Development of a Design System for Multi-Stage Gear Drives (2nd Report: Development of a Generalized New Design Algorithm)

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ABSTRACT

The design of multi-stage gear drives is a time-consuming process, since it includes more complicated problems, which are not considered in the design of single-stage gear drives. The designer has to determine the number of reduction stages and the gear ratios of each reduction stage. In addition, the design problems include not only the dimensional design but also the configuration design of gear drive elements. There is no definite rule and principle for these types of design problems. Thus the design practices largely depend on the sense and the experiences of the designer, and consequently result in undesirable design solution.

We propose a new and generalized design algorithm to support the designer at the preliminary design phase of multi-stage gear drives. The proposed design algorithm automates the design process by integrating the dimensional design and the configuration design process. The algorithm consists of four steps. In the first step, a designer determines the number of reduction stages. In the second step, gear ratios are chosen by using the random search method. In the third step, the values of basic design parameters are chosen by using the generate and test method. Then, the values of other dimensions, such as pitch diameter, outer diameter, and face width, are calculated for the configuration design in the final step. The strength and durability of a gear is guaranteed by the bending strength and the pitting resistance rating practices by using the AGMA rating formulas. In the final step, the configuration design is carried out by using the simulated annealing algorithm. The positions of gears and shafts are determined to minimize the geometrical volume (size) of a gearbox, while satisfying spatial constraints between them. These steps are carried out iteratively until a desirable solution is acquired.

The proposed design algorithm has been applied to the preliminary design of four-stage gear drives in order to validate the availability. The design solutions have shown considerably good results in both aspects of the dimensional and the configuration design.

Keywords : Gear, Multi-Stage Gear Drive, Dimensional Design, Configuration Design, Generalized New Design Algorithm

Nomenclature

b	Face width	[mm]
d_o	Outer diameter of a gear	[mm]
d_p	Pitch diameter of a gear	[mm]
d_s	Outer diameter of a shaft	[mm]
m	Normal module	[mm]
u	Gear ratio (> 1)	
z	Number of teeth	

1. Introduction

In order to design multi-stage gear drives, it is necessary to determine not only the number of gear stages and gear ratios, but also the dimension of gears according to transmission ratio (total gear ratio) and applied load. In addition, configuring drive elements properly into a limited space is also needed, while satisfying spatial constraints, such as meshing and interference between gears and shafts. The dimensional and the configuration design are highly coupled, and cannot be carried out independently. Until now, the design practices largely depend on the sense and the experiences of a designer, and consequently the design needs more time and often result in undesirable design solutions.

In the first report of our research[1], we have proposed the dimensional design processes, and have developed the configuration design algorithm using simulated annealing. However, the dimensional design processes are limited to the design of two- and three-stage gear drives, and has a demerit that the designer should determine some of the design parameters, such as module or number of teeth. Thus, it is necessary to offer a general methodology to design gear drives having more than four stages, which are needed commonly in high speed-reduction gear drives.

We propose a new and generalized design algorithm, which automates the determination of gear ratio, the dimensional design, and the configuration design of multi-stage gear drives. The design system can be applicable to the gear drives of having more than four stages, not to mention having two or three stages. The availability of the system will be validated by the design examples of four-stage gear drives.

2. Generalized New Design Algorithm for Multi-Stage Gear Drives

The design process of multi-stage gear drives consists of determining gear ratios and gear dimensions, and configuring elements of the gear drive, according to the input and output speed, transmitted power, and other design specifications. We propose a generalized new design algorithm to automate the preliminary design of multi-stage gear drives. The proposed algorithm is shown in Fig. 1. The algorithm consists of the following four steps; determination of the number of stages, determination of gear ratios, dimensional design, and configuration design. The design steps are carried out iteratively until a desirable solution is acquired.

2.1 Determination of Number of Stages: Step 1

Generally, the designer determines the number of reduction (increasing) stages and the gear ratio of each stage properly in consideration of total gear ratio, available space, and other design requirements. However, no definite rule and formal methodology have been proposed to determine the number of stage, except for several simple guides such as adding another stage to the gear train if the gear ratio of a stage is greater than 5[2]. The reason is that the number of stages is not a definite value, and moreover can be selected in a relatively small range. Thus, the designer sets provisionally the number of reduction stages from the ratio range recommended. It is rather inefficient to automate this step by integrating into the algorithm, since it unnecessarily increases computation time in most cases. The designer can decide optionally whether or not to proceed with another number of reduction stages, when the final design solution is not satisfactory or the iteration number exceeds the limit, i.e the design is regarded provisionally as having no feasible solution.

2.2 Determination of Gear Ratios: Step 2

In Step 2, the gear ratios of each reduction stage are determined by using a random search method within the specified ratio range.

As presented in the first report[1], the design formulas proposed by Niemann et al.[3] might be a practical one, in which gear ratios are determined based on the Hertz contact stress formula. However, this method is limited to the design of two- and three-stage gear drives, and the designer should determine previously the number of teeth or module. The general guides available for more than four-stage gear drives are to handle gear ratios up to 5 or 7 (maximum 10 in special cases) in a single reduction in ordinary spur and helical design practices, and to choose a greater value for a gear ratio of the former stage than that of the latter stage[4,5] from the relation of transmitted torque and gear size.

On the basis of the above guides, we assume two premises to propose a random search method to determine gear ratios. Firstly, we can set the upper and lower limits of a gear ratio to the generally acceptable values, although we do not know the definite value of the gear ratio. Secondly, we should choose a greater value for a gear ratio of the first stage than that of the second one, and of the second stage than of the third one, and so forth. From these premises, we can generate a random value for a gear ratio of the first reduction stage between the lower and the upper limits of it. Then, for the gear ratio of the second reduction stage, another random value is generated between the lower limit and the ratio of the first stage determined previously. The gear ratio for every reduction stage can be determined in the same way. The ratios are used in Step 3 and Step 4, and the algorithm iterates until the volume of the gearbox minimizes. Eventually, the gear dimensions and the gear ratios will have proper values, and this means that the ratios correlate with the dimensions and the configuration of gears closely. This will be verified by design examples in the later chapter.

2.3 Dimensional Design of Gears: Step 3

In Step 3, we set module m, number of teeth z, and face width b as basic design parameters, and determine the values of them by using the generate and test method. Then, we can calculate pitch circle diameter and outer diameter, which are used in the configuration design in Step 4. It is general to consider other design variables in practical cylindrical gear design, such as pressure angle, helix angle, and addendum modification. However, the variables do not affect largely on the overall size of a gear and may be determined in the detail design phase. It is, therefore, possible to set the variables to constant values, and to determine only the basic design parameters of module, number of teeth, and face width by using the generate and test method.

Although the efficiency of the method is not good in most cases, the search time of it can be reduced considerably by limiting the search space of the design variables. It is possible to treat module as a discrete variable, since we use only the standard values recommended by the ISO standards[6]. Moreover, the upper and the lower limits of it can be given according to the application of the gear drive. Secondly, the number of teeth in pinion is apparently an integer variable. The minimum value of it can be specified according to pressure angle, and the maximum value of it can be limited to a conventional value[3~5]. Finally, supposing that face width is specified by an integer multiple of module, as is in common practice, it can be treated as a discrete variable, although it is basically a continuous variable[5]. It is also possible to specify the upper and the lower limits of it to conventional values in accordance with the application.

The design solutions are validated by the strength

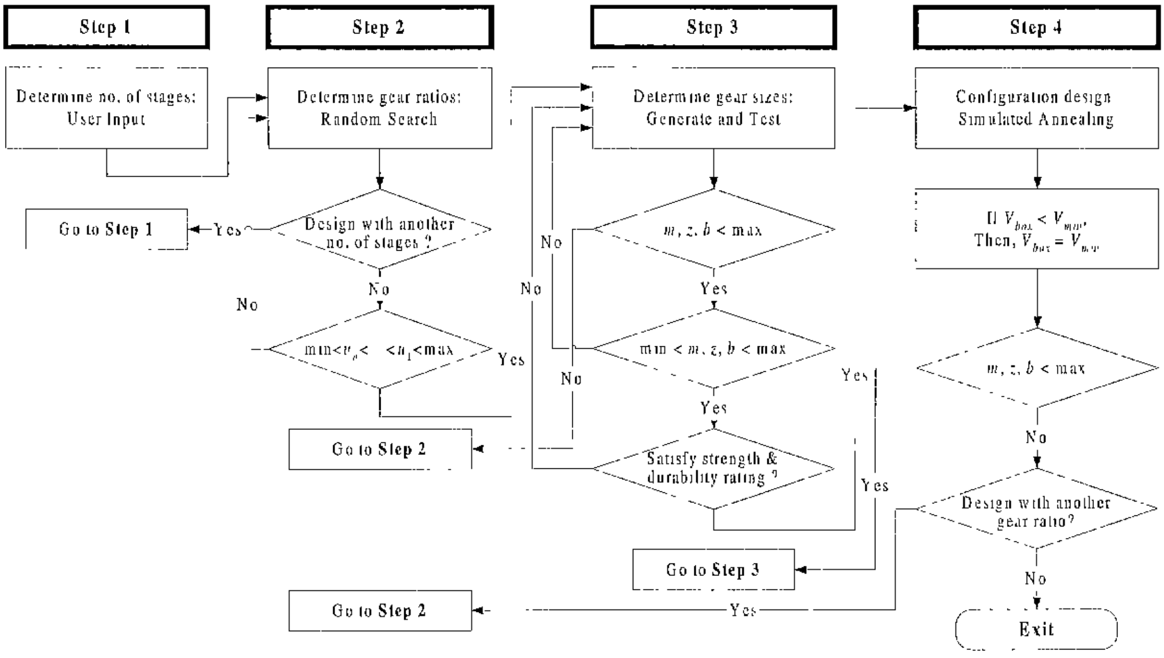


Fig. 1 Flow chart of the proposed design algorithm for multi-stage gear drives

rating practice, and the configuration design in Step 4 is carried out only for the gears satisfying the rating practice. The rating practice is carried out using the AGMA rating formulas(7) for bending strength and pitting resistance.

On the other hand, many researchers including the authors have developed the dimensional design methods of the gear pair by using the gradient-based optimal design techniques(8~10), and it might be considered to apply the techniques to the dimensional design in Step 3. However, the proposed algorithm can obtain a sufficiently good result in a short search time, and moreover, minimizing gear pairs does not guarantee the minimal volume of gearbox. Thus, we use the generate and test method in the algorithm for the dimensional design.

2.4 Configuration Design of a Gear Drive: Step4

Configuration design is carried out by using the simulated annealing algorithm to minimize the geometrical volume of a gearbox, while satisfying spatial constraints, such as gear meshing and interference. In the first report(1), the configuration design was considered as a problem of packing gears in three-dimensional space, supposing that the diameter and face width were fixed in the previous design steps.

The formulation of the objective function in the current report is the same as that of the first report, but the gear dimensions used in the proposed algorithm are no more than the provisional values. In other words,

Table 1 Constraints used for the design of four-stage gear drives

$C_1 = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2} - (d_{p1}+d_{p2})/2$	$C_{24} = (d_{o2}+d_{o7})/2 - \sqrt{(x_2-x_7)^2+(y_2-y_7)^2} > 0$
$C_2 = \sqrt{(x_3-x_4)^2+(y_3-y_4)^2} - (d_{p3}+d_{p4})/2$	$C_{25} = (d_{o2}+d_{o8})/2 - \sqrt{(x_2-x_8)^2+(y_2-y_8)^2} > 0$
$C_3 = \sqrt{(x_5-x_6)^2+(y_5-y_6)^2} - (d_{p5}+d_{p6})/2$	$C_{26} = (d_{o3}+d_{o7})/2 - \sqrt{(x_3-x_7)^2+(y_3-y_7)^2} > 0$
$C_4 = \sqrt{(x_7-x_8)^2+(y_7-y_8)^2} - (d_{p7}+d_{p8})/2$	$C_{27} = (d_{o3}+d_{o8})/2 - \sqrt{(x_3-x_8)^2+(y_3-y_8)^2} > 0$
$C_5 = z_1 - z_2$	$C_{28} = (d_{o4}+d_{o7})/2 - \sqrt{(x_4-x_7)^2+(y_4-y_7)^2} > 0$
$C_6 = z_3 - z_4$	$C_{29} = (d_{o4}+d_{o8})/2 - \sqrt{(x_4-x_8)^2+(y_4-y_8)^2} > 0$
$C_7 = z_5 - z_6$	$C_{30} = (d_{s1}+d_{o4})/2 - \sqrt{(x_{s1}-x_4)^2+(y_{s1}-y_4)^2} > 0$
$C_8 = z_7 - z_8$	$C_{31} = (d_{s1}+d_{o6})/2 - \sqrt{(x_{s1}-x_6)^2+(y_{s1}-y_6)^2} > 0$
$C_9 = x_2 - x_3$	$C_{32} = (d_{s1}+d_{o8})/2 - \sqrt{(x_{s1}-x_8)^2+(y_{s1}-y_8)^2} > 0$
$C_{10} = y_2 - y_3$	$C_{33} = (d_{s2}+d_{o6})/2 - \sqrt{(x_{s2}-x_6)^2+(y_{s2}-y_6)^2} > 0$
$C_{11} = x_4 - x_5$	$C_{34} = (d_{s2}+d_{o8})/2 - \sqrt{(x_{s2}-x_8)^2+(y_{s2}-y_8)^2} > 0$
$C_{12} = y_4 - y_5$	$C_{35} = (d_{s3}+d_{o1})/2 - \sqrt{(x_{s3}-x_1)^2+(y_{s3}-y_1)^2} > 0$
$C_{13} = x_6 - x_7$	$C_{36} = (d_{s3}+d_{o2})/2 - \sqrt{(x_{s3}-x_2)^2+(y_{s3}-y_2)^2} > 0$
$C_{14} = y_6 - y_7$	$C_{37} = (d_{s3}+d_{o8})/2 - \sqrt{(x_{s3}-x_8)^2+(y_{s3}-y_8)^2} > 0$
$C_{15} = \lvert z_2 - z_3 \rvert - (b_2+b_3)/2$	$C_{38} = (d_{s4}+d_{o1})/2 - \sqrt{(x_{s4}-x_1)^2+(y_{s4}-y_1)^2} > 0$
$C_{16} = \lvert z_4 - z_5 \rvert - (b_4+b_5)/2$	$C_{39} = (d_{s4}+d_{o2})/2 - \sqrt{(x_{s4}-x_2)^2+(y_{s4}-y_2)^2} > 0$
$C_{17} = \lvert z_6 - z_7 \rvert - (b_6+b_7)/2$	$C_{40} = (d_{s4}+d_{o4})/2 - \sqrt{(x_{s4}-x_4)^2+(y_{s4}-y_4)^2} > 0$
$C_{18} = (d_{o1}+d_{o5})/2 - \sqrt{(x_1-x_5)^2+(y_1-y_5)^2} > 0$	$C_{41} = (d_{s5}+d_{o1})/2 - \sqrt{(x_{s5}-x_1)^2+(y_{s5}-y_1)^2} > 0$
$C_{19} = (d_{o1}+d_{o6})/2 - \sqrt{(x_1-x_6)^2+(y_1-y_6)^2} > 0$	$C_{42} = (d_{s5}+d_{o2})/2 - \sqrt{(x_{s5}-x_2)^2+(y_{s5}-y_2)^2} > 0$
$C_{20} = (d_{o1}+d_{o7})/2 - \sqrt{(x_1-x_7)^2+(y_1-y_7)^2} > 0$	$C_{43} = (d_{s5}+d_{o4})/2 - \sqrt{(x_{s5}-x_4)^2+(y_{s5}-y_4)^2} > 0$
$C_{21} = (d_{o1}+d_{o8})/2 - \sqrt{(x_1-x_8)^2+(y_1-y_8)^2} > 0$	$C_{44} = (d_{s5}+d_{o6})/2 - \sqrt{(x_{s5}-x_6)^2+(y_{s5}-y_6)^2} > 0$
$C_{22} = (d_{o2}+d_{o5})/2 - \sqrt{(x_2-x_5)^2+(y_2-y_5)^2} > 0$	
$C_{23} = (d_{o2}+d_{o6})/2 - \sqrt{(x_2-x_6)^2+(y_2-y_6)^2} > 0$	

configuration solution is obtained by using the gear dimensions determined provisionally in Step 3, as shown in Fig. 1. In Step 4, the configuration solution is compared to the old optimal solution, and it is set as the new optimal if it is better than the old one. The design process iterates until the final optimal configuration is obtained. This iterative process means to automate the characteristics of the practical design process, in which the dimensional design and the configuration design are highly coupled.

The objective function f for the configuration design is the same as the formulation presented in the first report[1]. The formulation is a linear summation of the constraints and the volume of a virtual gearbox, as shown in equation (1).

$$f = W_0\, P_0\, V_{box} + \sum_{i=1}^{nc} W_i\, P_i \left|C_i\right| \qquad (1)$$

where, W_0 and P_0 are the weighting factor and the normalizing factor for the volume V_{box} of a virtual gearbox, respectively. W_i and P_i are the weighting factors and the normalizing factors for the i th constraints, C_i.

The constraints C_i include the center distance constraints for proper meshing between pinion and gear and the interference constraints for avoiding interference between gears and shafts. Table 1 shows the constraints for the design of four-stage gear drives $C_1 \sim C_{44}$, where (x_i , y_i , z_i) and (x_{si} , y_{si} , z_{si}) represent the coordinates of the center position of the i th gear and shaft, respectively. Design examples will be shown in the following chapter. The constraints $C_1 \sim C_4$ and $C_5 \sim C_8$ express the center distance constraints for meshing of gears and the center position constraints in z direction of the first, the second, the third, and the fourth stages, respectively. The constraints $C_9 \sim C_{14}$ and $C_{15} \sim C_{17}$ represent the center position constraints for the gears having a same shaft. The constraints $C_{18} \sim C_{29}$ are included to avoid interference between gears. The inequality signs of $C_{18} \sim C_{29}$ mean that the constraint is included in the objective function only when interference occurs, i.e. the interference constraint having larger value than zero. Similarly, the constraints $C_{30} \sim C_{44}$ are the interference constraints between gear and shaft, and are included in the objective function only when the interference constraint having larger value than zero.

3. Design Examples: Four-Stage Gear Drives

The proposed design algorithm has been applied to the preliminary design of four-stage gear drives. Table 2 shows the basic design specifications. The design practices have been applied to external spur gear drives with transmitted power of 8kW, total gear ratio of 300, and input speed of 6000 rpm. The design specifications for the strength rating practice are supposed to use the general values for the applications of commercial enclosed gear units.

Table 2 Design specifications for design example

Transmitted power [kW]	8
Input speed [rpm]	6000
Transmission ratio	300
Gear type	External spur
Pressure angle [deg]	20
Material	Steel
Heat treatment	Carburized & case hardened
Hardness [HRC]	55
AGMA quality number	11
Load cycles	1×10^7

Table 3 shows the minimum and the maximum values of the basic design parameters (gear ratio, module, number of teeth in pinion, and face width), and the types of them. The value for face width is determined by an integer multiple of module of the corresponding pinion, and treated as a discrete variable. We assume that pinion and gear of a reduction stage have the equal value for face width.

Table 3 Minimum and maximum values of the basic design parameters and their types

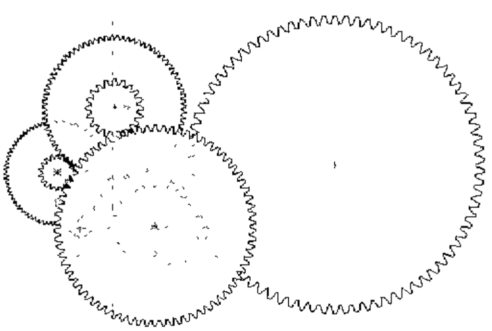
	Min.	Max.	Types
Gear ratios	1.0	9.0	Continuous
Module [mm]	1.0	6.0	Discrete
Number of teeth, pinion	14	25	Integer
Face width [module]	4	15	Discrete

Table 4 shows the design result for the case of not considering the interferences with shafts. Thus, the interference constraints $C_{30} \sim C_{44}$ in Table 1 are not included in the objective function formulation of equation (1).

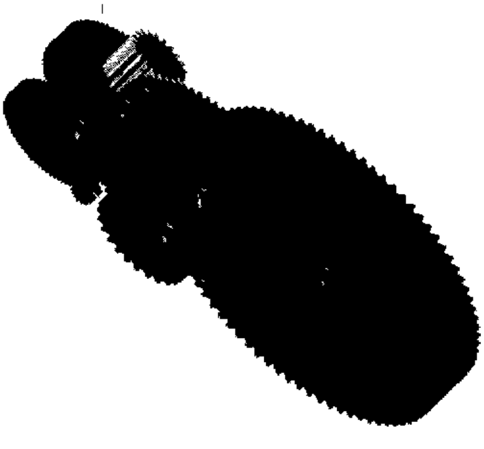
Table 4 Design result without considering shaft interference

Stage		1	2	3	4
Module	[mm]	1.5	2.0	3 0	4.0
Gear ratio		5 5	4.389	3 7	3 28
Number of teeth, pinion		14	18	20	25
Number of teeth, gear		77	79	74	82
Pitch diameter, pinion	[mm]	21.0	36.0	60.0	100.0
Pitch diameter, gear	[mm]	115.5	158.0	222 0	328.0
Face width	[mm]	18.0	30 0	45.0	60.0
Volume of gearbox	[mm^3]	19.3×10^6			

In Table 4. the gear ratios for each reduction stage are 5.5, 4 389, 3.7, and 3.28. The result seems to be quite good from an empirical point of view, although the algorithm uses the random search method to determine gear ratios. The values of other variables such as module, number of teeth, and face width also show reasonable results. This reveals that there is a direct relationship between gear ratios and the dimensional and the configuration design. This tendency might be viewed in Fig. 2 more clearly.



(a) two-dimensional representation



(b) three-dimensional representation

Fig. 2 Configuration of the gears in Table 4

Fig. 2 (a) shows the two-dimensional representation of the configuration of the gears in Table 4, and Fig. 2 (b) shows the three-dimensional view of the solid model of it. The gears reveal a marked tendency to gather around to minimize the volume of the virtual gearbox, i.e. a box completely bounding gears. In this case, the result indicates that a possible interference shall occur between the pinion shaft of the first stage and the gear of the third stage, since the interferences of gears with shafts have not been considered. This interference can be avoided by slightly changing the position of the pinion of the first stage without affecting the total volume of the gearbox. This means that the configuration design may also be a very useful guide to the designer for configuring gears, although it does not consider interferences with shafts. The volume of the gearbox is 19324379 mm^3.

Table 5 shows the design result for the case of considering the interference with shafts. In practical design practices, the interferences between gears and shafts should be avoided, and it is not sufficient to configure only the gears into the minimal space as the previous example. This problem can be easily solved in the proposed algorithm by inserting the interference constraints $C_{30} \sim C_{44}$ in Table 1 into the objective function.

Table 5 Design result considering shaft design

Stage		1	2	3	4
Module	[mm]	1.25	2.0	2.5	4.0
Number of teeth, pinion		19	18	24	24
Number of teeth, Gear		97	74	92	88
Gear ratio		5 105	5.05	3.833	3.667
Pitch diameter, pinion	[mm]	23.75	36.0	60.0	96.0
Pitch diameter, gear	[mm]	121.25	148.0	230.0	352.0
Face width	[mm]	18.75	28.0	37.5	56.0
Volume of gearbox	[mm^3]	22.7×10^6			

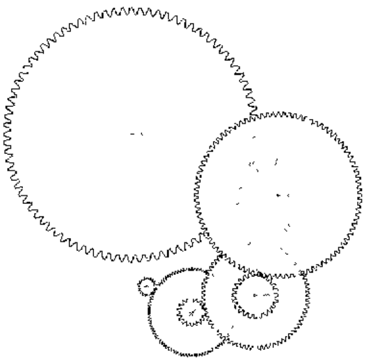
The design results of gear ratio. module, and face width in Table 5 also have reasonable values in accordance with the design intention. However, the configuration result in Fig. 3 has a different tendency from the case of not considering the shaft interference in Fig. 2. The gears also show a tendency to gather around to minimize the volume of the gearbox, but have configured with no interference between the gears and the shafts. The volume of the gearbox was 22689493 mm^3, which is slightly increased compared to the result of the previous example.

4. Conclusions

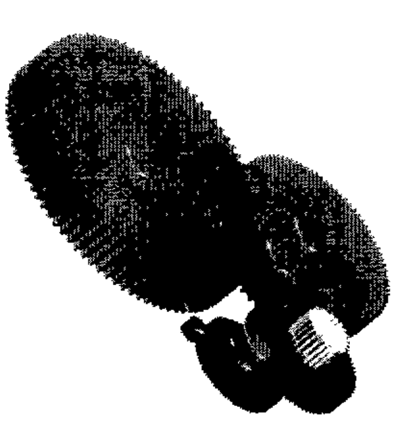
We have proposed a new and generalized design methodology for multi-stage gear drives. The proposed algorithm uses the random search method to determine gear ratios, and the generate and test method for the dimensional design of the gears. Then, the configuration design is carried out by using the simulated annealing algorithm to minimize the geometrical volume of the gearbox, while avoiding interferences between gears and shafts. As the design algorithm proceeds, the dimensional design and the configuration design iterates until the final optimal solution is obtained. This iterative process means to automate the characteristics of the practical design process, in which the dimensional design and the configuration design are highly coupled.

The proposed algorithm has been applied to the design of four-stage gear drives, and generated considerably good design results in both aspects of the dimensional and the configuration design.

From these results, we can conclude that the proposed algorithm can be very useful in the preliminary design of multi-stage gear drives, and reduce time and cost of the practical design process considerably. Moreover, the algorithm has offered an important foundation to develop an automatic design system for generic multi-stage gear drives, which is composed of worm gear, bevel gear and other types of gear drives.



(a) two-dimensional representation



(b) three-dimensional representation

Fig. 3 Configuration of the gears in Table 5

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