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# 직교이방성 장방형 후판의 탄성이론해석

## Elastic Analysis of Orthotropic Rectangular Thick Plates

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**Abstract**

A system of equations is developed for the theory of bending of thick orthotropic elastic plates which takes into account the transverse shear deformability of the plate. This system of equations is of such nature that three boundary conditions can and must be prescribed along the edge of the plate, i.e. $w = 0$, $M_x = 0$, $M_{xy} = 0$ ( $w = 0$, $M_y = 0$, $M_{xy} = 0$ ) at simple supported edges.

It can be obtained general solution that is added complementary solution $w^e$ and paticular solution $w^p$ by an assumption of solution function.

In the next paper, this analytical results will be obtained for perforated thick plates.

***Keywords : thick plate, orthotropic plate, perforated plate, transverse shear deformation***

## 1. 서 론

후판(厚板:thick plate)의 탄성이론은 1945년 Eric Reissner[1)]에 의하여, 응력분포에 근거를 두어 전단변형을 고려한 평판의 이론이 발표된 이래로, Kirchhoff의 가정 및 거기에 부수하는 환산 전단력을 검토하기 위한 연구는 그 동안 많이 발표되었다. 즉 기초방정식의 이론적 구성과 오차평가[2~6)], 얻어진 기초방정식의 수치해석법[7~8)], 경계조건 및 주변 반력에 대해서[9)], 그리고 유한요소법에 의한 해석[11~13)] 등이다.

이상의 연구에 대해서 기초방정식의 유도에 대해서 살펴보면 크게 두 가지로 분류할 수 있다. 즉 $\sigma_z$의 응력분포를 가정하는 Reissner 방법과 Kirchhoff 가정의 구속을 일부분 풀어 주어서 중립축의 회전을 독립변수로 하는 Marguerre의 방법으로 대별할 수 있다. 또한 위의 2가지 방법 모두가 그 결과는 거의 차이가 없다는 논문도 발표되었다[10)].

직교이방성후판(直交異方性厚板: Orthotropic thick plate)의 경우에는 기초방정식의 유도도 복잡할 뿐 아니라 유도된 편미분방정식의 일반해를 구하는 연구는 매우 난해하므로 대개 수치해석으로 취급하게 된다. 따라서 이 분야의 연구가 그렇게 활발하지는 못한 실정이다. 그러나 최근래는 전단변형을 고려한 적층판의 연구는 다소 진행되고 있다[14)].

본 연구에서 취급하는 직교이방성후판의 탄성해석의 경우에는 Marguerre의 방법의 일종으로서 $\sigma_z = 0$으로 두고 전단변형 $\gamma_{yz}$, $\gamma_{zx}$만을 고려해서 기초방정식을 유도하고 그 이론해법은 수직 전단력 $Q_x$, $Q_y$와 처짐 $w$로서 나타낸 휨모멘트의 평형방정식 2개를 자동으로 만족하는 해함수 $\Phi$를 새로 도입하여 수직 방향의 평형방정식으로부터 해함수 $\Phi$의 편미분방정식을 유도해서 그 편미분방정식의 일반해를 유도하게 된다. 즉, 본 논문은 직교이방성후판의 기초방정식을 새롭게 정립하고, 그 편미분방정식의 일반해를 수학적 기법으로 새롭게 해석하는 방법을 제시하게 되는 셈이다. 본고는 참고문헌[15]의 계속적인 연구이다[16)].

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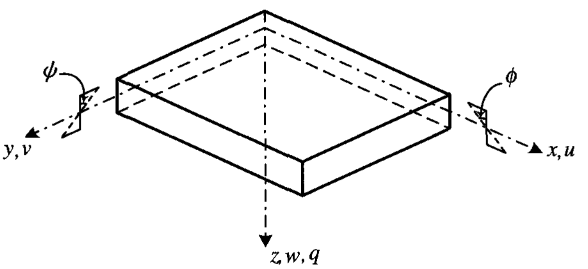
## 2. 직교이방성 후판의 기초방정식유도

<그림 1>에서 나타낸 바와 같이 회전각 $\phi$와 $\psi$를 미지량으로 하고 휨문제만을 고려하여 변위함수를 다음식과 같이 가정한다.

$$
\begin{aligned}
U(x,y.z) &= z\phi(x,y)\\
V(x,y.z) &= z\psi(x,y)\\
W(x,y.z) &= w(x,y) \qquad (2\text{-}1)
\end{aligned}
$$




〈그림 1〉 회전각 $\phi$와 $\psi$

그리고 직교이방성 탄성체의 응력도-변형율의 관계식은

$$
\begin{aligned}
\varepsilon_x &= \frac{1}{E_1}\sigma_x - \frac{v_{21}}{E_2}\sigma_y - \frac{v_{31}}{E_3}\sigma_z, \quad \gamma_{yz} = \frac{1}{G_{23}}\tau_{yz}\\
\varepsilon_y &= -\frac{v_{12}}{E_1}\sigma_x + \frac{1}{E_2}\sigma_y - \frac{v_{32}}{E_3}\sigma_z, \quad \gamma_{zx} = \frac{1}{G_{31}}\tau_{zx}\\
\varepsilon_z &= -\frac{v_{13}}{E_1}\sigma_x - \frac{v_{23}}{E_2}\sigma_y + \frac{1}{E_3}\sigma_z, \quad \gamma_{xy} = \frac{1}{G_{12}}\tau_{xy}
\end{aligned}
\qquad (2\text{-}2)
$$

과 같이 되고 그 역변환 식은 다음과 같이 된다.

$$
\begin{aligned}
\sigma_x &= \frac{E_1}{\overline{A}'}\left\{(1-v_{23}v_{32})\varepsilon_x + (v_{21}+v_{23}v_{31})\varepsilon_y + (v_{31}+v_{32}v_{21})\varepsilon_z\right\}\\
\sigma_y &= \frac{E_2}{\overline{A}'}\left\{(v_{21}+v_{13}v_{32})\varepsilon_x + (1-v_{31}v_{13})\varepsilon_y + (v_{32}+v_{31}v_{12})\varepsilon_z\right\}\\
\sigma_z &= \frac{E_3}{\overline{A}'}\left\{(v_{13}+v_{12}v_{23})\varepsilon_x + (v_{23}+v_{21}v_{13})\varepsilon_y + (1-v_{12}v_{21})\varepsilon_z\right\}\\
\tau_{yz} &= G_{23}\gamma_{yz}, \quad \tau_{zx} = G_{31}\gamma_{zx}, \quad \tau_{xy} = G_{12}\gamma_{xy} \qquad (2\text{-}3)
\end{aligned}
$$

단, $\overline{A}' = (1-v_{12}v_{21})(1-v_{32}v_{23}) - (v_{13}+v_{12}v_{23})(v_{31}+v_{32}v_{21})$이다.

그런데, 서론에서 가정한 바와 같이 판의 수직방향 응력도를 무시하여

$$\sigma_z = 0 \qquad (2\text{-}4)$$

라고 두면 식(2-3)의 제3식으로부터 $\varepsilon_z$는 다음과 같이 된다.

$$\varepsilon_z = -\frac{1}{1-v_{12}v_{21}}\left\{(v_{13}+v_{12}v_{23})\varepsilon_x + (v_{23}+v_{21}v_{13})\varepsilon_y\right\} \qquad (2\text{-}5)$$

식(2-5)를 식(2-3)에 대입하면 응력도-변형율의 관계식은 다음과 같이 된다.

$$
\begin{aligned}
\sigma_x &= \frac{E_1}{1-v_{12}v_{21}}\left\{\alpha_1\varepsilon_x + \alpha_2\varepsilon_y\right\}\\
\sigma_y &= \frac{E_2}{1-v_{12}v_{21}}\left\{\beta_2\varepsilon_x + \beta_1\varepsilon_y\right\}\\
\tau_{yz} &= G_{23}\gamma_{yz}, \quad \tau_{zx} = G_{31}\gamma_{zx}, \quad \tau_{xy} = G_{12}\gamma_{xy} \qquad (2\text{-}6)
\end{aligned}
$$

여기서,

$$
\begin{aligned}
\alpha_1 &= 1\\
\alpha_2 &= \frac{1}{\overline{A}'}\left\{(1-v_{12}v_{21})(v_{21}+v_{23}v_{31}) - (v_{31}+v_{32}v_{21})(v_{23}+v_{21}v_{13})\right\}\\
\beta_1 &= \frac{1}{\overline{A}'}\left\{(1-v_{12}v_{21})(1-v_{31}v_{13}) - (v_{32}+v_{31}v_{12})(v_{13}+v_{12}v_{23})\right\}\\
\beta_2 &= \frac{1}{\overline{A}'}\left\{(1-v_{12}v_{21})(v_{12}+v_{13}v_{32}) - (v_{32}+v_{31}v_{12})(v_{13}+v_{12}v_{23})\right\}
\end{aligned}
\qquad (2\text{-}7)
$$

여기에, 참고문헌[15]에서 유도한 직교이방성판의 등가강성에 대한 탄성정수(elastic constants)의 기호를 일치시키기 위해서 상반정리(相反定理)를 이용하여 식(2-6)을 다음과 같이 나타낼 수 있다.

$$
\begin{aligned}
\sigma_x &= d_{11}\varepsilon_x + d_{12}\varepsilon_y\\
\sigma_y &= d_{12}\varepsilon_x + d_{22}\varepsilon_y\\
\tau_{yz} &= G_{23}\gamma_{yz}, \quad \tau_{zx} = G_{31}\gamma_{zx}, \quad \tau_{xy} = G_{12}\gamma_{xy} \qquad (2\text{-}8)
\end{aligned}
$$

여기서,

$$
\begin{aligned}
d_{11} &= E_1\alpha_1/(1-v_{12}v_{21}),\\
d_{12} &= E_1\alpha_2/(1-v_{12}v_{21}) = E_2\beta_2/(1-v_{12}v_{21}) = d_{21}\\
d_{22} &= E_2\beta_1/(1-v_{12}v_{21})
\end{aligned}
$$

그리고 변형율-변위의 관계식은 다음과 같다.

$$
\begin{aligned}
\varepsilon_x &= \frac{\partial U}{\partial x}, \quad \gamma_{yz} = \frac{\partial W}{\partial y} + \frac{\partial V}{\partial z}\\
\varepsilon_y &= \frac{\partial V}{\partial y}, \quad \gamma_{zx} = \frac{\partial U}{\partial z} + \frac{\partial W}{\partial x}\\
\varepsilon_z &= \frac{\partial W}{\partial z}, \quad \gamma_{xy} = \frac{\partial V}{\partial x} + \frac{\partial U}{\partial y}
\end{aligned}
\qquad (2\text{-}9)
$$

식(2-9)는 식(2-1)을 이용하면 다음과 같이 된다.

$$\varepsilon_x = z\frac{\partial \phi}{\partial x}\ , \qquad \gamma_{yz} = \frac{\partial w}{\partial y} + \psi$$
$$\varepsilon_y = z\frac{\partial \psi}{\partial y}\ , \qquad \gamma_{zx} = \frac{\partial w}{\partial x} + \phi$$
$$\varepsilon_z = 0\ , \qquad \gamma_{xy} = z\left(\frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial y}\right) \qquad (2\text{-}10)$$

그런데, 휨모멘트 $M_x$, $M_y$, 비틀림모멘트 $M_{xy}$, 전단력 $Q_x$, $Q_y$는 다음과 같이 정의한다.

$$M_x = \int_{-h/2}^{h/2} \sigma_x z\,dz\ , \qquad M_y = \int_{-h/2}^{h/2} \sigma_y z\,dz$$
$$M_{xy} = \int_{-h/2}^{h/2} \tau_{xy}\, z\,dz$$
$$Q_x = \int_{-h/2}^{h/2} \tau_{zx}\,dz\ , \qquad Q_y = \int_{-h/2}^{h/2} \tau_{yz}\,dz \qquad (2\text{-}11)$$

한편, 가상일의 원리에 의하여 다음과 같은 평형방정식을 유도할 수 있다.

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = 0$$
$$\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y = 0$$
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \qquad (2\text{-}12)$$

식(2-10)을 식(2-8)에 대입하면 다음과 같이 된다.

$$\sigma_x = \left(d_{11}\frac{\partial \phi}{\partial x} + d_{12}\frac{\partial \psi}{\partial y}\right)z$$
$$\sigma_y = \left(d_{12}\frac{\partial \phi}{\partial x} + d_{22}\frac{\partial \psi}{\partial y}\right)z$$
$$\tau_{yz} = G_{23}\left(\frac{\partial w}{\partial y} + \psi\right)$$
$$\tau_{zx} = G_{31}\left(\frac{\partial w}{\partial x} + \phi\right)$$
$$\tau_{xy} = G_{12}\left(\frac{\partial \psi}{\partial x} + \frac{\partial \phi}{\partial y}\right)z \qquad (2\text{-}13)$$

식(2-13)을 식(2-11)에 대입하여 적분함과 동시에 전단력이 국부적인 평형방정식을 만족하도록 에너지의 보정을 수행하여 $h_s = h/1.2$를 이용하면 다음과 같은 식(2-14)를 얻을 수 있다.

$$M_x = \frac{h^3}{12}\left\{d_{11}\frac{\partial \phi}{\partial x} + d_{12}\frac{\partial \psi}{\partial y}\right\}$$
$$M_y = \frac{h^3}{12}\left\{d_{12}\frac{\partial \phi}{\partial x} + d_{22}\frac{\partial \psi}{\partial y}\right\}$$
$$M_{xy} = \frac{h^3}{12}G_{12}\left\{\frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y}\right\}$$
$$Q_x = G_{31}h_s\left\{\frac{\partial w}{\partial x} + \phi\right\}$$
$$Q_y = G_{23}h_s\left\{\frac{\partial w}{\partial y} + \psi\right\} \qquad (2\text{-}14)$$

식(2-14)에서 $\phi$, $\psi$를 소거하면 $M_x$, $M_y$ 및 $M_{xy}$는 다음과 같이 표현할 수 있다.

$$M_x = -\frac{h^3}{12}\left\{d_{11}\frac{\partial^2 w}{\partial x^2} + d_{12}\frac{\partial^2 w}{\partial y^2}\right\} + \frac{h^2}{10}\left\{\frac{d_{11}}{G_{31}} - \frac{d_{12}}{G_{23}}\right\}\frac{\partial Q_x}{\partial x} - \frac{h^2}{10}\frac{d_{12}}{G_{23}}q$$
$$M_y = -\frac{h^3}{12}\left\{d_{12}\frac{\partial^2 w}{\partial x^2} + d_{22}\frac{\partial^2 w}{\partial y^2}\right\} + \frac{h^2}{10}\left\{\frac{d_{22}}{G_{23}} - \frac{d_{12}}{G_{31}}\right\}\frac{\partial Q_y}{\partial y} - \frac{h^2}{10}\frac{d_{12}}{G_{31}}q$$
$$M_{xy} = -\frac{G_{12}h^3}{6}\frac{\partial^2 w}{\partial x \partial y} + \frac{G_{12}h^2}{10}\left\{\frac{1}{G_{23}}\frac{\partial Q_y}{\partial x} + \frac{1}{G_{31}}\frac{\partial Q_x}{\partial y}\right\} \qquad (2\text{-}15)$$

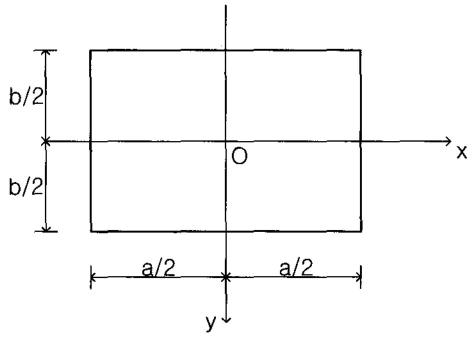
식(2-15)를 평형방정식(2-12)에 대입하면 다음과 같은 직교이방성 후판의 기초방정식이 얻어진다.

$$Q_x - \frac{h^2}{10}\left\{\left(\frac{d_{11}}{G_{31}} - \frac{d_{12}+G_{12}}{G_{23}}\right)\frac{\partial^2 Q_x}{\partial x^2} + \frac{G_{12}}{G_{31}}\frac{\partial^2 Q_x}{\partial y^2}\right\} + \frac{h^2}{10}\frac{d_{12}+G_{12}}{G_{23}}\frac{\partial q}{\partial x}$$
$$= -\frac{\partial}{\partial x}\left[\frac{h^3}{12}\left\{d_{11}\frac{\partial^2 w}{\partial x^2} + (d_{12}+2G_{12})\frac{\partial^2 w}{\partial y^2}\right\}\right]$$
$$Q_y - \frac{h^2}{10}\left\{\left(\frac{d_{22}}{G_{23}} - \frac{d_{12}+G_{12}}{G_{31}}\right)\frac{\partial^2 Q_y}{\partial y^2} + \frac{G_{12}}{G_{23}}\frac{\partial^2 Q_y}{\partial x^2}\right\} + \frac{h^2}{10}\frac{d_{12}+G_{12}}{G_{31}}\frac{\partial q}{\partial y}$$
$$= -\frac{\partial}{\partial y}\left[\frac{h^3}{12}\left\{d_{22}\frac{\partial^2 w}{\partial y^2} + (d_{12}+2G_{12})\frac{\partial^2 w}{\partial x^2}\right\}\right]$$
$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \qquad (2\text{-}16)$$

## 3. 직교이방성 장방형 후판의 일반해 유도

<그림 2>와 같은 좌표축을 갖고 등분포하중 $q$를 받는 주변단순지지판을 생각한다. 먼저 직교이방성

후판의 기초방정식 (2-16)을 간략화하기 위하여 다음과 같이 나타낸다.




〈그림 2〉 장방형판의 좌표계

$$Q_x - \left[ A_x \frac{\partial^2 Q_x}{\partial x^2} + B_x \frac{\partial^2 Q_x}{\partial y^2} \right] + C_x \frac{\partial q}{\partial x} + D_x \frac{\partial}{\partial x} \left[ \frac{\partial^2 w}{\partial x^2} + E_x \frac{\partial^2 w}{\partial y^2} \right] = 0$$

$$Q_y - \left[ A_y \frac{\partial^2 Q_y}{\partial y^2} + B_y \frac{\partial^2 Q_y}{\partial x^2} \right] + C_y \frac{\partial q}{\partial y} + D_y \frac{\partial}{\partial y} \left[ \frac{\partial^2 w}{\partial y^2} + E_y \frac{\partial^2 w}{\partial x^2} \right] = 0$$

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q = 0 \qquad (3\text{-}1)$$

여기서,

$$A_x = \frac{h^2}{10} \left\{ \frac{d_{11}}{G_{31}} - \frac{d_{12} + G_{12}}{G_{23}} \right\}$$

$$B_x = \frac{h^2}{10} \frac{G_{12}}{G_{31}}$$

$$C_x = \frac{h^2}{10} \frac{d_{12} + G_{12}}{G_{23}}$$

$$D_x = \frac{h^3}{12} d_{11}$$

$$E_x = \frac{d_{12} + 2G_{12}}{d_{11}}$$

$$A_y = \frac{h^2}{10} \left\{ \frac{d_{22}}{G_{23}} - \frac{d_{12} + G_{12}}{G_{31}} \right\}$$

$$B_y = \frac{h^2}{10} \frac{G_{12}}{G_{23}}$$

$$C_y = \frac{h^2}{10} \frac{d_{12} + G_{12}}{G_{31}}$$

$$D_y = \frac{h^3}{12} d_{22}$$

$$E_y = \frac{d_{12} + 2G_{12}}{d_{22}} \qquad (3\text{-}2)$$

다음, 모멘트 $M_x$, $M_y$ 및 비틀림 모멘트 $M_{xy}$를 전단력 $Q_x$, $Q_y$와 처짐 $w$ 및 단위 면적당 받는 하중 $q$로 나타내는 식(2-15)를 간략화하기 위하여 다음과 같이 나타낼 수 있다.

$$M_x = -D_x \left[ \frac{\partial^2 w}{\partial x^2} + F_x \frac{\partial^2 w}{\partial y^2} \right] + G_x \frac{\partial Q_x}{\partial x} + H_x q$$

$$M_y = -D_y \left[ \frac{\partial^2 w}{\partial y^2} + F_y \frac{\partial^2 w}{\partial x^2} \right] + G_y \frac{\partial Q_y}{\partial y} + H_y q$$

$$M_{xy} = I \frac{\partial^2 w}{\partial x \partial y} + J_x \frac{\partial Q_y}{\partial x} + J_y \frac{\partial Q_x}{\partial y} \qquad (3\text{-}3)$$

여기서,

$$F_x = \frac{d_{12}}{d_{11}}$$

$$G_x = \frac{h^2}{10} \left\{ \frac{d_{11}}{G_{31}} - \frac{d_{12}}{G_{23}} \right\}$$

$$H_x = -\frac{h^2}{10} \frac{d_{12}}{G_{23}}$$

$$F_y = \frac{d_{12}}{d_{22}}$$

$$G_y = \frac{h^2}{10} \left\{ \frac{d_{22}}{G_{23}} - \frac{d_{12}}{G_{31}} \right\}$$

$$H_y = -\frac{h^2}{10} \frac{d_{12}}{G_{31}}$$

$$I = -\frac{h^3 G_{12}}{6}$$

$$J_x = \frac{h^2}{10} \frac{G_{12}}{G_{23}}$$

$$J_y = \frac{h^2}{10} \frac{G_{12}}{G_{31}} \qquad (3\text{-}4)$$

경계조건을 주변단순지지로 생각하면 다음식을 얻게 된다.

$$x = \pm \frac{a}{2} \text{에서} \quad w = M_x = M_{xy} = 0$$

$$y = \pm \frac{b}{2} \text{에서} \quad w = M_y = M_{yx} = 0 \qquad (3\text{-}5)$$

여기서, 결국 $M_{xy} = M_{yx}$가 성립한다. 본 연구에서 유도된 기초방정식(3-1)의 일반해를 구하기 위해서 $w$, $Q_x$ 및 $Q_y$를 다음과 같이 나타낸다.

$$w = w^p + w^c$$

$$Q_x = Q_x^p + Q_x^c$$
$$Q_y = Q_y^p + Q_x^c \qquad (3\text{-}6)$$

여기서, 윗첨자 $p$는 특해(特解: particular solution)를, $c$는 여해(餘解: complementary solution)를 나타낸다. 또한, 위의 식(3-6)과 같은 형식으로 $M_x$, $M_y$ 및 $M_{xy}$를 다음과 같이 나타낸다.

$$M_x = M_x^p + M_x^c$$
$$M_y = M_y^p + M_y^c$$
$$M_{xy} = M_{xy}^p + M_{xy}^c \qquad (3\text{-}7)$$

### i ) 특해의 유도

Fourier급수에 의하여 하중함수 $q$를 다음과 같이 나타낸다.

$$q(x, y) = \sum_m \sum_n q_{mn} \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$
$$(m, n = 0, 1, 2, \cdots, \infty) \qquad (3\text{-}8)$$

여기서, $q_{mn}$은 상수이고 $m$, $n$은 정수이다.

또한, 기초방정식(3-1)의 특해 $w^p$, $Q_x^p$, $Q_y^p$는 경계조건을 고려하여 다음과 같은 2중 Fourier급수로 나타낼 수 있다.

$$w^p = \sum_m \sum_n w_{mn}^p \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$
$$(m, n = 1, 3, \cdots, \infty)$$
$$Q_x^p = \sum_m \sum_n Q_{xmn}^p \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$
$$(m, n = 1, 3, \cdots, \infty)$$
$$Q_y^p = \sum_m \sum_n Q_{ymn}^p \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$
$$(m, n = 1, 3, \cdots, \infty) \qquad (3\text{-}9)$$

윗 식(3-9)를 기초방정식(3-1)에 대입하면 계수 $w_{mn}^p$, $Q_{xmn}^p$, $Q_{ymn}^p$은 다음과 같이 얻어진다.

$$Q_{xmn}^p = \frac{C_x D_y\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\left\{\left(\frac{n\pi}{b}\right)^2 + E_y\left(\frac{m\pi}{a}\right)^2\right\}}{D_y\left(\frac{n\pi}{b}\right)\left\{1 + A_x\left(\frac{m\pi}{a}\right)^2 + B_x\left(\frac{n\pi}{b}\right)^2\right\}}$$
$$\frac{-C_y D_x\left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right)\left\{\left(\frac{m\pi}{a}\right)^2 + E_y\left(\frac{n\pi}{b}\right)^2\right\}}{\left\{\left(\frac{n\pi}{b}\right)^2 + E_y\left(\frac{m\pi}{a}\right)^2 + D_x\left(\frac{b}{n\pi}\right)\left(\frac{m\pi}{a}\right)^2\right\}}$$
$$\frac{-D_x\left(\frac{b}{n\pi}\right)\left(\frac{m\pi}{a}\right)\left\{1 + A_y\left(\frac{n\pi}{b}\right)^2 + B_y\left(\frac{m\pi}{a}\right)^2\right\}}{\left\{1 + A_y\left(\frac{n\pi}{b}\right)^2 + B_y\left(\frac{m\pi}{a}\right)^2\right\}}$$
$$\frac{\left\{\left(\frac{m\pi}{a}\right)^2 + E_x\left(\frac{n\pi}{b}\right)^2\right\}}{\left\{\left(\frac{m\pi}{a}\right)^2 + E_x\left(\frac{n\pi}{b}\right)^2\right\}} q_{mn}$$
$$Q_{ymn}^p = -\left(\frac{b}{n\pi}\right)\left\{\left(\frac{m\pi}{a}\right) Q_{xmn}^p + q_{mn}\right\}$$
$$w_{mn}^p = \frac{C_x\left(\frac{m\pi}{a}\right) q_{mn} - \left\{1 + A_x\left(\frac{m\pi}{a}\right)^2 + B_x\left(\frac{n\pi}{b}\right)^2\right\} Q_{xmn}^p}{D_x\left(\frac{m\pi}{a}\right)\left\{\left(\frac{m\pi}{a}\right)^2 + E_x\left(\frac{n\pi}{b}\right)^2\right\}}$$
$$(3\text{-}10)$$

따라서, 식(3-10)을 식(3-9)에 대입하면 특해를 얻을 수 있다.

다음 식(3-9)를 식(3-3)에 대입하면, 특해에 의한 모멘트 $M_x^p$, $M_y^p$ 및 비틀림모멘트 $M_{xy}^p$는 다음과 같이 된다.

$$M_x^p = \sum_m \sum_n M_{xmn}^p \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$
$$(m, n = 1, 3, \cdots, \infty)$$
$$M_y^p = \sum_m \sum_n M_{ymn}^p \cos\frac{m\pi x}{a} \cos\frac{n\pi y}{b}$$
$$(m, n = 1, 3, \cdots, \infty)$$
$$M_{xy}^p = \sum_m \sum_n M_{xymn}^p \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b}$$
$$(m, n = 1, 3, \cdots, \infty) \qquad (3\text{-}11)$$

여기서,

$$M_{xmn}^p = D_x w_{mn}^p \left\langle \left(\frac{m\pi}{a}\right)^2 + F_x\left(\frac{n\pi}{b}\right)^2 \right\rangle + G_x Q_{xmn}^p \left(\frac{m\pi}{a}\right) + H_x q_{mn}$$
$$M_{ymn}^p = D_y w_{mn}^p \left\langle \left(\frac{n\pi}{b}\right)^2 + F_y\left(\frac{m\pi}{a}\right)^2 \right\rangle + G_y Q_{ymn}^p \left(\frac{n\pi}{b}\right) + H_y q_{mn}$$
$$M_{xymn}^p = I w_{mn}^p \left(\frac{m\pi}{a}\right)\left(\frac{n\pi}{b}\right) - J_x Q_{ymn}^p \left(\frac{m\pi}{a}\right) - J_y Q_{xmn}^p \left(\frac{n\pi}{b}\right)$$
$$(3\text{-}12)$$

그리고 특해(3-9)를 이용하면 $w^p$, $M_x^p$, $M_y^p$ 및 $M_{xy}^p$는 경계조건식(3-5)를 다음과 같이 만족하고 있다.

$x = \pm\frac{a}{2}$에서 $w^p = 0$, $M_x^p = 0$

$$y=\pm\frac{b}{2}\text{에서}\quad w^{p}=0,\ M_y^{p}=0 \tag{3-13}$$

따라서, 경계조건식(3-5)는 식(3-7), (3-11)과 식(3-13)으로부터 다음과 같이 된다.

$$x=\pm\frac{a}{2}\text{에서}$$

$$w^c=0\ ,\ M_x^c=0,$$

$$M_{xy}^c=\pm\sum_m\sum_n(-1)^{\frac{m+1}{2}}M_{xymn}^p\sin\frac{n\pi y}{b}$$

$$(m,n=1,3,\cdots,\infty)$$

$$y=\pm\frac{b}{2}\text{에서}$$

$$w^c=0\ ,\ M_y^c=0,$$

$$M_{xy}^c=\pm\sum_m\sum_n(-1)^{\frac{n+1}{2}}M_{xymn}^p\sin\frac{m\pi x}{a}$$

$$(m,n=1,3,\cdots,\infty) \tag{3-14}$$

따라서, 경계조건식(3-5) 대신에 여해를 구해서 새로운 경계조건식(3-14)를 만족하도록 적분상수를 정하면 된다.

### ii) 여해의 유도

기초방정식(3-1)로부터 $q=0$으로 둠으로서 여해를 구하는 기초방정식은 다음과 같이 얻어진다.

$$Q_x-\left[A_x\frac{\partial^2Q_x}{\partial x^2}+B_x\frac{\partial^2Q_x}{\partial y^2}\right]+D_x\frac{\partial}{\partial x}\left[\frac{\partial^2w}{\partial x^2}+E_x\frac{\partial^2w}{\partial y^2}\right]=0$$

$$Q_y-\left[A_y\frac{\partial^2Q_y}{\partial y^2}+B_y\frac{\partial^2Q_y}{\partial x^2}\right]+D_y\frac{\partial}{\partial y}\left[\frac{\partial^2w}{\partial y^2}+E_y\frac{\partial^2w}{\partial x^2}\right]=0$$

$$\frac{\partial Q_x}{\partial x}+\frac{\partial Q_y}{\partial y}=0 \tag{3-15}$$

여해에 대한 모멘트 식은 다음과 같이 된다.

$$M_x=-D_x\left[\frac{\partial^2w}{\partial x^2}+F_x\frac{\partial^2w}{\partial y^2}\right]+G_x\frac{\partial Q_x}{\partial x}$$

$$M_y=-D_y\left[\frac{\partial^2w}{\partial y^2}+F_y\frac{\partial^2w}{\partial x^2}\right]+G_y\frac{\partial Q_y}{\partial y}$$

$$M_{xy}=I\frac{\partial^2w}{\partial x\partial y}+J_x\frac{\partial Q_y}{\partial x}+J_y\frac{\partial Q_x}{\partial y} \tag{3-16}$$

여기서, 기초방정식(3-15)를 만족하는 해 $w^c$, $Q_x^c$ 및 $Q_y^c$를 새로운 하나의 함수 $\Phi$로 나타내기 위해서 각각 다음과 같이 나타낸다.

$$w^c=A_xB_y\frac{\partial^4\Phi}{\partial x^4}+(A_xA_y+B_xB_y)\frac{\partial^4\Phi}{\partial x^2\partial y^2}+A_yB_x\frac{\partial^4\Phi}{\partial y^4}$$

$$-(A_x+B_y)\frac{\partial^2\Phi}{\partial x^2}-(A_y+B_x)\frac{\partial^2\Phi}{\partial y^2}+\Phi$$

$$Q_x^c=D_x\left[B_y\frac{\partial^5\Phi}{\partial x^5}+(A_y+E_xB_y)\frac{\partial^5\Phi}{\partial x^3\partial y^2}+E_xA_y\frac{\partial^5\Phi}{\partial x\partial y^4}\right.$$

$$\left.-\frac{\partial^3\Phi}{\partial x^3}-E_x\frac{\partial^3\Phi}{\partial x\partial y^2}\right]$$

$$Q_y^c=D_y\left[B_x\frac{\partial^5\Phi}{\partial y^5}+(A_x+E_yB_x)\frac{\partial^5\Phi}{\partial y^3\partial x^2}+E_yA_x\frac{\partial^5\Phi}{\partial y\partial x^4}\right.$$

$$\left.-\frac{\partial^3\Phi}{\partial y^3}-E_y\frac{\partial^3\Phi}{\partial y\partial x^2}\right] \tag{3-17}$$

그런데 윗 식(3-17)은 기초방정식(3-15)의 제1, 2식을 명백히 만족하고 있다. 식(3-15)의 제3식에 윗식(3-17)을 대입한 결과 다음 식을 얻는다.

$$\left[H_1\frac{\partial^6}{\partial x^6}+H_2\frac{\partial^6}{\partial x^4\partial y^2}+H_3\frac{\partial^6}{\partial x^2\partial y^4}+H_4\frac{\partial^6}{\partial y^6}\right.$$

$$\left.+H_5\frac{\partial^4}{\partial x^4}+H_6\frac{\partial^4}{\partial x^2\partial y^2}+H_7\frac{\partial^4}{\partial y^4}\right]\Phi=0 \tag{3-18}$$

여기서,

$$H_1=\frac{h^5}{120}\frac{G_{12}}{G_{23}}d_{11}$$

$$H_2=\frac{h^5}{120}\left\{\frac{d_{11}d_{22}-2G_{12}d_{12}-d_{12}^2}{G_{23}}+\frac{G_{12}}{G_{31}}d_{11}\right\}$$

$$H_3=\frac{h^5}{120}\left\{\frac{d_{11}d_{22}-2G_{12}d_{12}-d_{12}^2}{G_{31}}+\frac{G_{12}}{G_{23}}d_{22}\right\}$$

$$H_4=\frac{h^5}{120}\frac{G_{12}}{G_{31}}d_{22}$$

$$H_5=-\frac{h^3}{12}d_{11}$$

$$H_6=-\frac{h^3}{6}(d_{12}+2G_{12})$$

$$H_7=-\frac{h^3}{12}d_{22} \tag{3-19}$$

그런데 식(3-18)을 만족하는 $\Phi$를 구하면, $\Phi$는 기초방정식(3-15)의 3식 모두를 만족하게 된다. 따라서 $\Phi$가 얻어지면 여해 $w^c$, $Q_x^c$ 및 $Q_y^c$도 얻어지게 된다.

다음은 해함수 $\Phi$를 구한다. 혹자는 이해함수 $\Phi$를 Kwun's function이라고도 부르고 있다. 지금 $\Phi$는 다음 식과 같은 무한급수로서 나타낼 수 있다.

$$\Phi(x, y) = \sum_{K=1,3,\cdots}^{\infty} \Phi_k(x, y) \tag{3-20}$$

여기서, $\Phi_k$를 다음 식과 같이 가정한다.

$$\Phi_k(x, y) = f_1(y)\cos\frac{k\pi}{a}x + f_2(x)\cos\frac{k\pi}{b}y + f_3(y)\sin\frac{k\pi}{a}x + f_4(x)\sin\frac{k\pi}{b}y \tag{3-21}$$

식(3-21)을 식(3-18)에 대입하면 편미분방정식(3-1)은 다음과 같은 상미분방정식으로 만들 수 있다.

$$\begin{aligned}
&\left[H_4\frac{d^6f_1}{dy^6} + \left\{H_7 - \left(\frac{k\pi}{a}\right)^2 H_3\right\}\frac{d^4f_1}{dy^4} + \left(\frac{k\pi}{a}\right)^2\left\{\left(\frac{k\pi}{a}\right)^2 H_2 - H_6\right\}\frac{d^2f_1}{dy^2}\right.\\
&\quad\left. + \left(\frac{k\pi}{a}\right)^4\left\{H_5 - \left(\frac{k\pi}{a}\right)^2 H_1\right\}f_1\right]\cos\frac{k\pi}{a}x\\
&+\left[H_1\frac{d^6f_2}{dx^6} + \left\{H_5 - \left(\frac{k\pi}{b}\right)^2 H_2\right\}\frac{d^4f_2}{dx^4} + \left(\frac{k\pi}{b}\right)^2\left\{\left(\frac{k\pi}{b}\right)^2 H_3 - H_6\right\}\frac{d^2f_2}{dx^2}\right.\\
&\quad\left. + \left(\frac{k\pi}{b}\right)^4\left\{H_7 - \left(\frac{k\pi}{a}\right)^2 H_4\right\}f_2\right]\cos\frac{k\pi}{b}y\\
&+\left[H_4\frac{d^6f_3}{dy^6} + \left\{H_7 - \left(\frac{k\pi}{a}\right)^2 H_3\right\}\frac{d^4f_3}{dy^4} + \left(\frac{k\pi}{a}\right)^2\left\{\left(\frac{k\pi}{a}\right)^2 H_2 - H_6\right\}\frac{d^2f_3}{dy_2}\right.\\
&\quad\left. + \left(\frac{k\pi}{a}\right)^4\left\{H_5 - \left(\frac{k\pi}{a}\right)^2 H_1\right\}f_3\right]\sin\frac{k\pi}{a}x\\
&+\left[H_1\frac{d^6f_4}{dx^6} + \left\{H_5 - \left(\frac{k\pi}{b}\right)^2 H_2\right\}\frac{d^4f_4}{dx^4} + \left(\frac{k\pi}{b}\right)^2\left\{\left(\frac{k\pi}{b}\right)^2 H_3 - H_6\right\}\frac{d^2f_4}{dx^2}\right.\\
&\quad\left. + \left(\frac{k\pi}{b}\right)^4\left\{H_7 - \left(\frac{k\pi}{b}\right)^2 H_4\right\}f_4\right]\sin\frac{k\pi}{b}y = 0
\end{aligned} \tag{3-22}$$

따라서, 식(3-22)를 만족하는 $f_1(y)$, $f_2(x)$, $f_3(y)$ 및 $f_4(x)$를 구하면 된다. 식(3-22)로부터 알 수 있듯이 $f_1(y)$와 $f_3(y)$ 및 $f_2(x)$와 $f_4(x)$는 식의 형식이 같기 때문에 다음 식을 만족하는 $f_1(y)$및 $f_2(x)$를 구하면 된다.

$$\begin{aligned}
&H_4\frac{d^6f_1}{dy^6} + \left\{H_7 - \left(\frac{k\pi}{a}\right)^2 H_3\right\}\frac{d^4f_1}{dy^4} + \left(\frac{k\pi}{a}\right)^2\left\{\left(\frac{k\pi}{a}\right)^2 H_2 - H_6\right\}\frac{d^2f_1}{dy^2}\\
&\quad + \left(\frac{k\pi}{a}\right)^4\left\{H_5 - \left(\frac{k\pi}{a}\right)^2 H_1\right\}f_1 = 0\\
&H_1\frac{d^6f_2}{dx^6} + \left\{H_5 - \left(\frac{k\pi}{b}\right)^2 H_2\right\}\frac{d^4f_2}{dx^4} + \left(\frac{k\pi}{b}\right)^2\left\{\left(\frac{k\pi}{b}\right)^2 H_3 - H_6\right\}\frac{d^2f_2}{dx^2}\\
&\quad + \left(\frac{k\pi}{b}\right)^4\left\{H_5 - \left(\frac{k\pi}{b}\right)^2 H_4\right\}f_2 = 0
\end{aligned} \tag{3-23}$$

그런데 $f_1(y)$ 및 $f_2(x)$를 다음과 같이

$$\begin{aligned}
f_1(y) &= e^{\Lambda y}\\
f_2(x) &= e^{\lambda x}
\end{aligned} \tag{3-24}$$

로 두고, 윗 식(3-24)를 식(3-23)에 대입해서 정리하면 다음과 같은 특성방정식을 얻을 수 있다.

$$\begin{aligned}
&\Lambda^6 - \left(\frac{k\pi}{a}\right)^2\frac{H_3}{H_4}\Lambda^4 + \left(\frac{k\pi}{a}\right)^4\frac{H_2}{H_4}\Lambda^2 - \left(\frac{k\pi}{a}\right)^6\frac{H_1}{H_4}\\
&\quad + \frac{H_7}{H_4}\left\{\Lambda^4 - \left(\frac{k\pi}{a}\right)^2\frac{H_6}{H_7}\Lambda^2 + \left(\frac{k\pi}{a}\right)^4\frac{H_5}{H_7}\right\} = 0\\
&\lambda^6 - \left(\frac{k\pi}{b}\right)^2\frac{H_1}{H_2}\lambda^4 + \left(\frac{k\pi}{b}\right)^4\frac{H_3}{H_1}\lambda^2 - \left(\frac{k\pi}{b}\right)^6\frac{H_4}{H_1}\\
&\quad + \frac{H_5}{H_1}\left\{\lambda^4 - \left(\frac{k\pi}{b}\right)^2\frac{H_6}{H_5}\lambda^2 + \left(\frac{k\pi}{b}\right)^4\frac{H_7}{H_5}\right\} = 0
\end{aligned} \tag{3-25}$$

여기서, 식(3-25)의 근을 구해서 식(3-24)에 대입하면 $f_1(y)$, $f_2(x)$가 얻어진다. 따라서, $f_3(y)$, $f_4(x)$도 얻게 된다.

다음은 이 특성방정식(3-25)의 근을 구하는 방법을 설명한다.

먼저 식(3-25)에 식(3-19)를 대입하여 정리하면 다음 식과 같이 나타낼 수 있다.

$$\begin{aligned}
&\left[\Lambda^2 - \left(\frac{k\pi}{a}\right)^2\frac{G_{31}}{G_{23}}\right]\left[\Lambda^4 - \left(\frac{k\pi}{a}\right)^2\frac{d_{11}d_{22} - d_{12}(d_{12}+2G_{12})}{G_{12}d_{22}}\Lambda^2 + \left(\frac{k\pi}{a}\right)^4\frac{d_{11}}{d_{22}}\right]\\
&- \frac{10}{h^2}\frac{G_{31}}{G_{12}}\left[\Lambda^4 - \left(\frac{k\pi}{a}\right)^2\frac{2(d_{12}+2G_{12})}{d_{22}}\Lambda^2 + \left(\frac{k\pi}{a}\right)^4\frac{d_{11}}{d_{22}}\right] = 0\\
&\left[\lambda^2 - \left(\frac{k\pi}{b}\right)^2\frac{G_{23}}{G_{31}}\right]\left[\lambda^4 - \left(\frac{k\pi}{b}\right)^2\frac{d_{11}d_{22} - d_{12}(d_{12}+2G_{12})}{G_{12}d_{11}}\lambda^2 + \left(\frac{k\pi}{b}\right)^4\frac{d_{22}}{d_{11}}\right]\\
&- \frac{10}{h^2}\frac{G_{23}}{G_{12}}\left[\lambda^4 - \left(\frac{k\pi}{b}\right)^2\frac{2(d_{12}+2G_{12})}{d_{11}}\lambda^2 + \left(\frac{k\pi}{b}\right)^4\frac{d_{22}}{d_{11}}\right] = 0
\end{aligned} \tag{3-26}$$

여기서, 참고문헌[17]에서 설명한 바와 같이 $G_{12}$의 근사값으로서 $d_{11}$, $d_{12}$ 및 $d_{22}$로 나타내면 다음과 같이 쓸 수 있다.

$$G_{12} = \frac{1}{2}\left(\sqrt{d_{11}d_{22}} - d_{12}\right) \tag{3-27}$$

식(3-27)을 식(3-26)에 대입하면 식(3-26)은 다음과 같이 된다.

$$\begin{aligned}
&\left[\Lambda^2 - \left(\frac{k\pi}{a}\right)^2\sqrt{\frac{d_{11}}{d_{22}}}\right]^2\left[\Lambda^2 - \left\{\left(\frac{k\pi}{a}\right)^2\frac{G_{31}}{G_{23}} + \frac{10}{h^2}\frac{G_{31}}{G_{12}}\right\}\right] = 0\\
&\left[\lambda^2 - \left(\frac{k\pi}{b}\right)^2\sqrt{\frac{d_{22}}{d_{11}}}\right]^2\left[\lambda^2 - \left\{\left(\frac{k\pi}{b}\right)^2\frac{G_{23}}{G_{31}} + \frac{10}{h^2}\frac{G_{23}}{G_{12}}\right\}\right] = 0
\end{aligned} \tag{3-28}$$

따라서, 식(3-28)의 6개의 근은 각각 다음과 같이 된다.

$$\Lambda = \pm\left(\frac{d_{11}}{d_{22}}\right)^{\frac{1}{4}} \frac{k\pi}{a} \qquad (\text{각각, 重根})$$

$$\pm\left\{\frac{G_{31}}{G_{23}}\left(\frac{k\pi}{a}\right)^2 + \frac{10}{h^2}\frac{G_{31}}{G_{12}}\right\}^{\frac{1}{2}}$$

$$\lambda = \pm\left(\frac{d_{22}}{d_{11}}\right)^{\frac{1}{4}} \frac{k\pi}{b} \qquad (\text{각각, 重根})$$

$$\pm\left\{\frac{G_{23}}{G_{31}}\left(\frac{k\pi}{b}\right)^2 + \frac{10}{h^2}\frac{G_{23}}{G_{12}}\right\}^{\frac{1}{2}} \tag{3-29}$$

따라서, 함수 $f_1(y)$, $f_2(x)$는 다음과 같이 된다.

$$f_1(y) = (A_1' + A_2' y)\sinh\left\{\left(\frac{d_{11}}{d_{22}}\right)^{\frac{1}{4}} \frac{k\pi}{a} y\right\}$$
$$+ (A_3' + A_4' y)\cosh\left\{\left(\frac{d_{11}}{d_{22}}\right)^{\frac{1}{4}} \frac{k\pi}{a} y\right\}$$
$$+ A_5'\sinh\left[\left\{\frac{G_{31}}{G_{23}}\left(\frac{k\pi}{a}\right)^2 + \frac{10}{h^2}\frac{G_{31}}{G_{12}}\right\}^{\frac{1}{2}} y\right]$$
$$+ A_6'\cosh\left[\left\{\frac{G_{31}}{G_{23}}\left(\frac{k\pi}{a}\right)^2 + \frac{10}{h^2}\frac{G_{31}}{G_{12}}\right\}^{\frac{1}{2}} y\right]$$

$$f_2(x) = (B_1' + B_2' x)\sinh\left\{\left(\frac{d_{22}}{d_{11}}\right)^{\frac{1}{4}} \frac{k\pi}{b} x\right\}$$
$$+ (B_3' + B_4' x)\cosh\left\{\left(\frac{d_{22}}{d_{11}}\right)^{\frac{1}{4}} \frac{k\pi}{b} x\right\}$$
$$+ B_5'\sinh\left[\left\{\frac{G_{23}}{G_{31}}\left(\frac{k\pi}{b}\right)^2 + \frac{10}{h^2}\frac{G_{23}}{G_{12}}\right\}^{\frac{1}{2}} x\right]$$
$$+ B_6'\cosh\left[\left\{\frac{G_{23}}{G_{31}}\left(\frac{k\pi}{b}\right)^2 + \frac{10}{h^2}\frac{G_{23}}{G_{12}}\right\}^{\frac{1}{2}} y\right] \tag{3-30}$$

해함수 $\Phi$는 일반적으로 다음 식과 같이 된다.

$$\Phi_k = \left[(A_1' + A_2' y)\sinh\left(\Lambda_1 \frac{k\pi}{a} y\right) + (A_3' + A_4' y)\cosh\left(\Lambda_1 \frac{k\pi}{a} y\right)\right.$$
$$\left. + A_5'\sinh(\Lambda_{2k} y) + A_6'\cosh(\Lambda_{2k} y)\right]\cos\frac{k\pi}{a} x$$
$$+ \left[(B_1' + B_2' x)\sinh\left(\lambda_1 \frac{k\pi}{b} x\right) + (B_3' + B_4' x)\cosh\left(\lambda_1 \frac{k\pi}{b} x\right)\right.$$
$$\left. + B_5'\sinh(\lambda_{2k} x) + B_6'\cosh(\lambda_{2k} x)\right]\cos\frac{k\pi}{b} y$$
$$+ \left[(C_1' + C_2' y)\sinh\left(\Lambda_1 \frac{k\pi}{a} y\right) + (C_3' + C_4' y)\cosh\left(\Lambda_1 \frac{k\pi}{a} y\right)\right.$$
$$\left. + C_5'\sinh(\Lambda_{2k} y) + C_6'\cosh(\lambda_{2k} y)\right]\sin\frac{k\pi}{a} x$$
$$+ \left[(D_1' + D_2' x)\sinh\left(\lambda_1 \frac{k\pi}{b} x\right) + (D_3' + D_4' x)\cosh\left(\lambda_1 \frac{k\pi}{b} x\right)\right.$$
$$\left. + D_5'\sinh(\lambda_{2k} x) + D_6'\cosh(\lambda_{2k} x)\right]\sin\frac{k\pi}{b} y \tag{3-31}$$

식(3-31)에서 대칭성을 고려하면 $\Phi_k$는 결국 다음과 같은 식으로 나타난다.

$$\Phi_k = \left[A_2' y\sinh\left(\Lambda_1 \frac{k\pi}{a} y\right) + A_3'\cosh\left(\Lambda_1 \frac{k\pi}{a} y\right)\right.$$
$$\left. + A_6'\cosh(\Lambda_{2k} y)\right]\cos\frac{k\pi}{a} x$$
$$+ \left[B_2' x\sinh\left(\lambda_1 \frac{k\pi}{b} x\right) + B_3'\cosh\left(\lambda_1 \frac{k\pi}{b} x\right)\right.$$
$$\left. + B_6'\cosh(\Lambda_{2k} x)\right]\cos\frac{k\pi}{b} y \tag{3-32}$$

식(3-32)를 다음 식과 같이 쓸 수 있다.

$$\Phi_k = \left[a_k y\sinh\left(\Lambda_1 \frac{k\pi}{a} y\right) + b_k\cosh\left(\Lambda_1 \frac{k\pi}{a} y\right)\right.$$
$$\left. + c_k\cosh(\Lambda_{2k} y)\right]\cos\frac{k\pi}{a} x$$
$$+ \left[d_k x\sinh\left(\lambda_1 \frac{k\pi}{b} x\right) + e_k\cosh\left(\lambda_1 \frac{k\pi}{b} x\right)\right.$$
$$\left. + f_k\cosh(\lambda_{2k} x)\right]\cos\frac{k\pi}{b} y \tag{3-33}$$

여기서,

$$\Lambda_1 = \left(\frac{d_{11}}{d_{22}}\right)^{\frac{1}{4}}$$

$$\Lambda_{2k} = \left\{\frac{G_{31}}{G_{23}}\left(\frac{k\pi}{a}\right)^2 + \frac{10}{h^2}\frac{G_{31}}{G_{12}}\right\}^{\frac{1}{2}}$$

$$\lambda_1 = \left(\frac{d_{22}}{d_{11}}\right)^{\frac{1}{4}}$$

$$\lambda_{2k} = \left\{\frac{G_{23}}{G_{31}}\left(\frac{k\pi}{b}\right)^2 + \frac{10}{h^2}\frac{G_{23}}{G_{12}}\right\}^{\frac{1}{2}} \tag{3-34}$$

또한, $a_k$, $b_k$, $c_k$, $d_k$, $e_k$ 및 $f_k$는 상수이다.

다음, 식(3-33)을 이용하면 식(3-17)의 $w^c$, $Q_x^c$ 및 $Q_y^c$는 다음과 같이 된다.

$$w^c = \sum_{k=1,3,\cdots}^{\infty}\left[\left\{t_{1k} y\sinh\left(\Lambda_1 \frac{k\pi}{a} y\right) + t_{2k}\cosh\left(\Lambda_1 \frac{k\pi}{a} y\right)\right\}a_k\right.$$
$$\left. + \left\{t_{1k}\cosh\left(\Lambda_1 \frac{k\pi}{a} y\right)\right\}b_k + \{t_{3k}\cosh(\Lambda_{2k} y)\}c_k\right]\cos\frac{k\pi}{a} x$$

$$+\sum_{k=1,3,\cdots}^{\infty}\left[\left\{t_{4k}x\sinh\left(\lambda_1\frac{k\pi}{b}x\right)+t_{5k}\cosh\left(\lambda_1\frac{k\pi}{b}x\right)\right\}d_k\right.$$
$$\left.+\left\{t_{4k}\cosh\left(\lambda_1\frac{k\pi}{b}x\right)\right\}e_k+\{t_{6k}\cosh(\lambda_{2k}x)\}f_k\right]\cos\frac{k\pi}{b}y \tag{3-35}$$

여기서,

$$t_{1k}=\left(\frac{k\pi}{a}\right)^4\{A_xB_y-\Lambda_1^2(A_xA_y+B_xB_y)+\Lambda_1^4A_yB_x\}$$
$$+\left(\frac{k\pi}{a}\right)^2\{(A_x+B_y)-\Lambda_1^2(A_y+B_x)\}+1$$
$$t_{2k}=\left(\frac{k\pi}{a}\right)^3\{-2\Lambda_1(A_xA_y+B_xB_y)+4\Lambda_1^3A_yB_x\}-2\Lambda_1\left(\frac{k\pi}{a}\right)(A_y+B_x)$$
$$t_{3k}=\left(\frac{k\pi}{a}\right)^4A_xB_y-\Lambda_{2k}^2\left(\frac{k\pi}{a}\right)^2(A_xA_y+B_xB_y)+\Lambda_{2k}^4A_yB_x$$
$$+\left(\frac{k\pi}{a}\right)^2(A_x+B_y)-\Lambda_{2k}^2(A_y+B_x)+1$$
$$t_{4k}=\left(\frac{k\pi}{b}\right)^4\{\lambda_1^4A_xB_y-\lambda_1^2(A_xA_y+B_xB_y)+A_yB_x\}$$
$$+\left(\frac{k\pi}{b}\right)^2\{-\lambda_1^2(A_x+B_y)+(A_y+B_x)\}+1$$
$$t_{5k}=\left(\frac{k\pi}{b}\right)^3\{4\lambda_1^3A_xB_y-2\lambda_1(A_xA_y+B_xB_y)\}-2\lambda_1\left(\frac{k\pi}{b}\right)(A_x+B_y)$$
$$t_{6k}=\left(\frac{k\pi}{b}\right)^4A_yB_x-\lambda_{2k}^2\left(\frac{k\pi}{b}\right)^2(A_xA_y+B_xB_y)+\lambda_{2k}^4A_xB_y$$
$$+\left(\frac{k\pi}{b}\right)^2(A_y+B_x)-\lambda_{2k}^2(A_x+B_y)+1 \tag{3-36}$$

$$Q_x^c=\sum_{k=1,3,\cdots}^{\infty}\left[\left\{g_{1k}y\sinh\left(\Lambda_1\frac{k\pi}{a}y\right)+g_{2k}\cosh\left(\Lambda_1\frac{k\pi}{a}y\right)\right\}a_k\right.$$
$$\left.+\left\{g_{1k}\cosh\left(\Lambda_1\frac{k\pi}{a}y\right)\right\}b_k+\{g_{3k}\cosh(\Lambda_{2k}y)\}c_k\right]\sin\frac{k\pi}{a}x$$
$$+\sum_{k=1,3,\cdots}^{\infty}\left[\left\{g_{4k}x\cosh\left(\lambda_1\frac{k\pi}{b}x\right)+g_{5k}\sinh\left(\lambda_1\frac{k\pi}{b}x\right)\right\}d_k\right.$$
$$\left.+\left\{g_{4k}\sinh\left(\lambda_1\frac{k\pi}{b}x\right)\right\}e_k+\{g_{6k}\sinh(\lambda_{2k}x)\}f_k\right]\cos\frac{k\pi}{b}y \tag{3-37}$$

여기서,

$$g_{1k}=D_x\left(\frac{k\pi}{a}\right)^3\left[\left(\frac{k\pi}{a}\right)^2\{-B_y+\Lambda_1^2(A_y+E_xB_y)\right.$$
$$\left.-\Lambda_1^4E_xA_y\}-(1-\Lambda_1^2E_x)\right]$$
$$g_{2k}=D_x2\Lambda_1\left(\frac{k\pi}{a}\right)^2\left[\left(\frac{k\pi}{a}\right)^2\{(A_y+E_xB_y)\right.$$
$$\left.-2\Lambda_1^2E_xA_y\}+E_x\right]$$
$$g_{3k}=D_x\left(\frac{k\pi}{a}\right)\left[-\left(\frac{k\pi}{a}\right)^4B_y+\left(\frac{k\pi}{a}\right)^2\Lambda_{2k}^2(A_y+E_xB_y)\right.$$
$$\left.-\Lambda_{2k}^4E_xA_y-\left(\frac{k\pi}{a}\right)^2+\Lambda_{2k}^2E_x\right]$$
$$g_{4k}=D_x\lambda_1\left(\frac{k\pi}{b}\right)^3\left[\left(\frac{k\pi}{b}\right)^2\{\lambda_1^4B_y-\lambda_1^2(A_y+E_xB_y)\right.$$
$$\left.+E_xA_y\}-(\lambda_1^2-E_x)\right]$$
$$g_{5k}=D_x\left(\frac{k\pi}{b}\right)^2\left[\left(\frac{k\pi}{b}\right)^2(5\lambda_1^4B_y+E_xA_y)\right.$$
$$\left.-3\lambda_1\left(\frac{k\pi}{b}\right)(A_y+E_xB_y)-(3\lambda_1^2-E_x)\right]$$
$$g_{6k}=D_x\lambda_{2k}\left[\lambda_{2k}^4B_y-\left(\frac{k\pi}{b}\right)^2\lambda_{2k}^2(A_y+E_xB_y)\right.$$
$$\left.+\left(\frac{k\pi}{b}\right)^4E_xA_y-\lambda_{2k}^2+\left(\frac{k\pi}{b}\right)^2E_x\right] \tag{3-38}$$

$$Q_y^c=\sum_{k=1,3,\cdots}^{\infty}\left[\left\{h_{1k}y\cosh\left(\Lambda_1\frac{k\pi}{a}y\right)\right.\right.$$
$$\left.+h_{2k}\sinh\left(\Lambda_1\frac{k\pi}{a}y\right)\right\}a_k$$
$$+\left\{h_{1k}\sinh\left(\Lambda_1\frac{k\pi}{a}y\right)\right\}b_k$$
$$\left.+\{h_{3k}\sinh(\Lambda_{2k}y)\}c_k\right]\cos\frac{k\pi}{a}x$$
$$+\sum_{k=1,3,\cdots}^{\infty}\left[\left\{h_{4k}x\sinh\left(\lambda_1\frac{k\pi}{b}x\right)\right.\right.$$
$$\left.+h_{5k}\cosh\left(\lambda_1\frac{k\pi}{b}x\right)\right\}d_k$$
$$+\left\{h_{4k}\cosh\left(\lambda_1\frac{k\pi}{b}x\right)\right\}e_k$$
$$\left.+\{h_{6k}\cosh(\lambda_{2k}x)\}f_k\right]\sin\frac{k\pi}{b}y \tag{3-39}$$

여기서,

$$h_{1k}=D_y\Lambda_1\left(\frac{k\pi}{a}\right)^3\left[\left(\frac{k\pi}{a}\right)^2\{\Lambda_1^4B_x-\Lambda_1^2(A_x+E_yB_x)\right.$$
$$\left.+E_yA_x\}-(\Lambda_1^2-E_y)\right]$$
$$h_{2k}=D_y\left(\frac{k\pi}{a}\right)^2\left[\left(\frac{k\pi}{a}\right)^2\{5\lambda_1^4B_x+E_yA_x\}\right.$$
$$\left.-3\Lambda_1\left(\frac{k\pi}{a}\right)(A_x+E_yB_x)-(3\Lambda_1^2-E_y)\right]$$
$$h_{3k}=D_y\Lambda_{2k}\left[\Lambda_{2k}^4B_x-\left(\frac{k\pi}{a}\right)^2\Lambda_{2k}^2(A_x+E_yB_x)\right.$$
$$\left.+\left(\frac{k\pi}{a}\right)^4E_yA_x-\Lambda_{2k}^2+\left(\frac{k\pi}{a}\right)^2E_y\right]$$
$$h_{4k}=D_y\left(\frac{k\pi}{b}\right)^3\left[\left(\frac{k\pi}{b}\right)^2\{-B_x+\lambda_1^2(A_x+E_yB_x)\right.$$
$$\left.-\lambda_1^4E_yA_x\}-(1-\lambda_1^2E_y)\right]$$
$$h_{5k}=D_y2\lambda_1\left(\frac{k\pi}{b}\right)^2\left[\left(\frac{k\pi}{b}\right)^2\{(A_x+E_yB_x)\right.$$
$$\left.-2\lambda_1^2E_yA_x\}+E_y\right]$$
$$h_{6k}=D_y\left(\frac{k\pi}{b}\right)\left[-\left(\frac{k\pi}{b}\right)^4B_x+\left(\frac{k\pi}{b}\right)^2\lambda_{2k}^2(A_x+E_yB_x)\right.$$
$$\left.-\lambda_{2k}^4E_yA_x-\left(\frac{k\pi}{b}\right)^2+\lambda_{2k}^2E_y\right] \tag{3-40}$$

다음으로, 식(3-35), (3-37) 그리고 식(3-39)를 이용하면 식(3-16)의 휨모멘트 $M_x^c$, $M_y^c$ 및 비틀림모멘트 $M_{xy}^c$는 다음과 같이 된다.

$$M_x^c=\sum_{k=1,3,\cdots}^{\infty}\Big[\Big\{p_{1k}y\sinh\Big(\Lambda_1\frac{k\pi}{a}y\Big)+p_{2k}\cosh\Big(\Lambda_1\frac{k\pi}{a}y\Big)\Big\}a_k+\Big\{p_{1k}\cosh\Big(\Lambda_1\frac{k\pi}{a}y\Big)\Big\}b_k+\{p_{3k}\cosh(\Lambda_{2k}y)\}c_k\Big]\cos\frac{k\pi}{a}x$$
$$+\sum_{k=1,3,\cdots}^{\infty}\Big[\Big\{p_{4k}x\sinh\Big(\lambda_1\frac{k\pi}{b}x\Big)+p_{5k}\cosh\Big(\lambda_1\frac{k\pi}{b}x\Big)\Big\}d_k+\Big\{p_{4k}\cosh\Big(\lambda_1\frac{k\pi}{b}x\Big)\Big\}e_k+\{p_{6k}\cosh(\lambda_{2k}x)\}f_k\Big]\cos\frac{k\pi}{b}y \quad (3\text{-}41)$$

여기서,

$$p_{1k}=D_x\Big(\frac{k\pi}{a}\Big)\Big\{\Big(\frac{k\pi}{a}\Big)(1-\Lambda_1^2F_x)t_{1k}+G_xg_{1k}\Big\}$$
$$p_{2k}=D_x\Big(\frac{k\pi}{a}\Big)\Big\{-2\Lambda_1t_{1k}+\Big(\frac{k\pi}{a}\Big)(1-\Lambda_1^2F_x)t_{2k}+G_xg_{2k}\Big\}$$
$$p_{3k}=D_x\Big(\frac{k\pi}{a}\Big)\Big\{\Big(\frac{k\pi}{a}\Big)(1-\Lambda_{2k}^2F_x)t_{3k}+G_xg_{3k}\Big\}$$
$$p_{4k}=D_x\Big(\frac{k\pi}{b}\Big)\Big\{-\Big(\frac{k\pi}{b}\Big)(\lambda_1^2-F_x)t_{4k}+G_x\lambda_1g_{4k}\Big\}$$
$$p_{5k}=D_x\Big\{-2\lambda_1\Big(\frac{k\pi}{b}\Big)t_{4k}-\Big(\frac{k\pi}{b}\Big)^2(\lambda_1^2-F_x)t_{5k}+G_x\Big(\lambda_1\frac{k\pi}{b}g_{5k}+g_{4k}\Big)\Big\}$$
$$p_{6k}=D_x\Big\{-\Big(\frac{k\pi}{b}\Big)^2(\lambda_{2k}^2-F_x)t_{6k}+G_x\lambda_{2k}g_{6k}\Big\} \quad (3\text{-}42)$$

$$M_y^c=\sum_{k=1,3,\cdots}^{\infty}\Big[\Big\{q_{1k}y\sinh\Big(\Lambda_1\frac{k\pi}{a}y\Big)+q_{2k}\cosh\Big(\Lambda_1\frac{k\pi}{a}y\Big)\Big\}a_k+\Big\{q_{1k}\cosh\Big(\Lambda_1\frac{k\pi}{a}y\Big)\Big\}b_k+\{q_{3k}\cosh(\Lambda_{2k}y)\}c_k\Big]\cos\frac{k\pi}{a}x$$
$$+\sum_{k=1,3,\cdots}^{\infty}\Big[\Big\{q_{4k}x\sinh\Big(\lambda_1\frac{k\pi}{b}x\Big)+q_{5k}\cosh\Big(\lambda_1\frac{k\pi}{b}x\Big)\Big\}d_k+\Big\{q_{4k}\cosh\Big(\lambda_1\frac{k\pi}{b}x\Big)\Big\}e_k+\{q_{6k}\cosh(\lambda_{2k}x)\}f_k\Big]\cos\frac{k\pi}{b}y \quad (3\text{-}43)$$

여기서,

$$q_{1k}=D_y\Big(\frac{k\pi}{a}\Big)\Big\{-\Big(\frac{k\pi}{a}\Big)(\Lambda_1^2-F_y)t_{1k}+G_y\Lambda_1h_{1k}\Big\}$$
$$q_{2k}=D_y\Big\{-2\Lambda_1\Big(\frac{k\pi}{a}\Big)t_{1k}-\Big(\frac{k\pi}{a}\Big)^2(\Lambda_1^2-F_y)t_{2k}+G_y\Big(\Lambda_1\frac{k\pi}{a}h_{2k}+h_{1k}\Big)\Big\}$$
$$q_{3k}=D_y\Big\{-\Big(\frac{k\pi}{a}\Big)^2(\Lambda_{2k}^2-F_y)t_{3k}+G_y\Lambda_{2k}h_{3k}\Big\}$$
$$q_{4k}=D_y\Big(\frac{k\pi}{b}\Big)\Big\{\Big(\frac{k\pi}{b}\Big)(1-\lambda_1^2F_y)t_{4k}+G_yh_{4k}\Big\}$$
$$q_{5k}=D_y\Big(\frac{k\pi}{b}\Big)\Big\{-2\lambda_1t_{4k}+\Big(\frac{k\pi}{b}\Big)(1-\lambda_1^2F_y)t_{5k}+G_yh_{5k}\Big\}$$
$$q_{6k}=D_y\Big(\frac{k\pi}{b}\Big)\Big\{\Big(\frac{k\pi}{b}\Big)(1-\lambda_{2k}^2F_y)t_{6k}+G_yh_{6k}\Big\} \quad (3\text{-}44)$$

$$M_{xy}^c=\sum_{k=1,3,\cdots}^{\infty}\Big[\Big\{l_{1k}y\cosh\Big(\Lambda_1\frac{k\pi}{a}y\Big)+l_{2k}\sinh\Big(\Lambda_1\frac{k\pi}{a}y\Big)\Big\}a_k+\Big\{l_{1k}\sinh\Big(\Lambda_1\frac{k\pi}{a}y\Big)\Big\}b_k+\{l_{3k}\sinh(\Lambda_{2k}y)\}c_k\Big]\sin\frac{k\pi}{a}x$$
$$+\sum_{k=1,3,\cdots}^{\infty}\Big[\Big\{l_{4k}x\cosh\Big(\lambda_1\frac{k\pi}{b}x\Big)+l_{5k}\sinh\Big(\lambda_1\frac{k\pi}{b}x\Big)\Big\}d_k+\Big\{l_{4k}\sinh\Big(\lambda_1\frac{k\pi}{b}x\Big)\Big\}e_k+\{l_{6k}\sinh(\lambda_{2k}x)\}f_k\Big]\sin\frac{k\pi}{b}y \quad (3\text{-}45)$$

여기서,

$$l_{1k}=-\Big(\frac{k\pi}{a}\Big)\Big\{\Lambda_1\Big(\frac{k\pi}{a}\Big)It_{1k}+J_xD_yh_{1k}-\Lambda_1J_yD_xg_{1k}\Big\}$$
$$l_{2k}=-\Big(\frac{k\pi}{a}\Big)I\Big\{t_{1k}+\Lambda_1\Big(\frac{k\pi}{a}\Big)t_{2k}\Big\}-\Big(\frac{k\pi}{a}\Big)J_xD_yh_{2k}+J_yD_x\Big\{g_{1k}+\Lambda_1\Big(\frac{k\pi}{a}\Big)g_{2k}\Big\}$$
$$l_{3k}=-\Lambda_{2k}\Big(\frac{k\pi}{a}\Big)It_{3k}-\Big(\frac{k\pi}{a}\Big)J_xD_yh_{3k}+\Lambda_{2k}J_yD_xg_{3k}$$
$$l_{4k}=-\Big(\frac{k\pi}{b}\Big)\Big\{\lambda_1\Big(\frac{k\pi}{b}\Big)It_{4k}+J_yD_xg_{4k}-\lambda_1J_xD_yh_{4k}\Big\}$$
$$l_{5k}=-\Big(\frac{k\pi}{b}\Big)I\Big\{t_{4k}+\lambda_1\Big(\frac{k\pi}{b}\Big)t_{5k}\Big\}-\Big(\frac{k\pi}{b}\Big)J_yD_xg_{5k}+J_xD_y\Big\{h_{4k}+\lambda_1\Big(\frac{k\pi}{b}\Big)h_{5k}\Big\}$$
$$l_{6k}=-\lambda_{2k}\Big(\frac{k\pi}{b}\Big)It_{6k}-\Big(\frac{k\pi}{b}\Big)J_yD_xg_{6k}+\lambda_{2k}J_xD_yh_{6k} \quad (3\text{-}46)$$

다음, 계수 $a_k$, $b_k$, $c_k$, $d_k$, $e_k$ 및 $f_k$를 구한다. 식(3-35), (3-41) 및 (3-43)에 경계조건식(3-14)의 제 1, 2식 및 제 4, 5식을 이용하면 다음과 같은 4개의 식을 얻을 수 있다.

$$\sum_{k=1,3,\cdots}^{\infty}(\alpha_{1k}a_k+\alpha_{2k}b_k+\alpha_{3k}c_k)=0$$
$$\sum_{k=1,3,\cdots}^{\infty}(\alpha_{4k}d_k+\alpha_{5k}e_k+\alpha_{6k}f_k)=0$$
$$\sum_{k=1,3,\cdots}^{\infty}(\beta_{1k}a_k+\beta_{2k}b_k+\beta_{3k}c_k)=0$$

$$\sum_{k=1,3,\cdots}^{\infty}(\beta_{4k}d_k+\beta_{5k}e_k+\beta_{6k}f_k)=0 \qquad (3\text{-}47)$$

여기서,

$$\alpha_{1k}=t_{1k}\left(\frac{b}{2}\right)\sinh\left(\Lambda_1\frac{k\pi b}{2a}\right)+t_{2k}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)$$
$$\alpha_{2k}=t_{1k}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)$$
$$\alpha_{3k}=t_{3k}\cosh\left(\Lambda_{2k}\frac{b}{2}\right)$$
$$\alpha_{4k}=t_{4k}\left(\frac{a}{2}\right)\sinh\left(\lambda_1\frac{k\pi a}{2b}\right)+t_{5k}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)$$
$$\alpha_{5k}=t_{4k}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)$$
$$\alpha_{6k}=t_{6k}\cosh\left(\lambda_{2k}\frac{a}{2}\right)$$
$$\beta_{1k}=q_{1k}\left(\frac{b}{2}\right)\sinh\left(\Lambda_1\frac{k\pi b}{2a}\right)+q_{2k}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)$$
$$\beta_{2k}=q_{1k}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)$$
$$\beta_{3k}=q_{3k}\cosh\left(\Lambda_{2k}\frac{b}{2}\right)$$
$$\beta_{4k}=p_{4k}\left(\frac{a}{2}\right)\sinh\left(\lambda_1\frac{k\pi a}{2b}\right)+p_{5k}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)$$
$$\beta_{5k}=p_{4k}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)$$
$$\beta_{6k}=p_{6k}\cosh\left(\lambda_{2k}\frac{a}{2}\right) \qquad (3\text{-}48)$$

또, 식(3-45)에 경계조건식 (3-14)의 제 3, 6식을 이용하여 나머지 2개의 식을 구해야 한다.

따라서, 식(3-45)에 경계조건식(3-14)의 제 3, 6식을 이용해서 정리하면 다음과 같이 된다.

먼저 $x=a/2$의 경우에는,

$$\sum_m\sum_n(-1)^{\frac{m+1}{2}}M^p_{xymn}\sin\frac{n\pi y}{b}$$
$$+\sum_k\left[\left\{l_{4k}\left(\frac{a}{2}\right)\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)+l_{5k}\sinh\left(\lambda_1\frac{k\pi a}{2b}\right)\right\}d_k\right.$$
$$\left.+\left\{l_{4k}\sinh\left(\lambda_1\frac{k\pi a}{2b}\right)\right\}e_k+\left\{l_{6k}\sinh\left(\lambda_{2k}\frac{a}{2}\right)\right\}f_k\right]\sin\frac{k\pi y}{b}$$
$$=\sum_k(-1)^{\frac{k+1}{2}}\left[\left\{l_{1k}y\cosh\left(\Lambda_1\frac{k\pi y}{a}\right)\right.\right.$$
$$\left.+l_{2k}\sinh\left(\Lambda_1\frac{k\pi y}{a}\right)\right\}a_k+\left\{l_{1k}\sinh\left(\Lambda_1\frac{k\pi y}{a}\right)\right\}b_k$$
$$\left.+\{l_{3k}\sinh(\Lambda_{2k}y)\}c_k\right]$$
$$(m,n,k=1,3,\cdots,\infty) \qquad (3\text{-}49)$$

로 되고, $y=b/2$의 경우에는 다음과 같이 된다.

$$\sum_m\sum_n(-1)^{\frac{n-1}{2}}M^p_{xymn}\sin\frac{m\pi x}{a}$$
$$+\sum_k\left[\left\{l_{1k}\left(\frac{b}{2}\right)\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)+l_{2k}\sinh\left(\Lambda_1\frac{k\pi b}{2a}\right)\right\}a_k\right.$$
$$+\left\{l_{1k}\sinh\left(\Lambda_1\frac{k\pi b}{2a}\right)\right\}b_k$$
$$\left.+\left\{l_{3k}\sinh\left(\Lambda_{2k}\frac{b}{2}\right)\right\}c_k\right]\sin\frac{k\pi x}{a}$$
$$=\sum_k(-1)^{\frac{k+1}{2}}\left[\left\{l_{4k}x\cosh\left(\lambda_1\frac{k\pi x}{b}\right)\right.\right.$$
$$\left.+l_{5k}\sinh\left(\lambda_1\frac{k\pi x}{b}\right)\right\}d_k$$
$$\left.+\left\{l_{4k}\sinh\left(\lambda_1\frac{k\pi x}{b}\right)\right\}e_k+\{l_{6k}\sinh(\lambda_{2k}x)\}f_k\right]$$
$$(m,n,k=1,3,\cdots,\infty) \qquad (3\text{-}50)$$

여기서, 식(3-49), (3-50)의 좌변은 각각 $y$, $x$에 대한 sine의 무한급수임과 동시에, 우변 같은 변수의 쌍곡선함수이다.

그런데 이들 우변의 쌍곡선함수와 sine만을 포함하는 삼각함수로 Fourier급수 전개하면 식(3-49)의 우변은 다음과 같이 얻어진다.

$$우변=\sum_{\gamma=1,3,\cdots}^{\infty}\left[\left[(-1)^{\frac{\gamma-1}{2}}\sin\frac{\gamma\pi y}{b}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}\right.\right.$$
$$\cdot\left\langle\left[l_{1k}\frac{b}{\pi^2}\left\{\frac{\left(\frac{\gamma}{2}\right)^2-\left(\Lambda_1\frac{kb}{2a}\right)^2}{\left[\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{\gamma}{2}\right)^2\right]^2}\right.\right.\right.$$
$$\left.\cdot\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)+\left(\Lambda_1\frac{k\pi b}{2a}\right)\frac{\sinh\left(\Lambda_1\frac{k\pi b}{2a}\right)}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right\}$$
$$\left.+l_{2k}\frac{\Lambda_1kb}{\pi a}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)\frac{1}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right]a_k$$
$$+\left[l_{1k}\frac{\Lambda_1kb}{\pi a}\frac{\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right]b_k$$
$$\left.\left.\left.+\left[l_{3k}\frac{b}{\pi^2}\Lambda_{2k}\frac{\cosh\left(\Lambda_{2k}\frac{b}{2}\right)}{\left(\Lambda_{2k}\frac{b}{2\pi}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right]c_k\right\rangle\right]\right]$$
$$(3\text{-}51)$$

그리고, 식(3-50)의 우변은 다음과 같이 얻어진다.

$$좌변=\sum_{\gamma=1,3,\cdots}^{\infty}\left[\left[(-1)^{\frac{\gamma-1}{2}}\sin\frac{\gamma\pi x}{a}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}\right.\right.$$

$$\cdot\left\langle\left[l_{4k}\frac{a}{\pi^2}\left\{\frac{\left(\frac{\gamma}{2}\right)^2-\left(\lambda_1\frac{ka}{2b}\right)^2}{\left[\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{\gamma}{2}\right)^2\right]^2}\right.\right.\right.$$

$$\left.\cdot\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)+\left(\lambda_1\frac{k\pi a}{2b}\right)\frac{\sinh\left(\lambda_1\frac{k\pi a}{2b}\right)}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right\}$$

$$\left.+\ l_{5k}\frac{\lambda_1 ka}{\pi b}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)\frac{1}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right]d_k$$

$$+\left[l_{4k}\frac{\lambda_1 ka}{\pi b}\frac{\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right]e_k$$

$$\left.\left.+\left[l_{6k}\frac{a}{\pi^2}\lambda_{2k}\frac{\cosh\left(\lambda_{2k}\frac{a}{2}\right)}{\left(\lambda_{2k}\frac{a}{2\pi}\right)^2+\left(\frac{\gamma}{2}\right)^2}\right]f_k\right\rangle\right]\right] \qquad (3\text{-}52)$$

윗 식(3-51), (3-52)로부터 $\sin\frac{n\pi y}{b}$ 와 $\sin\frac{m\pi x}{a}$ 의 계수를 비교하기 위하여 식(3-51)은 좌변의 $k$를 $n$으로, 우변의 $\gamma$는 $n$으로, 또 식(3-52)는 좌변의 $k$를 $m$으로, 우변의 $\gamma$는 $m$으로 두면 식(3-51)과 식(3-52)는 다음과 같이 된다.

식(3-51)로부터, 다음 식을 얻을 수 있다.

$$\sum_m\sum_n(-1)^{\frac{m-1}{2}}M^p_{xymn}+\sum_n\left[\left\{l_{4n}\left(\frac{a}{2}\right)\cosh\left(\lambda_1\frac{n\pi a}{2b}\right)\right.\right.$$

$$\left.+\ l_{5n}\sinh\left(\lambda_1\frac{n\pi a}{2b}\right)\right\}d_n$$

$$\left.+\left\{l_{4n}\sinh\left(\lambda_1\frac{n\pi a}{2b}\right)\right\}e_n+\left\{l_{6n}\sinh\left(\lambda_{2n}\frac{a}{2}\right)\right\}f_n\right]$$

$$=\sum_n\left[\left[(-1)^{\frac{n-1}{2}}\sum_k(-1)^{\frac{k+1}{2}}\right.\right.$$

$$\cdot\left\langle\left[l_{1k}\frac{b}{\pi^2}\left\{\frac{\left(\frac{n}{2}\right)^2-\left(\lambda_1\frac{kb}{2a}\right)^2}{\left[\left(\lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2\right]^2}\cosh\left(\lambda_1\frac{k\pi b}{2a}\right)\right.\right.\right.$$

$$\left.+\left(\Lambda_1\frac{k\pi b}{2a}\right)\frac{\sinh\left(\Lambda_1\frac{k\pi b}{2a}\right)}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2}\right\}$$

$$\left.+\ l_{2k}\frac{\Lambda_1 kb}{\pi a}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)\frac{1}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2}\right]a_k$$

$$+\left[l_{1k}\frac{\Lambda_1 kb}{\pi a}\frac{\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2}\right]b_k$$

$$\left.\left.+\left[l_{3k}\frac{b}{\pi^2}\Lambda_{2k}\frac{\cosh\left(\Lambda_{2k}\frac{b}{2}\right)}{\left(\Lambda_{2k}\frac{b}{2\pi}\right)^2+\left(\frac{n}{2}\right)^2}\right]c_k\right\rangle\right]\right] \qquad (3\text{-}53)$$

또한 식(3-52)로부터, 다음 식을 얻을 수 있다.

$$\sum_m\sum_n(-1)^{\frac{n-1}{2}}M^p_{xymn}$$

$$+\sum_k\left[\left\{l_{1m}\left(\frac{a}{2}\right)\cosh\left(\Lambda_1\frac{m\pi b}{2a}\right)+l_{2m}\sinh\left(\Lambda_1\frac{m\pi b}{2a}\right)\right\}a_m\right.$$

$$\left.+\left\{l_{1m}\sinh\left(\Lambda_1\frac{m\pi b}{2a}\right)\right\}b_m+\left\{l_{3m}\sinh\left(\Lambda_{2m}\frac{b}{2}\right)\right\}c_m\right]$$

$$=\sum_m\left[\left[(-1)^{\frac{m-1}{2}}\sum_k(-1)^{\frac{k+1}{2}}\right.\right.$$

$$\cdot\left\langle\left[l_{4k}\frac{a}{\pi^2}\left\{\frac{\left(\frac{m}{2}\right)^2-\left(\lambda_1\frac{ka}{2b}\right)^2}{\left[\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2\right]^2}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)\right.\right.\right.$$

$$\left.+\left(\lambda_1\frac{k\pi a}{2b}\right)\frac{\sinh\left(\lambda_1\frac{k\pi a}{2b}\right)}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2}\right\}$$

$$\left.+\ l_{5k}\frac{\lambda_1 ka}{\pi b}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)\frac{1}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2}\right]d_k$$

$$+\left[l_{4k}\frac{\lambda_1 ka}{\pi b}\frac{\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2}\right]e_k$$

$$\left.\left.+\left[l_{6k}\frac{a}{\pi^2}\lambda_{2k}\frac{\cosh\left(\lambda_{2k}\frac{a}{2}\right)}{\left(\lambda_{2k}\frac{a}{2\pi}\right)^2+\left(\frac{m}{2}\right)^2}\right]f_k\right\rangle\right]\right] \qquad (3\text{-}54)$$

식(3-53), (3-54)를 간단화 하기 위하여 다음과 같이 두면 편리하다.

$$\sum_n\{\sum_k(\gamma_{1kn}a_k+\gamma_{2kn}b_k+\gamma_{3kn}c_k)+\gamma_{4n}d_n+\gamma_{5n}e_n$$

$$+\gamma_{6n}f_n\}=\sum_n\sum_m\gamma_{mn}$$

$$\sum_m\{\delta_{1m}a_m+\delta_{2m}b_m+\delta_{3m}c_m+\sum_k(\delta_{4km}d_k+\delta_{5km}e_k$$

$$+\delta_{6km}f_k)\}=\sum_n\sum_m\delta_{mn}$$

$$(m,n=1,3,\cdots,\infty) \qquad (3\text{-}55)$$

여기서,

$$\gamma_{1kn}=(-1)^{\frac{n+1}{2}}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}$$

$$\cdot\left[l_{1k}\frac{b}{\pi^2}\left\{\frac{\left(\frac{n}{2}\right)^2-\left(\Lambda_1\frac{kb}{2a}\right)^2}{\left[\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2\right]^2}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)\right.\right.$$

$$\left.+\Lambda_1\frac{k\pi b}{2a}\frac{\sinh\left(\Lambda_1\frac{k\pi b}{2a}\right)}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2}\right\}$$

$$\left.+\ l_{2k}\frac{\Lambda_1}{\pi}\frac{kb}{a}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)\frac{1}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2}\right]$$

$$\gamma_{2kn}=(-1)^{\frac{n+1}{2}}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}$$

$$\cdot\left[l_{1k}\frac{\Lambda_1}{\pi}\frac{kb}{a}\cosh\left(\Lambda_1\frac{k\pi b}{2a}\right)\frac{1}{\left(\Lambda_1\frac{kb}{2a}\right)^2+\left(\frac{n}{2}\right)^2}\right]$$

$$\gamma_{3kn}=(-1)^{\frac{n+1}{2}}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}$$

$$\cdot\left[l_{3k}\frac{b}{\pi^2}\Lambda_{2k}\cosh\left(\Lambda_{2k}\frac{b}{2}\right)\frac{1}{\left(\Lambda_{2k}\frac{b}{2\pi}\right)^2+\left(\frac{n}{2}\right)^2}\right]$$

$$\gamma_{4n}=l_{4n}\left(\frac{a}{2}\right)\cosh\left(\lambda_1\frac{n\pi a}{2b}\right)+l_{5n}\sinh\left(\lambda_1\frac{n\pi a}{2b}\right)$$

$$\gamma_{5n}=l_{4n}\sinh\left(\lambda_1\frac{n\pi a}{2b}\right)$$

$$\gamma_{6n}=l_{6n}\sinh\left(\lambda_{2n}\frac{a}{2}\right)$$

$$\gamma_{mn}=(-1)^{\frac{m+1}{2}}M^p_{xymn}$$

$$\delta_{1m}=l_{1m}\left(\frac{b}{2}\right)\cosh\left(\Lambda_1\frac{m\pi b}{2a}\right)+l_{2m}\sinh\left(\Lambda_1\frac{m\pi b}{2a}\right)$$

$$\delta_{2m}=l_{1m}\sinh\left(\Lambda_1\frac{m\pi b}{2a}\right)$$

$$\delta_{3m}=l_{3m}\sinh\left(\Lambda_{2m}\frac{b}{2}\right)$$

$$\delta_{4km}=(-1)^{\frac{m+1}{2}}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}$$

$$\cdot\left[l_{4k}\frac{a}{\pi^2}\left\{\frac{\left(\frac{m}{2}\right)^2-\left(\lambda_1\frac{ka}{2b}\right)^2}{\left[\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2\right]^2}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)\right.\right.$$

$$\left.+\lambda_1\frac{k\pi a}{2b}\frac{\sinh\left(\lambda_1\frac{k\pi a}{2b}\right)}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2}\right\}$$

$$\left.+\ l_{5k}\frac{\lambda_1}{\pi}\frac{ka}{b}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)\frac{1}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2}\right]$$

$$\delta_{5km}=(-1)^{\frac{m+1}{2}}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}$$

$$\cdot\left[l_{4k}\frac{\lambda_1}{\pi}\frac{ka}{b}\cosh\left(\lambda_1\frac{k\pi a}{2b}\right)\frac{1}{\left(\lambda_1\frac{ka}{2b}\right)^2+\left(\frac{m}{2}\right)^2}\right]$$

$$\delta_{6km}=(-1)^{\frac{m+1}{2}}\sum_{k=1,3,\cdots}^{\infty}(-1)^{\frac{k+1}{2}}$$

$$\cdot\left[l_{6k}\frac{a}{\pi^2}\lambda_{2k}\cosh\left(\lambda_{2k}\frac{a}{2}\right)\frac{1}{\left(\lambda_{2k}\frac{a}{2\pi}\right)^2+\left(\frac{m}{2}\right)^2}\right]$$

$$\delta_{mn}=(-1)^{\frac{n+1}{2}}M^p_{xymn} \qquad (3\text{-}56)$$

다음 식(3-47), (3-55)로부터 계수 $a_k$, $b_k$, $c_k$, $d_k$, $e_k$ 그리고 $f_k$를 구하여 식(3-33)에 대입함으로서 해함수 $\Phi$가 얻어짐과 동시에 처짐 $w^c$, 전단력 $Q^c_x$, $Q^c_y$ , 휨모멘트 $M^c_x$, $M^c_y$, 비틀림모멘트 $M^c_{xy}$를 구할 수 있다.

이상으로 특해와 여해가 얻어졌기 때문에, 식(3-6)에서 $w$, $Q_x$, $Q_y$를 얻을 수 있고, 식(3-7)에서는 $M_x$, $M_y$, $M_{xy}$를 구할 수 있다.

## 4. 결 론

본 연구는 직교이방성 장방형 후판의 이론해석을 탄성이론과 수학적 해석방법을 적용하여 기초방정식을 유도하고, 유도된 기초방정식의 일반해를, 해함수 $\Phi$를 가정하여 새로운 수학적 이론을 적용하고 정밀한 이론해를 구하는데 성공한 셈이다. 수식은 다소 복잡하나 컴퓨터 프로그램으로 쉽게 결과를 얻을 수 있다. 다음 호에 계속되는 논문에는 수치해석 결과를 제시할 예정이다.

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