Development of a Design System for Multi-Stage Gear Drives (1st Report: Proposal of Formal Processes for Dimensional Design)

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ABSTRACT

In recent years, the concern of designing multi-stage gear drives has increased with more application of them in high-speed and high-load. Until now, however, the researches on the design of gear drives have been focused on single gear pairs. Thus, the design practice for multi-stage gear drives has been depended on experiences and expertise of designers and carried out commonly by trial and error. We propose an automation algorithm for the design of two- and three-stage cylindrical gear drives. The two types of dimensional design processes have been proposed to determine gear dimensions in a formal way. The first design process (Process I) uses the total volume of gears to determine gear ratio, and uses K factor, unit load, and aspect ratio to determine gear dimensions. The second one (Process II) makes use of Niemann's formula and center distance to calculate gear ratio and gear dimensions. Process I and Process II employ material data from AGMA and ISO standards, respectively. The configuration design determines the positions of gears with minimizing the volume of gearbox by using a simulated annealing algorithm. The availability of the design algorithm is validated by the design examples of two- and three-stage gear drives.

Keywords : Gear, Multi-Stage Gear Drive, Dimensional Design, Configuration Design, Simulated Annealing Algorithm

Nomenclature

a	Center distance	[mm]
b	Face width	[mm]
d_o	Outer diameter of a gear	[mm]
d_p	Pitch diameter of a gear	[mm]
d_s	Outer diameter of a shaft	[mm]
K	K factor	[MPa]
m_a	Aspect ratio	
m	Normal module	[mm]
n	Revolutionary speed	[rpm]
u	Gear ratio (> 1)	
U_l	Unit load	[MPa]
V	Total geometric volume of gears	[mm^3]
V_{box}	Volume of gearbox	[mm^3]
W_t	Transmitted tangential load	[N]
ϕ	Pressure angle	[deg.]
σ_H	Hertz stress	[MPa]
$\sigma_{H\lim}$	Allowable contact stress	[MPa]

Subscripts

1	First stage of a gear drive
2	Second stage of a gear drive
3	Third stage of a gear drive

1. Introduction

Recently, the concern of designing multi-stage gear drives has increased consistently with more application of them in high-speed and high-load. Until now, however, the researches on the design of gear drives have been mainly focused on the dimensional design of single gear pairs. The common design practice of multi-stage gear drives is carried out by rule of thumb, which largely depends on experience and intuition of the designer.

Thus, the design practices are time-consuming even for expert designers, and often result in unsatisfactory design solutions.

It is necessary to determine the number of gear stages and gear ratios of each stage according to transmission ratio (total gear ratio) in order to design multi-stage gear drives. In addition, it is important to configure the gear drive elements with satisfying spatial constraints related to the gear arrangement. However, no formal methodology has been proposed to solve the problems, except that a few empirical methods have been used to determine only the number of stages and gear ratios[1,2].

As a preliminary work for the development of automatic design system for generic multi-stage gear drives, we propose two types of formal design processes for the dimensional design of two- and three-stage gear drives based on the comprehensive analysis of existing empirical design methods. In addition, we propose a configuration design algorithm to locate the proper positions of gears and shafts by using a simulated annealing algorithm[3]. We will compare the design solution by the proposed dimensional design processes with an existing elevator helical gear drive. The configuration design algorithm also will be validated through a design example for cylinders. Finally, we will present design examples for a three-stage gear drive.

2. Formal Processes for Dimensional Design

The dimensional design of multi-stage gear drives is to determine the number of stages, gear ratios and gear dimensions, according to the design specifications, such as transmitted power, input speed, total gear ratio, and etc. We propose formal design processes (Process I and Process II), as shown in Fig. 1.

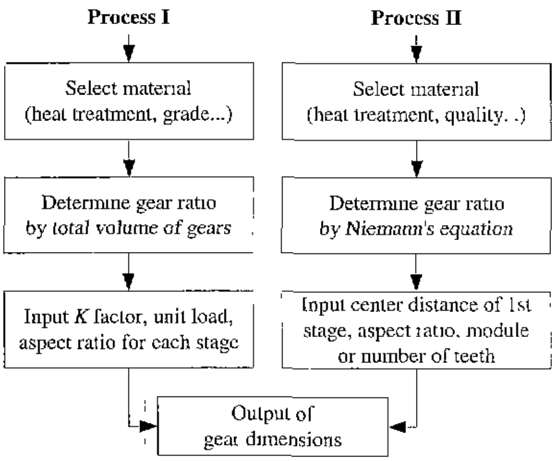


Fig. 1 Proposed formal processes for dimensional design

2.1 Process I

Process I is shown in the left-hand side of Fig. 1. At first, a designer selects material, heat treatment method and grade of the material to find allowable contact stress. The designer can determine the material properties experientially in consideration of the application of the gear drive and transmitted load. Material data used in Process I is employed from the AGMA standards[4], and is used to determine gear ratios and dimensions in the following design phase. Secondly, the designer determines the gear ratio of every stage from the relation between gear ratio and the total volume of the gears. We have induced the following equations to show the relation between them.

The total volume of the gears is calculated by introducing Hertz contact stress formula, as shown in equation (1).

$$\sigma_H = \sqrt{\frac{0.7}{(1/E_p + 1/E_g)\cos\phi\sin\phi}}\sqrt{\frac{W_t}{b\cdot d_p}\frac{u+1}{u}} \tag{1}$$

where, E_p and E_g are modulus of elasticity of pinion and gear, respectively.

We can acquire equation (2) from equation (1), assuming that the transmitted power does not change in any stage.

$$\sigma_{H\lim 1}^2 \frac{u_1 n_1}{(u_1+1)^3} b_1 a_1^2 = \sigma_{H\lim 2}^2 \frac{u_2 n_2}{(u_2+1)^3} b_2 a_2^2 = \cdots \tag{2}$$

By rearranging equation (2), we can obtain equations (3-1) and (3-2) for the first and the second stages, and the second and the third stages, respectively.

$$f_{12} = \frac{u_1^2(u_2+1)^3}{u_2(u_1+1)^3} = \left(\frac{\sigma_{H\lim 2}}{\sigma_{H\lim 1}}\right)^2 \frac{b_2}{b_1}\left(\frac{a_2}{a_1}\right)^3 \tag{3-1}$$

$$f_{23} = \frac{u_2^2(u_3+1)^3}{u_3(u_2+1)^3} = \left(\frac{\sigma_{H\lim 3}}{\sigma_{H\lim 2}}\right)^2 \frac{b_3}{b_2}\left(\frac{a_3}{a_2}\right)^3 \tag{3-2}$$

We may express the volume of pinion and gear of the first stage in terms of gear ratio and operating center distance, as shown in equation (4).

$$V_1 = \pi b_1 a_1^2 \frac{u_1^2+1}{(u_1+1)^2} \tag{4}$$

Then, the total volume of gears may be represented as equation (5).

$$V = \pi \left\{ b_1 a_1^2 \frac{u_1^2+1}{(u_1+1)^2} + b_2 a_2^2 \frac{u_2^2+1}{(u_2+1)^2} + \cdots \right\} \tag{5}$$

If we rearrange equation (5) by using equation (3) and total gear ratio u , we can derive equation (6) for a two-stage gear drive. We represent the volume in dimensionless form $V/b_1 a_1^2$ as equation (6), since, in general, a designer can determine the face width b_1 and the center distance a_1, and consequently the volume V becomes the function of the gear ratio u_1 only.

$$\frac{V}{b_1 a_1^2} = \frac{u_1^2+1}{(u_1+1)^2} + \left(\frac{\sigma_{\mathrm{Hlim1}}}{\sigma_{\mathrm{Hlim2}}} \right)^2 \frac{u_1^3 + u u_1^2 + u^2 u_1 + u^3}{u(u_1+1)^3} \tag{6}$$

We generate a graph automatically to show the relation between them. Thus, the designer can determine gear ratio easily in consideration of the relation of it with volume V . For example, Fig. 2 shows the change of the dimensionless value of $V/b_1 a_1^2$ with u_1 of the two-stage gear drive of total gear ratio 21, supposing that the material properties are the same for pinion and gear.

Finally, gear dimensions (module, number of teeth, and face width) are determined by the input of K factor, unit load and aspect ratio. The specific values of these factors can be referred from the gear design manual and/or texts according to application, load cycles, and operation environment of the gear drive. K factor in equation (7) is related to contact stress of a gear, while unit load U_l in equation (8) relates gear dimensions with bending strength

$$K = \frac{W_t}{d_p \cdot b} \left(\frac{u+1}{u} \right) \tag{7}$$

$$U_1 = \frac{W_t}{b \ m_n} \tag{8}$$

$$m_n = \frac{d_p}{b} \tag{9}$$

2.2 Process II

Process II is shown in the right-hand side of Fig. 1. Firstly, allowable contact stress is obtained by using material, heat treatment, quality, and hardness, in the same way as Process I. ISO material data[5] are used in Process II. We introduce the formula proposed by Niemann to determine gear ratios[1]. Niemann suggested equations (10) and (11) to determine gear ratios for two- and three-stage gear drives, respectively.

$$u_1 = 0.8 \left(u \frac{\sigma_{\mathrm{Hlim1}}}{\sigma_{\mathrm{Hlim2}}} \right)^{2/3} \tag{10}$$

$$u_1 = 0.6 u^{4/7} \left(\frac{\sigma_{\mathrm{Hlim1}}}{\sigma_{\mathrm{Hlim2}}} \right)^{2/7} \left(\frac{\sigma_{\mathrm{Hlim1}}}{\sigma_{\mathrm{Hlim3}}} \right)^{4/7} \tag{11-1}$$

$$u_2 = 1.1 u^{2/7} \left(\frac{\sigma_{\mathrm{Hlim2}}}{\sigma_{\mathrm{Hlim1}}} \right)^{4/7} \left(\frac{\sigma_{\mathrm{Hlim2}}}{\sigma_{\mathrm{Hlim3}}} \right)^{2/7} \tag{11-2}$$

Then, gear dimensions are obtained from the input values of center distance and aspect ratio of the first stage. Operating pitch diameter is expressed by equation (12) using aspect ratio and face width.

$$b = \frac{2a}{u+1} \cdot \frac{1}{m_a} \tag{12}$$

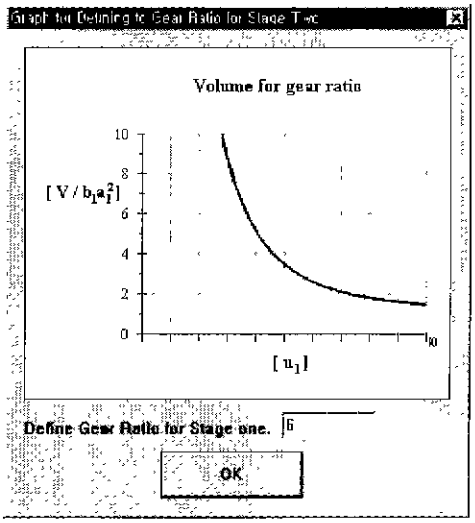


Fig. 2 Graph showing the relation between volume and gear ratio

If equation (3) is rearranged by substituting equation (12) into it, the center distances of the second and the third stages are expressed as follows:

$$a_2 = a_1 \cdot \left(f_{12} \frac{\sigma_{\mathrm{Hlim}1}}{\sigma_{\mathrm{Hlim}2}} \frac{u_2+1}{u_1+1} \frac{m_{a1}}{m_{a2}} \right)^{1/3} \qquad (13\text{-}1)$$

$$a_3 = a_2 \cdot \left(f_{23} \frac{\sigma_{\mathrm{Hlim}2}}{\sigma_{\mathrm{Hlim}3}} \frac{u_3+1}{u_2+1} \frac{m_{a2}}{m_{a3}} \right)^{1/3} \qquad (13\text{-}2)$$

Pitch diameter may be obtained by using the center distance from equation (13) and the gear ratio from equation (10) or (11). Then, the remaining gear dimensions may be calculated easily by determining module or number of teeth.

3. Configuration Design using the Simulated Annealing Algorithm

Supposing that the gear dimensions have been determined, the arrangement of gears, that is, the

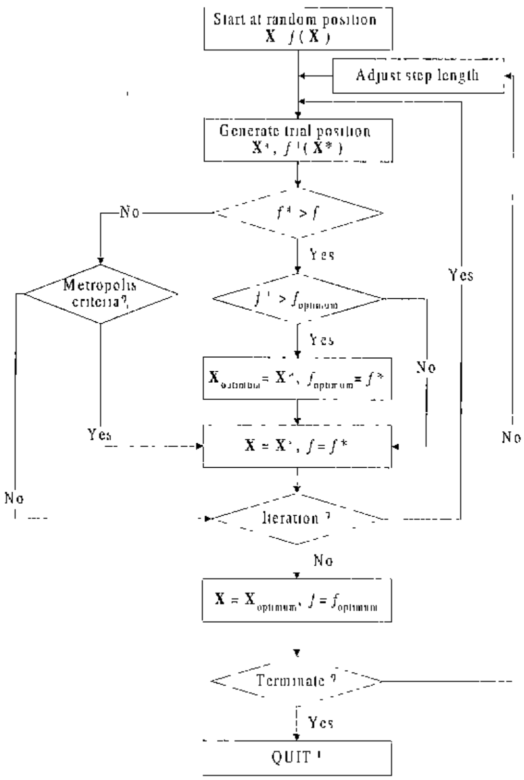


Fig. 3 Flow chart of the simulated annealing algorithm for the configuration design of gear drives

configuration design of gears may be treated as a problem of packing elements into a three-dimensional space. Several research works have been reported to solve the packing problems by using the global optimization techniques[6,7] for the past decades. In particular, Szykman et al. have reported that the simulated annealing may be applied in solving three-dimensional packing problems effectively[7]. On the basis of this research, we employ the simulated annealing algorithm to configure gear drive elements.

Fig. 3 shows the simulated annealing algorithm used in our research. Simulated annealing is an adaptive search technique based on physical annealing process, and is basically a kind of steepest descent methods. Starting from an initial random point, the algorithm takes a step and the function is evaluated. When minimizing a function, any downhill step is accepted and the process repeats from this new point. An uphill step may be accepted. Thus, it can escape from the local optima. This uphill decision is made by the Metropolis criteria. As the optimization process proceeds, the length of the step declines and the algorithm closes in on the global optimum.

The objective function f for the configuration design is shown in equation (14). It is formulated simply as the linear summation of the constraints and the volume of a virtual gearbox, i.e. a box completely bounding gears.

$$f = W_0 P_0 V_{box} + \sum_{i=1}^{m} W_i P_i |C_i| \qquad (14)$$

where, W_0 and P_0 are the weighting and the normalizing factors for the volume V_{box} of the virtual gearbox, respectively. W_i and P_i are the weighting and normalizing factors for i th constraints C_i, respectively. The spatial constraints C_i include center distance constraints for proper meshing of pinion and gear, and interference constraints for avoiding interference between gears and shafts. As the objective function f minimizes, the value of the constraints approaches zero.

4. Verification of the Design Algorithms

4.1 Dimensional Design Algorithm

In order to verify the dimensional design algorithm, design results by the algorithm have been compared to the design of an existing two-stage helical gear drive used in an elevator reduction drive. The design specifications are the same for both gear drives, as shown in Table 1. We have used 2.07 MPa for K factor, 48 MPa for unit load, and 335 HB and 300 HB for the minimum hardness of pinion and gear, respectively. Material grades are assumed to be AGMA 1 for Process I, and ISO ML for Process II. The center distance of the first stage in Process II starts from 150 mm.

Table 2 shows the dimension of existing elevator gear drive (EG) and the design results by using the two design processes. Both the results of Process I and Process II show smaller pitch diameters in pinion and gear than those in EG. The face widths have increased in the second stages, but have been reduced much smaller in the first stages. From these results, it may be confirmed that the proposed design processes can be used in the dimensional design of two- and three-stage gear drives for good and reliable design solutions.

Table 1 Design specifications for two-stage gear drives

Transmitted power [kW]	7.5
Input speed [rpm]	1450
Transmission ratio	21.722
Material	Carbon steel
Heat treatment	Carburized & case hardened
Gear type	External helical
Pressure angle [deg.]	20
Helix angle [deg.]	25
Aspect ratio	0.6(stage 1), 0.75(stage 2)

Table 2 Design results of two-stage gear drives

	EG*		Process I		Process II	
Stage	1	2	1	2	1	2
Normal module [mm]	2.5	3.5	2	3	2	3
Number of teeth, pinion	24	21	25	25	23	25
Number of teeth, gear	119	92	124	109	114	109
Gear ratio	4.958	4.381	4.96	4.36	4.96	4.36
Pitch dia., pinion [mm]	66.2	81.1	55.17	82.75	50.76	82.75
Pitch dia., gear [mm]	328.3	355.2	273.6	360.8	251.6	360.8
Center distance [mm]	198	219	164.4	221.8	151.2	221.8
Face width [mm]	40	60	33.1	62.1	30.5	62.1

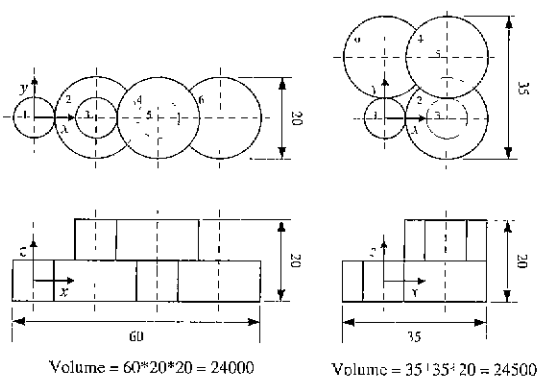
* gear dimensions of existing elevator reduction drive

4.2 Configuration Design Algorithm

In order to confirm the validity and availability of the proposed configuration design algorithm, we have carried out a configuration example of six cylinders, which we can estimate the optimal configuration. The cylinders consist of three cylinders with the same diameter of 10 mm, and three cylinders of the diameter of 20 mm. The height of every cylinder is set to 10 mm. The example may be treated as an analogy of gear meshing of a three-stage reduction gear drive.

Fig. 4 shows possible optimal configurations of the cylinders. The global optimal configuration is shown in Fig. 4 (a) with its bounding box having volume of 24000 mm^3. We may expect another configuration shown in Fig. 4 (b). This configuration might be regarded also as a good design practically, though it is not the global optimum.

The constraints for configuration include the center distance constraints for proper meshing and the center position constraints for avoiding interference between cylinders, as shown in List 1. The constraints C_1, C_2, and C_3 represent the center distance constraints for meshing of the cylinders in the first, the second, and the third stages, respectively, where, x, y and z represent the coordinate of the center position of a cylinder. The constraints C_4, C_5, and C_6 represent the center position constraints in z direction of the first, the second, and the third stages, respectively. C_7, C_8, and C_9 represent the center position constraints for cylinders 2 and 3 in xy plane and in z direction. Similarly, C_{10}, C_{11}, and C_{12} represent the center position constraints for cylinders 4 and 5 in xy plane and in z direction. The constraints



(a) a global optimal (b) a local optimal

Fig. 4 Optimal configurations of six cylinders

$C_{13} \sim C_{16}$ are included to avoid interferences between the cylinders. The inequality signs of $C_{13} \sim C_{16}$ mean that the constraint is included in the objective function of equation (14), only when interference occurs. i.e. the interference constraint have larger value than zero.

Fig. 5 shows one of the configuration design result using the proposed algorithm. In this case, we have used unity value for the weighting factors to evaluate efficiency of the algorithm. Final volume of the bounding box is 24962 mm^3, and the number of function evaluation is 61201. The configuration result shows that the simulated annealing algorithm with the objective function formulation of equation (14) can be used to configure gears effectively.

List 1 Constraints for the configuration design of six cylinders

$$C_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} - (d_1 + d_2)/2$$
$$C_2 = \sqrt{(x_3 - x_4)^2 + (y_3 - y_4)^2} - (d_3 + d_4)/2$$
$$C_3 = \sqrt{(x_5 - x_6)^2 + (y_5 - y_6)^2} - (d_5 + d_6)/2$$
$$C_4 = z_1 - z_2$$
$$C_5 = z_3 - z_4$$
$$C_6 = z_5 - z_6$$
$$C_7 = x_2 - x_3$$
$$C_8 = y_2 - y_3$$
$$C_9 = x_4 - x_5$$
$$C_{11} = |z_2 - z_3| - (b_2 + b_3)/2$$
$$C_{12} = |z_4 - z_5| - (b_4 + b_5)/2$$
$$C_{13} = (d_1 + d_5)/2 - \sqrt{(x_1 - x_5)^2 + (y_1 - y_5)^2} > 0$$
$$C_{14} = (d_2 + d_5)/2 - \sqrt{(x_2 - x_5)^2 + (y_2 - y_5)^2} > 0$$
$$C_{15} = (d_1 + d_6)/2 - \sqrt{(x_1 - x_6)^2 + (y_1 - y_6)^2} > 0$$
$$C_{16} = (d_2 + d_6)/2 - \sqrt{(x_2 - x_6)^2 + (y_2 - y_6)^2} > 0$$

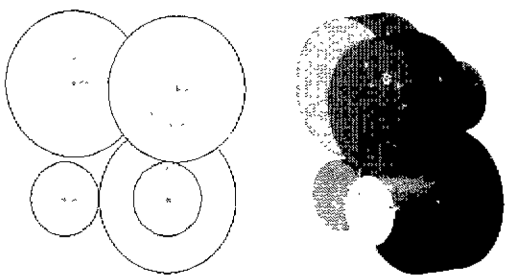


Fig. 5 Configuration result of six cylinders

5. Design Examples

The proposed design algorithm has been applied to the preliminary design of three-stage helical gear drives. Table 3 shows the design specifications for dimensional design. The design practices have been applied to external helical gear drives with transmitted power of 8 kW and transmission ratio (total gear ratio) of 110. We have assumed that K factor and unit load had the values for the applications of commercial enclosed gear units. The values of material hardness for pinion and gear have been 335 HB and 300 HB, respectively.

Table 3 Design specifications for three-stage gear drives

Transmitted power [kW]	8.0
Input speed [rpm]	3520
Transmission ratio	110
Material	Carbon steel
Heat treatment	Carburized & case hardened
Gear type	External helical
Pressure angle [deg.]	20
Helix angle [deg.]	25
Aspect ratio	0.6 (stage 1, 2), 0.75 (stage 3)

The constraints for the configuration design include not only the constraints in List 1, but also the constraints to avoid interferences between gears and shafts, as shown in List 2. The inequality signs of $C_{17} \sim C_{24}$ mean

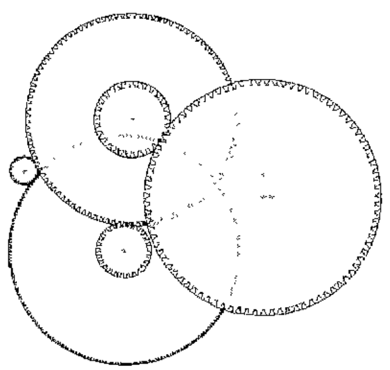
List 2 Constraints for avoiding shaft interference

$$C_{17} = (d_{s1} + d_{o4})/2 - \sqrt{(x_{s1} - x_4)^2 + (y_{s1} - y_4)^2} > 0$$
$$C_{18} = (d_{s1} + d_{o6})/2 - \sqrt{(x_{s1} - x_6)^2 + (y_{s1} - y_6)^2} > 0$$
$$C_{19} = (d_{s2} + d_{o6})/2 - \sqrt{(x_{s2} - x_6)^2 + (y_{s2} - y_6)^2} > 0$$
$$C_{20} = (d_{s3} + d_{o1})/2 - \sqrt{(x_{s3} - x_1)^2 + (y_{s3} - y_1)^2} > 0$$
$$C_{21} = (d_{s3} + d_{o2})/2 - \sqrt{(x_{s3} - x_2)^2 + (y_{s3} - y_2)^2} > 0$$
$$C_{22} = (d_{s4} + d_{o1})/2 - \sqrt{(x_{s4} - x_1)^2 + (y_{s4} - y_1)^2} > 0$$
$$C_{23} = (d_{s4} + d_{o2})/2 - \sqrt{(x_{s4} - x_2)^2 + (y_{s4} - y_2)^2} > 0$$
$$C_{24} = (d_{s4} + d_{o4})/2 - \sqrt{(x_{s4} - x_4)^2 + (y_{s4} - y_4)^2} > 0$$

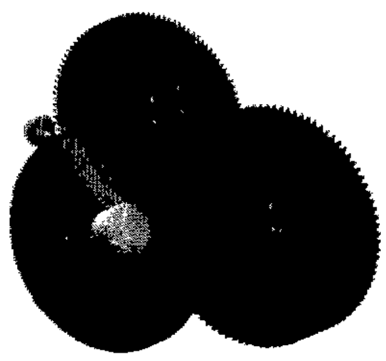
that the constraint is included in the objective function only when the interference occurs, i.e. the constraint have larger value than zero. In List 2, x_s , y_s and z_s represent the coordinate of the center position of a shaft.

Table 4 Dimensional design of three-stage gear drives

	Process I			Process II		
Stage	1	2	3	1	2	3
Normal module [mm]	1.5	3	4	1.25	3	4
Number of teeth, pinion	27	26	27	27	26	27
Number of teeth, gear	229	105	87	237	109	80
Gear ratio	8.481	4.038	3.22	8.777	4.192	2.963
Pitch dia., pinion [mm]	44.7	86.06	119.2	37.24	86.06	119.2
Pitch dia., gear [mm]	379	347.6	385.0	326.9	360.8	353.1
Center distance [mm]	211.9	216.8	251.6	182.1	223.4	236.1
Face width [mm]	26.8	47.8	95.3	22.34	51.6	95.3



(a) Two-dimensional representation



(b) Three-dimensional representation

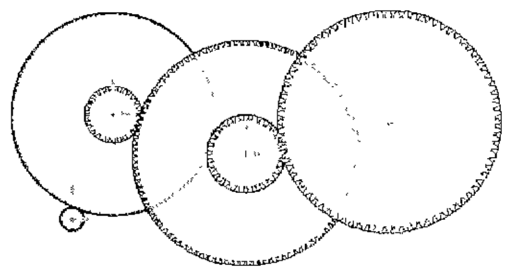
Fig. 6 Configuration design using the gears of Process I

Table 4 shows the dimensional design results of Process I and Process II. The results show small differences in module, number of teeth, face width, and other dimensions, but the differences have no significant meaning from the viewpoint of practical design. In addition, the gear ratios of Process I and Process II also have different values of 8.481, 4.038, 3.22 and 8.777, 4.192, 2.963, but both the results may be regarded as being well consistent with the design intention.

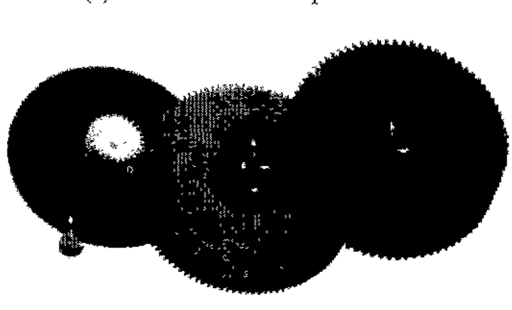
Fig. 6 and Fig. 7 show the configuration design results using the gears of Process I and Process II in Table 4, respectively. In Fig. 6, the gears show a tendency to gather around to the center, while the gears in Fig. 7 arrange in a row. Both the results show that the gears avoid interference with the shafs. This tendency might have been expected from the optimal configurations of Fig. 4. The volume of the gearbox of Fig. 6 is 60042685 mm^3, which is slightly larger than that of Fig. 7 (54201519 mm^3). However, this result does not mean that the dimensional design result of Process II is superior to that of Process I.

6. Conclusions

In this paper, we have proposed the dimensional



(a) Two-dimensional representation



(b) Three-dimensional representation

Fig. 7 Configuration design using the gears of Process II design processes to design two- and three-stage gear drives in a formalized way. In addition, we have proposed the configuration design algorithm using the simulated annealing algorithm to locate the proper positions of gears and shafts. The developed design system has been validated by the design examples, and applied to the design of three-stage gear drives. We have acquired considerably good design results in both aspects of the dimensional and the configuration design.

It might be said that this research has offered a new and consistent design methodology for the design of the two- and three-stage gear drives. This may be used as a foundation for the development of the automatic design system of multi-stage gear drives. However, the dimensional design is limited to two- and three-stage gear drives, and has a demerit that a designer should determine some of the design parameters, such as module or number of teeth. Thus, it is necessary to offer a general methodology to design gear drives having more than four stages, which are needed commonly in practical design situation. We will present a new and generalized design algorithm to design multi-stage gear drives in the second report of our research.

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