FX)^*, Y^*)\xi, \quad [X^*, \xi] = 0. \tag{3.11}$$

Using these equations and taking account of (2.4) and (2.10) with $c = 4$, we can easily see that

(3.12)
$$\begin{aligned} {}'R(X^*, Y^*)Z^* &= g(Y, Z)^* X^* - g(X, Z)^* Y^* \\ &+ \sum_\alpha \{g(A_\alpha Y, Z)^* (A_\alpha X)^* - g(A_\alpha X, Z)^* (A_\alpha Y)^*\} \\ &+ g'(\{u(Y)A_1 X - u(X)A_1 Y\}^*, Z^*)\xi, \end{aligned}$$

where ${}'R$ denotes the Riemannian curvature tensor of $\pi^{-1}(M)$. Making also use of (3.1) and (3.2), we have

$$\begin{aligned} {}'\nabla_{X^*}{}'\nabla_{Y^*}\xi &= -\{(\nabla_X F)Y + F(\nabla_X Y)\}^* - g'((FX)^*, (FY)^*)\xi, \\ {}'\nabla_\xi{}'\nabla_{Y^*} X^* &= -(F\nabla_Y X)^* - \{g(F^2 Y, X)^* + g(FY, FX)^*\}\xi, \\ {}'\nabla_{Y^*}{}'\nabla_\xi X^* &= -\{(\nabla_Y F)X + F(\nabla_Y X)\}^* - g(FY, FX)^*\xi, \\ {}'\nabla_\xi{}'\nabla_{X^*}\xi &= \{-X + u(X)U\}^*, \end{aligned}$$

from which, using (1.8), (2.4), (3.11) and the fibre being totally geodesic, we can easily obtain

$${}'R(X^*, Y^*)\xi = -\{u(Y)A_1 X - u(X)A_1 Y\}^*, \tag{3.13}$$

$$\begin{aligned} {}'R(Y^*, \xi)X^* = &- \{u(X)A_1 Y\}^* \\ &- \{g(Y, X)^* - u(Y)^* g(U, X)^*\}\xi, \end{aligned} \tag{3.14}$$

$${}'R(\xi, X^*)\xi = \{-X + u(X)U\}^*. \tag{3.15}$$

Putting $Z = U$ in (3.12) and using (2.9), we have

$$
\begin{aligned}
'R(X^*, Y^*)U^* = & u(Y)^* X^* - u(X)^* Y^* \\
& + g(A_1 Y, U)^* (A_1 X)^* - g(A_1 X, U)^* (A_1 Y)^* \\
& + g(u(Y) A_1 X - u(X) A_1 Y, U)^* \xi,
\end{aligned} \tag{3.16}
$$

from which, differentiating covariantly in the horizontal direction Z^* and using (3.1), (3.2) and (3.16), we find

$$
\begin{aligned}
& ('\nabla_{Z^*} 'R)(X^*, Y^*)U^* + 'R(X^*, Y^*)(FA_1 Z)^* \\
& + g(FZ, X)^* (A_1 Y)^* - g(FZ, Y)^* (A_1 X)^* \\
= & g(FA_1 Z, Y)^* X^* - g(FA_1 Z, X)^* Y^* \\
& + \{g((\nabla_Z A_1)Y, U)^* + g(A_1 Y, FA_1 Z)^*\}(A_1 X)^* \\
& - \{g((\nabla_Z A_1)X, U)^* + g(A_1 X, FA_1 Z)^*\}(A_1 Y)^* \\
& + g(A_1 Y, U)^* \{(\nabla_Z A_1)X\}^* - g(A_1 X, U)^* \{(\nabla_Z A_1)Y\}^* \\
& - g(u(Y) A_1 X - u(X) A_1 Y, U)^* (FZ)^* \\
& + \{u(Y)^* g(FZ, X)^* - u(X)^* g(FZ, Y)^* \\
& + u(A_1 Y)^* g(A_1 FZ - FA_1 Z, X)^* \\
& - u(A_1 X)^* g(A_1 FZ - FA_1 Z, Y)^* \\
& + u(Y)^* g(\nabla_Z A_1)X, U)^* - u(X)^* g(\nabla_Z A_1)Y, U)^* \\
& + g(u(Y) A_1 X - u(X) A_1 Y, FA_1 Z)^*\}\xi.
\end{aligned} \tag{3.17}
$$

Differentiating (3.13)-(3.15) covariantly in the horizontal direction Z^* respectively and using (3.1), (3.2) and (3.13)-(3.15) themselves, we have

$$
\begin{aligned}
& ('\nabla_{Z^*} 'R)(X^*, Y^*)\xi + g(FZ, X)^* \{-Y + u(Y)U\}^* \\
& + g(FZ, Y)^* \{-X + u(X)U\}^* - 'R(X^*, Y^*)(FZ)^* \\
= & - \{g(FA_1 Z, Y) A_1 X - g(FA_1 Z, X) A_1 Y + u(Y)(\nabla_Z A_1)X \\
& - u(X)(\nabla_Z A_1)Y\}^* - g(FZ, u(Y) A_1 X - u(X) A_1 Y)^* \xi,
\end{aligned} \tag{3.18}
$$

(3.19)

$$
\begin{aligned}
& ('\nabla_{Z^*} 'R)(Y^*, \xi)X^* - 'R(Y^*, (FZ)^*)X^* + g(FZ, X)^* \{Y - u(Y)U\}^* \\
= & - \{g(FA_1 Z, X) A_1 Y + u(X)(\nabla_Z A_1)Y\}^* + \{g(Y, X) \\
& - u(Y)u(X)\}^* (FZ)^* + \{g(FA_1 Z, Y)u(X) \\
& + g(FA_1 Z, X)u(Y) - g(FZ, A_1 Y)u(X)\}^* \xi,
\end{aligned}
$$

$$
\begin{aligned}
& ('\nabla_{Z^*} 'R)(\xi, X^*)\xi \\
= & \{g(FA_1 Z, X)U + u(X)(FA_1 Z - A_1 FZ)\}^*.
\end{aligned} \tag{3.20}
$$

4. Main results

Let M be as in section 2 and use the same notations as shown in that section. We assume that $('\nabla_{Z^*}{}'R)(Y^*,\xi)X^* = 0$. Then (3.19) implies

$$\begin{aligned} &-{}'R(Y^*,(FZ)^*)X^* + g(FZ,X)^*\{Y-u(Y)U\}^* \\ &= -\{g(FA_1Z,X)A_1Y + u(X)(\nabla_Z A_1)Y\}^* \\ &\quad + \{g(Y,X)-u(Y)u(X)\}^*(FZ)^* + \{g(FA_1Z,Y)u(X) \\ &\quad + g(FA_1Z,X)u(Y) - g(FZ,A_1Y)u(X)\}^*\xi. \end{aligned} \tag{4.1}$$

Taking the vertical part of (4.1) with $X = U$ and using (1.6), (1.9) and (3.16), we can easily obtain

$$u(Y)g(A_1FZ,U) = -g(A_1FZ,Y) + g(FA_1Z,Y),$$

from which, putting $Y = U$ and using (1.6), it follows that

$$g(A_1FZ,U) = 0$$

and consequently

$$A_1F - FA_1 = 0.$$

Combining this equation and Theorem O-V, we have

THEOREM 1. *Let M be an n-dimensional CR-submanifold of $(n-1)$ CR-dimension isometrically immersed in a complex projective space $CP^{(n+p)/2}$ and let the normal field N_1 be parallel with respect to the normal connection. If $('\nabla_{Z^*}{}'R)(Y^*,\xi)X^* = 0$ for any vector fields Z,Y,X on M, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some odd-dimensional spheres.*

COROLLARY. *Let M be an n-dimensional CR-submanifold of $(n-1)$ CR-dimension isometrically immersed in a complex projective space $CP^{(n+p)/2}$ and let the normal field N_1 be parallel with respect to the normal connection. If $'\nabla_{Z^*}{}'R = 0$ for any vector field Z on M, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some odd-dimensional spheres.*

We next assume that $'\nabla_{Z^*}{}'R)(\xi,X^*)\xi = 0$ identically on $\pi^{-1}(M)$. Then (3.20) gives

$$g(FA_1Z,X)U + u(X)(FA_1Z - A_1FZ) = 0,$$

from which, putting $X = U$ and using (1.6) and (1.9), we have

$$FA_1 - A_1F = 0.$$

Combining this equation and Theorem O-V, we have

THEOREM 2. *Let M be an n-dimensional CR-submanifold of $(n-1)$ CR-dimension isometrically immersed in a complex projective space $CP^{(n+p)/2}$ and let the normal field N_1 be parallel with respect to the normal connection. If $'\nabla_{Z^*}{}'R)(\xi, X^*)\xi = 0$ for any vector fields Z, X on M, then $\pi^{-1}(M)$ is locally a product of $M_1 \times M_2$ where M_1 and M_2 belong to some odd-dimensional spheres.*

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