Partially Coherent MC-CDMA Downlink Performance in Rayleigh Fading Channels

Nam-Soo Kim

Abstract

Multicarrier code-division multiple access (MC-CDMA) is one of the promising technique for high capacity wireless communication. However the carrier phase error and frequency offset cause the performance degradation of MC-CDMA due to the inter-carrier interference.

In this work, downlink performance of the partially coherent MC-CDMA is analytically derived in Rayleigh fading channels. The bit error rate sensitivity by combining method, Maximal ratio combining (MRC) and Equal gain combining (EGC), is compared as functions of phase errors, multi-user interference, and received signal-to-noise ratio.

The results show that the susceptibility for the effect of the phase errors of MC-CDMA with MRC is more robust than that with EGC. However, it is also noticed that the performance degradation of EGC and MRC is negligible for loop SNR's of above 15 dB and above 10 dB, respectively.

Ⅰ. INTRODUCTION

In recent days, a lot of attention has been paid to the Multicarrier code division multiple access (MC-CDMA) scheme for high bit rate application, because at the same processing gain its required chip rate is much lower than that of a corresponding DS-CDMA system[1]~[3]. And MC-CDMA provides resistance against frequency selective fading due to the fact that each subcarrier will be narrow bandwidth respect to the coherence bandwidth. Compared with wideband transmission based on the DS-CDMA, the MC-CDMA will not need to employ the RAKE receiver as each subcarrier is subject to flat fading. However, similar to OFDM, the carrier phase error and the frequency offset cause the performance degradation of MC-CDMA because the lost of the subcarrier's orthogonality generates inter carrier interference[4],[5].

Most of the coherent receiver adopts phase locked loop (PLL) for the carrier synchronization. In that case the output of PLL is not perfectly synchronized to the carrier phase and may contain random errors. These phase errors, which are not only from the phase noise of an oscillator but from the multi-user interference, and AWGN noise, affect the system performance. Generally, the phase errors from PLL are assumed to have been derived from second-order PLL's and follow the Tikhonov distribution when the PLL's are in lock[6].

The effect of phase noise on communication system performance has been of interest up to now. Viterbi[7],[8] derived the probability density function (pdf) of the phase error caused from a first-order PLL. This pdf , which is often referred to as the Tikhonov distribution, can be applied for a second-order PLL when the loop signal-to-noise ratio is

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청주대학교 전정반 공학부(Chongju Univ. Computer and Communication Eng.)
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large[9].

In [9], Eng and Milstein analyzed the effect of the phase errors from PLL on DS-CDMA in frequency selective fading. For the multi-user performance of cdma in the presence of phase noise was is detailed in [10] where a differential modulation at the chip level was adapted to improve system performance. On MC-CDMA, the effect of phase noise of an oscillator and frequency offset has been presented analytically and semi-analytically (both analysis and simulation) for the additive white Gaussian noise channel and for the multipath fading channels, respectively[4]. And the sensitivity of multicarrier transmission over multipath channel was presented in 1996, but it does not include combining effect[11]. To analyze the phase error sensitivity on MC-CDMA, we adopt the Linnartz's model[2]. And we focuse on the downlink performance because a subscriber unit, which has stringent power consumption and size, may have much unstable local oscillator than that of a base station. Downlink performance of partially coherent MC-CDMA is analytically derived in Rayleigh fading channels. The bit error rate sensitivity of MC-CDMA with Maximal ratio combining (MRC) and with Equal gain combining (EGC) is compared as functions of phase errors, multi-user interference, and received signal-to-noise ratio.

The organization of this paper is as follows: In section Ⅱ the MC-CDMA system and channel model is described and the pdf of the phase error is introduced. Phase error for EGC and MRC is investigated in section Ⅲ. In section Ⅳ, we derived the bit error rate caused by the phase error on MC-CDMA system with EGC and with MRC, respectively. Degradations of the performance of the two systems to this phase error are compared and reviewed in section V by numerical results. Section Ⅵ concluding remarks are provided.

Ⅱ. SYSTEM AND CHANNEL MODEL

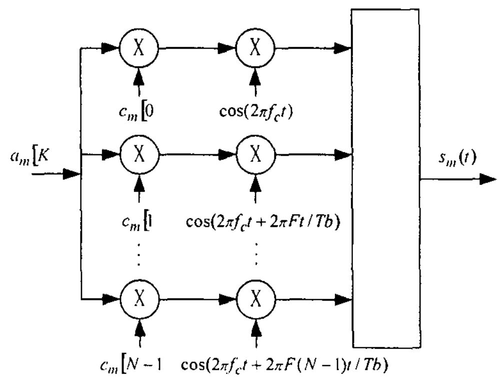

Fig. 1. Transmitter model.

The input information sequence from an user is replicated into N parallel branches. We assume input data symbols, $a_m(k)$, are binary antipodal where k and m denote k th bit interval and mth user respectively. Each branch of the parallel stream is multiplied by a chip from a spreading code of length N. After multiplication, each branch is binary phase shift keying modulated to a subcarrier spaced apart from its neighboring subcarriers by F/ T_b Hz where F is an integer number and T_b is data bit duration[2]. The transmitted signal is the sum of the output of these branches. Fig. 1 shows the transmitter model[2],[12].

We assumed frequency selective Rayleigh fading channel, and the channel delay spread is much shorter than the data bit duration such that each subcarrier signal is assumed to be flat fading during a bit period. Moreover we also assume independent fading for each subcarrier. With these assumptions, the transmitted signal corresponding of the m th user is [2]

$$s_m(t) = \sum_{i=0}^{N-1} c_m(i)\, a_m(k) \cos\left(2\pi f_c t + 2\pi i \frac{F}{T_b} t\right) \cdot P_{T_b}(t - kT_b) \qquad (1)$$

where $c_m(i)$ is i th chip of the m th user's spreading code. And $P_{T_b}(t)$ is rectangular pulse defined on $[0,\ T_b]$.

Applying the receiver model of Fig. 2 to the received signal, the received signal for M active users can be written as

$$r(t)=\sum_{m=0}^{M-1}\sum_{i=0}^{N-1}\rho_{m,i}c_m(i)a_m(k)\cdot \cos\left(2\pi f_c t+2\pi i\frac{F}{T_b}t+\theta_{m,i}\right)P_{T_b}(t-kT_b)+n(t) \quad (2)$$

where $\rho_{m,i}$ and $\theta_{m,i}$ is the received amplitude and phase of the ith branch of the m th user, respectively. $n(t)$ denotes additive white Gaussian noise(AWGN) with zero mean and double sided power spectral density $N_0/2$. The local mean power at the ith subcarrier is $\overline{p_i}=\frac{1}{2}E[\rho_i^2]$. And assuming that the local mean powers of the subcarriers are equal, the total local mean power of the m th user is $\overline{p}=N\overline{p_i}$. To obtain the decision variables, demodulated and combined subcarrier signals are integrated and sampled [2]. The decision variable of k th data symbol of 0^{th} user is obtained as

$$v_0=\sum_{m=0}^{M-1}\sum_{i=0}^{N-1}\rho_{m,i}\ c_m(i)d_{0,i}\ a_m(k)\frac{2}{T_b}\cdot\int_{kT_b}^{(k+1)T_b}\cos\left(2\pi f_c t+2\pi i\frac{F}{T_b}t+\theta_{m,i}\right)\cdot\cos\left(2\pi f_c t+2\pi i\frac{F}{T_b}t+\hat{\theta}_{0,i}\right)dt+\eta \quad (3)$$

where the phase estimation of the i th subcarrier

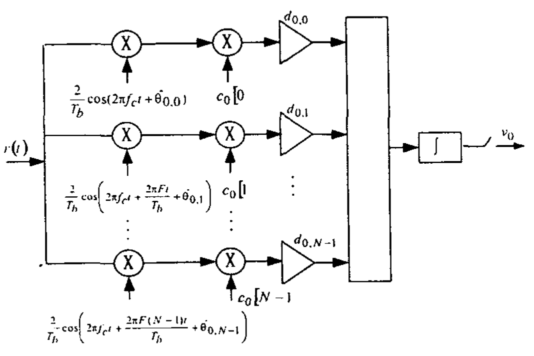

Fig. 2. Receiver model.

of the 0^{th} user, $\hat{\theta}_{m,i}$ is given as

$$\hat{\theta}_{m,i}=\theta_{0,i}+\Delta\theta_{0,i} \quad (4)$$

where $\Delta\theta_{0,i}$ is the phase errors. This phase errors are random variable and assumed to have been derived from second order PLL's. Its pdf has Tikhonov distribution [6];

$$p(\Delta\theta_{0,i})=\frac{\exp(\gamma_{0,i}\cos\Delta\theta_{0,i})}{2\pi I_0(\gamma_{0,i})},\ -\pi<\Delta\theta_{0,i}<\pi \quad (5)$$

where $I_0(\bullet)$is the zeroth-order modified Bessel function of the first kind. $\gamma_{0,i}$ is the loop signal-to-noise ratio of the PLL. At a large loop signal-to-noise ratio (i.e. γ>5 dB), the variance of the phase estimator error becomes

$$\sigma_p^2=\frac{1}{\gamma} \quad (6)$$

With this assumption, which implies the phase estimates have small variances conditioned on γ, the random variable $\cos(\Delta\theta)$ may be replaced by its expected value[9].

After some manipulation we can rewrite the decision variable,

$$v_0=\sum_{m=0}^{M-1}\sum_{i=0}^{N-1}\rho_{m,i}c_m(i)c_0(i)d_{0,i}a_m(k)\cdot\cos\{\theta_{m,i}-(\theta_{0,i}+\Delta\theta_{0,i})\}+\eta \quad (7)$$

and it can be divided into three terms,

$$v_0=a_0(k)\sum_{i=0}^{N-1}\rho_{0,i}d_{0,i}\cos(\Delta\theta_{0,i})+\sum_{m=1}^{M-1}\sum_{i=0}^{N-1}\rho_{m,i}c_m(i)c_0(i)d_{0,i}a_m(k)\cos(\tilde{\theta}_{m,i})+\eta \quad (8)$$

where

$$\tilde{\theta}_{m,i}=\theta_{0,i}+\Delta\theta_{0,i}-\theta_{m,i} \quad (9)$$

And the mean and the variance of η is zero and $N\frac{N_0}{T_b}E[d_{0,i}^2]$, respectively. Where $E[\bullet]$ means expected value.

Ⅲ. PHASE ERROR ANALYSIS FOR EGC AND MRC

Because of the orthogonality of the spreading codes, the decision variables can be written as[2]

$$v_0 = a_0(k)\sum_{i=0}^{N-1}\rho_{0,i}\,d_{0,i}\cos(\Delta\theta_{0,i})$$
$$+\sum_{m=1}^{M-1}a_m(k)\left\{\sum_{j=0}^{\frac{N}{2}-1}\rho_{0,a_j}-\sum_{j=0}^{\frac{N}{2}-1}\rho_{0,b_j}\right\}\cos(\Delta\theta_{0,i})$$
$$+\eta$$
$$= U_S + U_I + \eta \qquad (10)$$

where U_S and U_I denotes desired signal component and interfering component, respectively. The decision variable can be obtained from (10) by replacing $d_{0,i}$ with one for EGC and $\rho_{0,i}$ for MRC.

3-1 Equal Gain Combining

Variance of the desired signal component is

$$\mathrm{var}(U_S) = \mathrm{var}\left[\sum_{i=0}^{N-1}\rho_{0,i}I(\gamma_{0,i})\right]$$
$$=\left(2-\frac{\pi}{2}\right)\overline{P_0}\,I^2(\gamma_{0,i}) \qquad (11)$$

where $I(\gamma_{0,i})$, the expectation of the r.v. of $(\Delta\theta_{0,i})$, can be obtained from linear approximation to derive bit error rate with closed form[9].

The average of the interfering component is zero and the variance is given by

$$\mathrm{var}(U_I) = E\left[\left(\sum_{m=1}^{M-1}a_m(k)\left\{\sum_{j=0}^{\frac{N}{2}-1}\rho_{0,a_j}\right.\right.\right.$$
$$\left.\left.\left.-\sum_{j=0}^{\frac{N}{2}-1}\rho_{0,b_j}\right\}I(\gamma_{0,i})\right)^2\right]$$
$$=(M-1)N\sigma^2\rho_{0,i}I^2(\gamma_{0,i})$$
$$=(M-1)NI^2(\gamma_{0,i})\{E[\rho_{0,i}^2]-E^2[\rho_{0,i}]\}$$
$$=(M-1)2\left(1-\frac{\pi}{4}\right)I^2(\gamma_{0,i})\overline{P_0} \qquad (12)$$

And the mean and the variance of η is zero and $N\frac{N_0}{T_b}$, respectively.

3-2 MRC

Variance of the desired signal component is

$$\mathrm{var}(U_s) = \mathrm{var}\left[\sum_{i=0}^{N-1}\rho^2 0, iI(\gamma_{0,i})\right]$$
$$=\frac{4\overline{P_0}^2}{N}I^2(\gamma_{0,i}) \qquad (13)$$

The average of the interfering component is zero and the variance is given by

$$\mathrm{var}[U_I] = (M-1)NI^2(\gamma_{0,i})\{E[\rho_{0,i}^4]-E^2[\rho_{0,i}]\}$$
$$=\frac{M-1}{N}4\overline{P_0}^2I^2(\gamma_{0,i}) \qquad (14)$$

The mean and the variance of η is zero and $2\frac{N_0}{T_b}\overline{p_0}$, respectively.

Ⅳ. PERFORMANCE ANALYSIS

The probability of decision error conditioned on the amplitudes of the desired signal, $\sum_{i=0}^{N-1}\rho_{0,i}$, and the local mean interference power, $\mathrm{var}\{U_I\}$, given $a_0[k]=-1$ is

$$P_r\left(error\middle|\sum_{i=0}^{N-1}\rho_{0,i},\ \mathrm{var}\{U_I\}\right)$$
$$=P_r\left(\sum_{i=0}^{N-1}\rho_{0,i}I(\gamma_{0,i})<U_I+\eta\right) \qquad (15)$$

4-1 Equal Gain Combining

Because of the interference component and noise component are independent and Gaussian, the sum to the right of the inequality has a zero mean Gaussian distribution with a variance equal to the sum of their variances. Therefore the probability of decision error can be written as

$$P_r\left(error\middle|\sum_{i=0}^{N-1}\rho_{0,i},\ \mathrm{var}\{U_I\}\right)$$

$$=\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\frac{1}{2}\left\{\sum_{i=0}^{N-1}\rho_{0,i}\,I(\gamma_{0,i})\right\}^2}{(M-1)2\left(1-\frac{\pi}{4}\right)I^2(\gamma_{0,i})\overline{P_0}+N\frac{N_0}{T_b}}}\right) \tag{16}$$

where the complementary error function is defined to be

$$erfc(x)=\frac{2}{\sqrt{\pi}}\int_x^{\infty}\exp(-t^2)\,dt \tag{17}$$

To drive an expression for the average probability of error, (16) should be averaged over the distribution of the instantaneous amplitudes $\sum_{i=0}^{N-1}\rho_{0,i}$. If we apply the Central limit theorem, then (16) can be represented by,

$$P_r(error\mid\overline{P_0})$$
$$=\int_{-\infty}^{\infty}\frac{1}{2}\,\mathrm{erfc}\left(\sqrt{\frac{\frac{1}{2}\left\{\sum_{i=0}^{N-1}\rho_{0,i}\,I(r_{0,}i)\right\}^2}{(M-1)2\left(1-\frac{\pi}{4}\right)I^2(r_{0,}i)\overline{P_0}+N\frac{N_0}{T_b}}}\right)\times\frac{1}{\sqrt{2\pi\sigma^2\rho_0}}\exp\left[-\frac{(\rho_0-\mu\rho_0)^2}{2\sigma^2\rho_0}\right]d\rho_0 \tag{18}$$

where $\rho_0=\sum_{i=0}^{N-1}\rho_{0,i}$ and average of ρ_0, μ_{ρ_0} (N times of average of $\rho_{0,i}$) can be given as

$$\mu\rho_0=\sqrt{\frac{\pi}{2}}\,\sigma=\sqrt{\frac{\pi}{2}}\sqrt{N\overline{P_0}}=\sqrt{N\overline{P_0}} \tag{19}$$

The variance of ρ_0 is

$$\sigma^2\rho_0=\left(2-\frac{\pi}{2}\right)\overline{P_0} \tag{20}$$

If we replace those values, average and variance of ρ_0, the bit error rate becomes

$$P_r(error\mid\overline{P_0})=\frac{1}{2}\,erfc\left(\sqrt{\frac{\frac{\pi}{4}\,T_b\overline{P_0}\,I^2(r_{0,i})}{2M\frac{T_b}{N}\left(1-\frac{\pi}{4}\right)\overline{P_0}\,I^2(r_{0,i})+N_0}}\right) \tag{21}$$

4-2 MRC

Apply to the same method, the error probability can be written by

$$P_r(error\mid\overline{P_0})=\frac{1}{2}\,erfc\left(\sqrt{\frac{T_b\overline{P_0}\,I^2(r_{0,}i)}{2M\frac{T_b}{N}\overline{P_0}\,I^2(r_{0,}i)+N_0}}\right) \tag{22}$$

Ⅴ. NUMERICAL RESULTS

Fig. 3 shows the bit error probability versus loop signal-to-noise ratio, which is the inverse of the phase error variance, for N=128 and M=15 in the downlink Rayleigh fading channel. The performance of EGC and that of MRC are compared for the

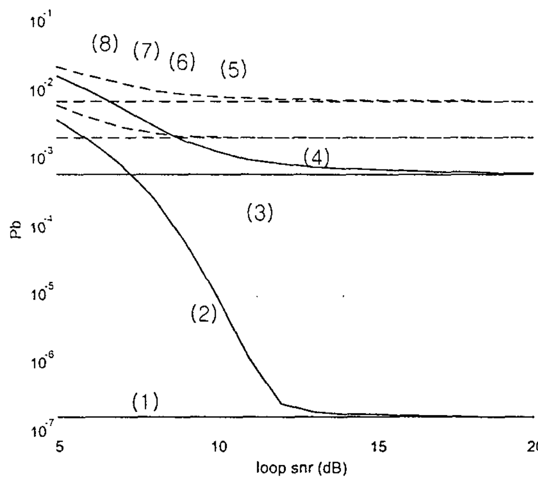

Fig. 3. Bit error probability versus loop signal-to-noise ratio with N=128, M=15; (1) EGC without phase errors (E_b/N_0=20 dB), (2) EGC with phase errors (E_b/N_0=20 dB), (3) EGC without phase errors (E_b/N_0=10 dB), (4) EGC with phase errors (E_b/N_0=10 dB), (5) MRC without phase errors (E_b/N_0=20 dB), (6) MRC with phase errors (E_b/N_0=20 dB), (7) MRC without phase errors (E_b/N_0=10 dB), (8) MRC with phase errors (E_b/N_0=10dB).

perfect and partial coherent cases. In the case of no phase errors, it is seen that the performance of MC-CDMA with EGC is better than that with MRC under the same conditions; this means that the orthogonality loss for EGC is less serious than that of MRC. However the susceptibility for the effect of the phase errors of MRC is more robust than that of EGC.

Also, it is noticed that the performance degradation of EGC and MRC is negligible for loop SNR's of above 15 dB and above 10 dB, respectively.

Fig. 4 illustrates the behavior of the bit error probability as a function of the number of simultaneous users M when N=128, E_b/N_0=20 dB, and loop signal-to-noise ratio=10 dB. It is noticed the effect of the phase error decreases as the number of simultaneous users increases. The performance degradation can be neglected when the number of the simultaneous users is larger than 30 for EGC, while 9 for MRC.

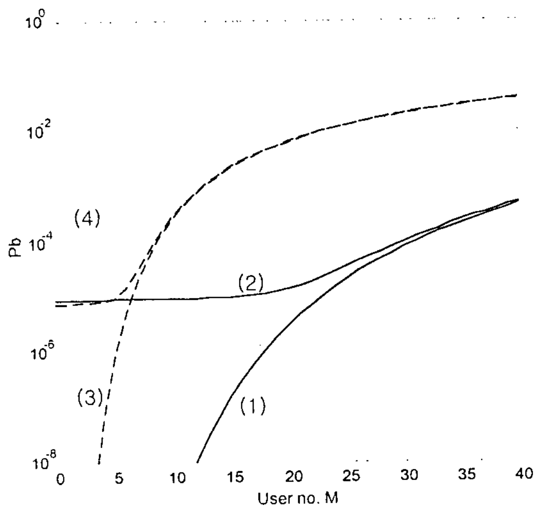

Fig. 4. Bit error probability versus user number with N=128, E_b/N_0=20 dB, and loop signal-to-noise ratio=10 dB; (1) EGC without phase errors, (2) EGC with phase errors, (3) MRC without phase errors, (4) MRC with phase errors.

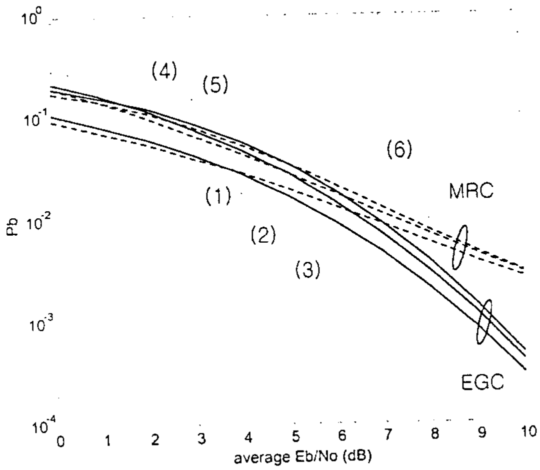

Fig. 5. Bit error probability versus average E_b/N_0 with N=128, M=15; (1) EGC without phase errors, (2) EGC with phase errors-linear approx., (3) EGC with phase errors-exact, (4) MRC without phase errors, (5) MRC with phase errors- linear approx., (6) MRC with phase errors-exact.

The bit error probability as a function of average E_b/N_0 when N=128, M=10, and loop signal-to-noise ratio 2 dB above the average E_b/N_0 is shown in Fig. 5. In this figure the "linear approx." and "exact" curve obtained from the linear approximation and from the expectation of $I(\gamma_{0,i})$, the expectation of the r.v. of $\cos(\Delta\theta_{0,i})$, respectively[9]. The differences between the linear approximation and statistical average are within 0.5 dB regardless of the combining method. Also we concluded that the effect of phase error on system performance is less as the loop SNR increases.

VI. CONCLUSIONS

In this work, we evaluate the impact of the phase errors on the system performance of MC-CDMA scheme in Rayleigh fading. The phase errors are caused by various reasons; phase noise of an oscillator, multi-user interference, and white noise, etc. It is high possibility that an oscillator of a subscriber unit has more phase noise than that of a base station. Therefore we focus on the downlink performance.

With given loop SNR, inverse of the variance of phase errors, we notice the performance of MC-CDMA with EGC is more sensitive than with MRC. However, it is also noticed that the performance degradation with EGC and with MRC is negligible for loop SNR's of above 15 dB and above 10 dB, respectively.

Similar conclusions hold for the multi-users. The effect of the phase error can be neglected when the number of the simultaneous users is larger than 30 for EGC and 9 for MRC with the processing gain N=128, E_b/N_0=20 dB, and loop signal-to-noise ratio=10 dB.

As the received signal-to-noise ratio increases, the bit error rate is rapidly decreased. Also we concluded that the effect of phase error on system performance is less as the loop SNR increases.

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Nam-Soo Kim

He is currently with Chongju University, Chongju, where he is an associate professor of Computer and Communication Engineering. His research interests include spread spectrum communication, RF system design, digital modulation. From 1986 to 1994, he was with ETRI, where he developed an DS-CDMA cellular radio system as a head of radio technology division of ETRI. And he was a visiting researcher at the BNR(Bell-Northern Research), Dallas, Texas in 1991.