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COMMON FIXED POINT THEOREMS FOR MANN TYPE ITERATIONS

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ABSTRACT In this paper, we give some common fixed point theorems for five and six mappings satisfying the Mann-type iteration in Banach spaces We improve some results of Gornicki and Rhoades, Khan and Imdad, Cho, Fisher and Kang, Ciric and many others

Introduction and Preliminaries

Let $(X, \|\cdot\|)$ be a Banach space and F be a mapping from a nonempty closed convex subset C of X into itself Let I denote the identity mapping. If F is nonexpansive, i.e.

$$\|Fx - Fy\| \leq \|x - y\|$$

for all $x, y \in C$, then Krasnoselskii [21] proved that, for some $x_0 \in C$, the sequence $\{F^n x_0\}$ does not converge necessarily to a fixed point of F, whereas the sequence $\{F_\lambda^n x_0\}$, where

$$(*) \qquad F_\lambda = (1-\lambda)I + \lambda F, \qquad 0 < \lambda \leq 1,$$

may converge to a fixed point of F as shown by Krasnoselskii [21] which assumed that $\lambda = \frac{1}{2}$, X is uniformly convex and C is compact subset of X. Schaefer [32], extended this result for a general number λ.

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The scheme $(*)$ has been extended by the so called "Mann iterative process" [22] associated with F, which is described in the following way :

$$(**) \qquad x_{n+1} = (1 - c_n)x_n + c_n F x_n$$

for $n = 0, 1, 2, \ldots$, where $\{c_n\}$ is a sequence of real numbers such that

$$0 < c_n \leq 1 \quad and \quad \sum_{n=0}^{\infty} c_n = \pm\infty.$$

The scheme $(**)$ has been studied by many authors [1],[2],[5]-[8],[11], [14],[15],[17],[23]-[25] and [27]-[31].

In this paper, we show that a sequence in C defined by the Mann-type iterations converges to a unique common fixed point of five and six mappings on C, satisfying some conditions. Our results extend and improve some results of Gornicki and Rhoades [10], Iseki [12],[13], Khan and Imdad [18]-[20], Rehman and Ahmad [26], Rhoades [29]-[31], Cho, Fisher and Kang [3]

In [16], Jungck defined the concept of compatibility of two mappings which inculdes weakly commuting mappings as a proper subclass.

DEFINITION. Let A and S be two mappings from a normed linear space $(X, \|\cdot\|)$ into itself. The mappings A snd S are said to be *compatible* if

$$\lim_{n\to\infty} \|ASx_n - SAx_n\| = 0$$

where $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = z$$

for some $z \in X$.

LEMMA 1 [16] *Let A and S be compatible mappings of a normed linear space $(X, \|\cdot\|)$ into itself. If $Az = Sz$ for some $z \in X$, then*

$$ASz = S^2 z = SAz = A^2 z.$$

Main Results

THEORM 1. *Let C be a nonempty closed convex subset of a Banach space $(X, \|\cdot\|)$ and A, B, S, T and P be mappings from C into itself satisfying the following conditions:*

(1.1) *there exist constants $\alpha, \beta, \gamma, \delta \geq 0$ such that*

$$\begin{aligned}\|Px - Py\| \leq \alpha\|ABx - STy\| &+ \beta\|ABx - Px\| \\ &+ \gamma max\{\|STy - Py\|, \|ABx - Py\|\} \\ &+ \delta\|STy - Px\|\end{aligned}$$

for all $x, y \in C$, where $0 \leq \alpha + \gamma + \delta < 1$ and $0 \leq \gamma < 1$,

(1.2) *for some $x_0 \in C$, there exists a constant $k \in [0,1)$ such that*

$$\|x_{n+2} - x_{n+1}\| \leq k\|x_{n+1} - x_n\|$$

for $n = 0, 1, 2, \ldots$ where $\{x_n\}$ is a sequence in C defined by

(1.3) $ABx_{2n+1} = \frac{1}{2}Px_{2n} + \frac{1}{2}ABx_{2n}$, $STx_{2n+2} = \frac{1}{2}Px_{2n+1} + \frac{1}{2}STx_{2n+1}$,

(1.4) *the pairs $\{P, AB\}$ and $\{P, ST\}$ are compatible,*

(1.5) $PB = BP$, $PT = TP$, $AB = BA$, $ST = TS$,

(1.6) *A, B, S and T are continuous at $z \in C$.*

Then the sequence $\{x_n\}$ defined by (1.3) converges to $z \in C$ and Pz is a unique common fixed point of A, B, S, T and P.

PROOF. From (1.2), it follows that

$$\|x_{n+2} - x_{n+1}\| \leq k^{n+1}\|x_1 - x_0\|,$$

for $n = 0, 1, 2, \ldots$ and so $\{x_n\}$ is a Cauchy sequence in C. Since C is closed subspace of a complete space X, it is also complete and hence the sequence $\{x_n\}$ converges to a point $z \in C$.

We will prove that Pz is a unique common fixed point of A, B, S, T and P.

From (1.3), it follows that

$$\frac{1}{2}Px_{2n} = ABx_{2n+1} - \frac{1}{2}ABx_{2n}$$

and since A and B are continuous at z, we have

$$\lim_{n\to\infty} ABx_n = \lim_{n\to\infty} Px_{2n} = ABz.$$

Similarly, we also have

$$\lim_{n\to\infty} STx_n = \lim_{n\to\infty} Px_{2n+1} = STz.$$

By (1.1), we have

$$\begin{aligned}\|Px_{2n} - Px_{2n+1}\| \leq \alpha\|ABx_{2n} - STx_{2n+1}\| + \beta\|ABx_{2n} - Px_{2n}\| \\ + \gamma max\{\|STx_{2n+1} - Px_{2n+1}\|, \|ABx_{2n} - Px_{2n+1}\|\} \\ + \delta\|STx_{2n+1} - Px_{2n}\|.\end{aligned}$$

This implies that, as $n \to \infty$

$$\begin{aligned}\|ABz - STz\| &\leq \alpha\|ABz - STz\| + \beta\|ABz - ABz\| \\ &+ \gamma max\{\|STz - STz\|, \|ABz - STz\|\} \\ &+ \delta\|STz - ABz\| \\ &= (\alpha + \gamma + \delta)\|ABz - STz\|,\end{aligned}$$

which implies that $ABz = STz$ since $0 \leq \alpha + \gamma + \delta < 1$.
By (1.1), we have

$$\begin{aligned}\|Px_{2n} - Pz\| &\leq \alpha\|ABx_{2n} - STz\| + \beta\|ABx_{2n} - P_x 2n\| \\ &+ \gamma max\{\|STz - Pz\|, \|ABx_{2n} - Pz\|\} \\ &+ \delta\|STz - Px_{2n}\|.\end{aligned}$$

This implies that, as $n \to \infty$

$$\begin{aligned}\|ABz - Pz\| &\leq \alpha\|ABz - STz\| + \beta\|ABz - ABz\| \\ &+ \gamma max\{\|STz - Pz\|, \|ABz - Pz\|\} + \delta\|STz - ABz\| \\ &= \gamma\|ABz - Pz\|,\end{aligned}$$

which implies that $ABz = Pz$ since $0 \leq \gamma < 1$. Combining the above results, we have

(1.7) $\quad ABz = STz = Pz.$

Since the pair $\{P, AB\}$ is compatible and $ABz = Pz$ for some $z \in X$, then by Lemma 1, we obtain

(1.8) $\quad (AB)Pz = P^2z.$

From (1.1), (1.7) and (1.8), it follows that

$$\begin{aligned}\|P^2z - Pz\| &\leq \alpha\|AB(Pz) - STz\| + \beta\|AB(Pz) - P^2z\| \\ &+ \gamma max\{\|STz - Pz\|, \|AB(Pz) - Pz\|\} \\ &+ \delta\|STz - P^2z\| \\ &= (\alpha + \gamma + \delta)\|P^2z - Pz\|,\end{aligned}$$

which implies that $P^2z = Pz$ since $0 \leq \alpha + \gamma + \delta < 1$.

On the other hand, from (1.1), (1.5) and (1.7) it follows that

$$\begin{aligned}\|PBz - Pz\| &\leq \alpha\|AB(Bz) - STz\| + \beta\|AB(Bz) - PBz\| \\ &+ \gamma max\{\|STz - Pz\|, \|AB(Bz) - Pz\|\} \\ &+ \delta\|STz - PBz\| \\ &\leq (\alpha + \gamma + \delta)\|BPz - PZ\|,\end{aligned}$$

which implies that $BPz = Pz$ since $0 \leq \alpha + \gamma + \delta < 1$.

By (1.8), we have $AB(Pz) = P^2z$. Therefore, $APz = Pz$.

Since the pair $\{P, ST\}$ is compatible and $Pz = STz$ for some $z \in X$, then again by Lemma 1, we obtain

(1.9) $\quad ST(Pz) = P^2z.$

From (1.1), (1.5) and (1.7), it follows that

$$\begin{aligned}\|Pz - PTz\| &\leq \alpha\|ABz - ST(Tz)\| + \beta\|ABz - Pz\| \\ &+ \gamma max\{\|ST(Tz) - PTz\|, \|ABz - PTz\|\} \\ &+ \delta\|ST(Tz) - Pz\| \\ &\leq (\alpha + \gamma + \delta)\|TPz - Pz\|,\end{aligned}$$

which implies that $TPz = Pz$ since $0 \leq \alpha + \gamma + \delta < 1$.

By (1 9), we have $ST(Pz) = P^2z$. Therefore, $SPz = Pz$. Combining the above results we obtain

$$APz = BPz = SPz = TPz = P^2z = Pz.$$

Therefore, Pz is a common fixed point of A, B, S, T and P.

The uniqueness of the common fixed point Pz follows easily from (1.1). This completes the proof.

If we put $B = T = I$(the identity mapping on C) in Theorem 1, we obtain the following:

COROLLARY 1. *Let C be a nonempty closed convex subset of a Banach space $(X, \|\cdot\|)$ and A, S and P be mappings from C into itself satisfying the following conditions:*

(i) *there exist constants $\alpha, \beta, \gamma, \delta \leq 0$, such that*

$$\|Px - Py\| \leq \alpha\|Az - Sy\| + \beta\|Ax - Px\| + \gamma max\{\|Sy - Py\|, \|Ax - Py\|\} + \delta\|Sy - Px\|$$

for all $x, y \in C$, where $0 \leq \alpha + \gamma + \delta < 1$,

(ii) *for some $x_0 \in C$, there exists a constant $k \in [0,1)$ such that*

$$\|x_{n+2} - x_{n+1}\| \leq k\|x_{n+1} - x_n\|$$

for all $n = 1, 2, 3, \ldots$, where $\{x_n\}$ is a sequence in C defined by

(iii) $Ax_{2n+1} = \frac{1}{2}Px_{2n} + \frac{1}{2}Ax_{2n}, Sx_{2n+2} = \frac{1}{2}Px_{2n+1} + \frac{1}{2}Sx_{2n+1}$,

(iv) *the pair $\{P, A\}$ and $\{P, S\}$ are compatible,*

(v) *A and S are continuous at $z \in C$.*

Then the sequence $\{x_n\}$ defined by (iii) converges to $z \in C$ and Pz is a unique common fixed point of A, S and P.

If we put $B = T = A = S = I$ in Theorem 1, we have the following result due to Gornicki and Rhoades [10].

COROLLARY 2. *Let C be a nonempty closed convex subset of a Banach space $(X, \|\cdot\|)$ and P be a mapping from C into itself satisfying the following conditions.*

(vi) *there exist constants $\alpha, \beta, \gamma, \delta \geq 0$, $0 \leq \gamma < 1$ such that*

$$\|Px - Py\| \leq \alpha\|x - y\| + \beta\|x - Px\| + \gamma\max\{\|y - Py\|, \|x - Px\|\} + \delta\|y - Px\|$$

for all $x, y \in C$.

(vii) *for some $x_0 \in C$, there exists a constant $k \in [0,1)$ such that*

$$\|x_{n+2} - x_{n+1}\| \leq k\|x_{n+1} - x_n\|$$

for $n = 0, 1, 2, \ldots$, *where* $\{x_n\}$ *is a sequence in* C *defined by*

(viii) $x_{n+1} = \frac{1}{2}Px_n + \frac{1}{2}x_n$.

Then the sequence $\{x_n\}$ *defined by (viii) converges to a point* $z \in C$ *and* z *is a unique fixed point of* P.

From Corollary 2, we have the following result due to Ciric [4].

COROLLARY 3 *Let* C *be a nonempty closed convex subset of a Banach space* $(X, \|\cdot\|)$ *and* P *be a mapping from* C *into itself satisfying the following condition:*
there exists a constant $k \in [0, 1)$ *such that*

$$\|Px-Py\| \leq k \max\{\|x-y\|, \frac{1}{2}\|x-Py\|, \frac{1}{2}\|y-Py\|, \frac{1}{2}\|x-Px\|, \frac{1}{2}\|y-Px\|\}$$

for all $x, y \in C$ *and*

$$\left(\frac{k}{2}\right)^{\alpha} \|x - y\| \leq k\|P^2x - y\| \leq \left(\frac{k}{2}\right)^{\beta} \|x - y\|$$

for all $x \in C$ *and* $y \in \{Fx, Px, PFx\}$ *where* $Fx = \frac{1}{2}(x+Px)$ *and* $0 \leq \beta \leq \alpha < 1$. *Then* P *has a unique fixed point in* C.

REMARK 1 *Theorem 1 contains some results as special cases, i.e. Corollary 3 contains Theorem 1 of Goebel and Zlotkiwicz [19] theorems of Iseki [12], [13]. Theorem 2.1 of Khan and Imdad [19].*

If we replace the condition (1.4) in Theorem 1. by the following condition:

(1.10) $AB = P = I$ and $ST = P = I$,

we obtain the following.

COROLLARY 4 *Let* C *be a nonempty closed convex subset of a Banach space* $(X, \|\cdot\|)$ *and* A, B, S, T *and* P *be mappings from* C *into itself satisfying the conditions (1.1), (1.2), (1.3), (1.5), (1.6) and (1.10). Then the sequence* $\{x_n\}$ *defined by (1.3) converges to a point* $z \in C$ *and* z *is a unique common fixed point of* A, B, S, T *and* P.

REMARK 2. *Corollary 4, improves results of Gornicki and Rhoades [10], Khan and Imdad [19], Rehman and Ahmad [26].*

REMARK 3 *In Theorem 1, if we replace conditions (1.4) and (1.6) by the following conditions.*

(1.11) $\|x - ABx\| \geq \|x - STx\|$, *for all* $x \in X$

(1.12) A *and* B *are continuous,*

(1.13) *the pair* $\{P, AB\}$ *is compatible.*

Then Theorem 1, is still true.

By using the Theorem 1, we have the following:

THEOREM 2 *Let* C *be a nonempty closed convex subset of a Banach space* $(X, \|\cdot\|)$ *and* A, B, S, T *and* $\{P_i\}_{i\in\Lambda}$ *be mappings from* C *into itself satisfying conditions (1.2) and (1.6) of Theorem 1 and the following conditions.*

(2 1) *there exist constants* $\alpha, \beta, \gamma, \delta \geq 0$ *such that*

$$\|P_i x - P_i y\| \leq \alpha\|ABx - STy\| + \beta\|ABx - P_i x\| + \gamma\max\{\|STy - P_i y\|, \|ABx - P_i y\|\} + \delta\|STy - P_i x\|$$

for all $x, y \in C$, *for all* $i \in \Lambda$ *where* Λ *is an index set,* $0 \leq \alpha+\gamma+\delta < 1$ *and* $0 \leq \gamma < 1$, *a sequence* $\{x_n\}$ *in* C *is defined by*

(2.2) $ABx_{2n+1} = \frac{1}{2}P_i x_{2n} + \frac{1}{2}ABx_{2n}$,
$STx_{2n+2} = \frac{1}{2}P_i x_{2n+1} + \frac{1}{2}STx_{2n+1}$

for all $i \in \Lambda$,

(2.3) *for all* $i \in \Lambda$, *the pairs* $\{P_i, AB\}$ *and* $\{P_i, ST\}$ *are compatible,*

(2.4) *for all* $i \in \Lambda$, $P_i B = BP_i$, $P_i T = TP_i$, $AB = BA$, $ST = TS$.

Then the sequence $\{x_n\}$ *defined by (2.2) converges to* $z \in C$ *and* $P_i z$ *for all* $i \in \Lambda$ *is a unique common fixed point of* A, B, S, T *and* $\{P_i\}_{i\in\Lambda}$.

PROOF. The proof of Theorem 2 is similar to that of Theorem 1.

Now, we extend Theorem 1, for six mappings. We prove the following:

THEOREM 3. *Let C be a nonempty closed convex subset of a Banach space $(X, \|\cdot\|)$ and A, B, S, T, P and Q be mappings from C into itself satisfying conditions (1.2), (1.6) of Theorem 1 and the following conditions:*

(3.1) *there exist constants $\alpha, \beta, \gamma, \delta \geq 0$ such that*

$$\|Px - Qy\| \leq \alpha\|ABx - STy\| + \beta\|ABx - Px\| \\ + \gamma \max \|STy - Qy\|, \|ABx - Qy\|\} + \delta\|STy - Px\|$$

for all $x, y \in C$, where $0 \leq \max\{\alpha + \gamma + \delta, \beta + \delta\} < 1$ and $0 \leq \gamma < 1$, a sequence $\{x_n\}$ in C is defined by

(3.2) $ABx_{2n+1} = \frac{1}{2}Px_{2n} + \frac{1}{2}ABx_{2n}$,
$STx_{2n+2} = \frac{1}{2}Qx_{2n+1} + \frac{1}{2}STx_{2n+1}$,

(3.3) *the pairs $\{P, AB\}$ and $\{Q, ST\}$ are compatible,*

(3.4) $PB = BP, AB = BA, ST = TS, TQ = QT$.

Then the sequence $\{x_n\}$ defined by (3.2) converges to a point $z \in C$ and Qz is a unique common fixed point of A, B, S, T, P and Q

PROOF. From (1.2) it is clear that $\{x_n\}$ is a Cauchy sequence in C. Since C is closed subspace of a complete space X, it is also complete and hence the sequence $\{x_n\}$ converges to a point $z \in C$. We will prove that Qz is a unique common fixed point of A, B, S, T, P and Q. From (3.2) it follows that

$$\frac{1}{2}Px_{2n} = ABx_{2n+1} - \frac{1}{2}ABx_{2n}$$

and since A and B are continuous at z, we have

$$\lim_{n \to \infty} ABx_n = \lim_{n \to \infty} Px_{2n} = ABz.$$

Similary, we also have

$$\lim_{n \to \infty} STx_n = \lim_{n \to \infty} Qx_{2n+1} = STz.$$

By (3.1), we have

$$\begin{aligned}\|Px_{2n} - Qx_{2n+1}\| \le \alpha\|ABx_{2n} - STx_{2n+1}\| &+ \beta\|ABx_{2n} - Px_{2n}\| \\ &+ \gamma\max\{\|STx_{2n+1} - Qx_{2n+1}\|, \|ABx_{2n} - Qx_{2n+1}\|\} \\ &+ \delta\|STx_{2n+1} - Px_{2n}\|.\end{aligned}$$

This implies that, as $n \to \infty$

$$\|ABz - STz\| \le (\alpha + \gamma + \delta)\|ABz - STz\|,$$

which implies that $ABz = STz$ since $0 \le \alpha + \gamma + \delta < 1$.

By (3.1), we have

$$\begin{aligned}\|Px_{2n} - Qz\| \le \alpha\|ABx_{2n} - STz\| &+ \beta\|ABx_{2n} - Px_{2n}\| \\ &+ \gamma\max\{\|STz - Qz\|, \|ABx_{2n} - Qz\|\} \\ &+ \delta\|STz - Px_{2n}\|.\end{aligned}$$

This implies that, as $n \to \infty$

$$\|ABz - Qz\| \le \gamma\|ABz - Qz\|,$$

which implies that $ABz = Qz$ since $0 \le \gamma < 1$.

Again by (3.1), we have

$$\begin{aligned}\|Pz - Qx_{2n+1}\| \le \alpha\|ABz - STx_{2n+1}\| &+ \beta\|ABz - Pz\| \\ &+ \gamma\max\{\|STx_{2n+1} - Qx_{2n+1}\|, \|ABz - Qx_{2n+1}\|\} \\ &+ \delta\|STx_{2n+1} - Pz\|.\end{aligned}$$

This implies that, as $n \to \infty$

$$\|Pz - Qz\| \le (\beta + \delta)\|Pz - Qz\|,$$

which implies that $Pz = Qz$ since $0 \le \beta + \delta < 1$. Combining the results we have

(3.5) $\quad ABz = STz = Pz = Qz.$

Since $\{P, AB\}$ is compatible and $ABz = Pz$ for some $z \in X$, then by Lemma 1, we obtain

(3.6) $(AB)Pz = P^2z.$

Similarly,

(3.7) $(ST)Qz = Q^2z.$

From (3.1), (3.5) and (3.6), it follows that

$$\|P^2z - Qz\| \leq (\alpha + \gamma + \delta)\|P^2z - Qz\|,$$

which implies that $P^2z = PQz = Qz$, since $0 \leq \alpha + \beta + \gamma < 1$. By (3.1), (3.4) and (3.5), we have

$$\|PBz - Qz\| \leq (\alpha + \gamma + \delta)\|PBz - Qz\|.$$

Since $0 \leq \alpha + \gamma + \delta < 1$, therefore, we have $BPz = BQz = Qz$.

By (3.6), we have $(AB)Pz = P^2z$. Therefore, $AQz = Qz$.

From (3.1), (3.5), (3.7), we have

$$\|Pz - Q^2z\| \leq (\alpha + \gamma + \delta)\|Pz - Q^2z\|.$$

Since $0 \leq \alpha + \gamma + \delta < 1$,therefore, we have $Q^2z = Pz = Qz$.

Finally from (3.1), (3.4) and (3.5), it follows that

$$\|Pz - QTz\| \leq (\alpha + \gamma + \delta)\|TQz - Pz\|,$$

which implies that $TQz = Pz = Qz$, since $0 \leq \alpha + \gamma + \delta < 1$.

By (3.7), we have $(ST)Qz = Q^2z$. Therefore, we have $SQz = Qz$.

Combining the above results we obtain

$$AQz = BQz = SQz = TQz = PQz = Q^2z = Qz.$$

Therefore, Qz is a common fixed point of A, B, S, T, P and Q. The uniqueness of the common fixed point Qz follows easily from (3.1). This completes the proof.

In Theorem 3, if we put $B = T = I$ (the identity map on C) we obtain the following result due to Cho, Fisher and Kang [3].

COROLLARY 5 *Let C be a nonempty closed convex subset of a Banach space $(X, \|\cdot\|)$ and A, S, P and Q be the mappings from C into itself satisfying the following conditions :*

(1) *there exists constants $\alpha, \beta, \gamma, \delta \geq 0$ such that*

$$\|Px - Qy\| \leq \alpha\|Ax - Sy\| + \beta\|Ax - Px\| \\ + \gamma max\{\|Sy - Qy\|, \|Ax - Qy\|\} + \delta\|Sy - Px\|.$$

for all $x, y \in C$, where $0 \leq max\{\alpha + \gamma + \delta, \beta + \delta\} < 1$, $0 \leq \gamma < 1$,

(2) *for some $x_0 \in C$, there exists a constant $k \in [0, 1)$ such that*

$$\|x_{n+2} - x_{n+1}\| \leq k\|x_{n+1} - x_n\|$$

for $n = 0, 1, 2, \ldots,$ where $\{x_n\}$ is a sequence in C defined by

(3) $Ax_{2n+1} = \frac{1}{2}Px_{2n} + \frac{1}{2}Ax_{2n}, Sx_{2n+2} = \frac{1}{2}Qx_{2n+1} + \frac{1}{2}Sx_{2n+1},$

(4) *the pairs $\{P, A\}$ and $\{Q, S\}$ are compatible,*

(5) *A and S are continuous at the point $z \in C$.*

Then the sequence $\{x_n\}$ defined by (3) converges to a point $z \in C$ and Qz is a unique common fixed point of A, S, P and Q.

Remark 3. If we put $P = Q$ in Theorem 3, it reduces to Theorem 1.

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