East Asian Math J 17(2001), No 2, pp 297-301

CORRECTION TO EXISTENCE OF SOLUTION FOR GENERALIZED MULTIVALUED VECTOR VARIATIONAL INEQUALITIES WITHOUT CONVEXITY

MEE-KWANG KANG

In [3], the authors proved the following main result using Bardaro and Ceppitelli's generalization [1] of F-KKM Theorem in [2], for H-KKM multifunctions on H-spaces.

THEOREM *Let $(X, \{\Gamma_K\})$ be an H-Banach space, and $\{C(x) . x \in X\}$ be a family of closed pointed convex cones with nonempty interior $int\, C(x)$ in ordered Banach spaces Y.*

Assume that

1^0. *$A : L(X,Y) \to L(X,Y)$ is a continuous mapping.*

2^0. *$T : X \to 2^{L(X,Y)}$ be a compact valued, continuous multivalued mapping, where $L(X,Y)$ is equipped with the weak topology.*

3^0. *The multivalued mapping $W(x) = Y \setminus \{-int\, C(x)\}$ is upper semicontinuous.*

4^0. *For each $y \in X$, $B_y = \{x \in X : \exists w \in T(y)$ such that $\langle Aw, x - y\rangle \in -int\, C(x)\}$ is H-convex or empty.*

Received March 2, 2001 Revised October 12, 2001

2000 Mathematics Subject Classification 49J40

Key words and phrases vector variational inequalities, H-space, H-KKM mapping, upper semicontinuous, set-valued mapping, closed graphed

This work was supported by The Research Institute of Basic Science, Dongeui University (2001)

5^0. *There exists a compact set $L \subset X$ and an H-compact set $E \subset X$ such that for every weakly H-convex set D with $E \subset D \subset X$*

$$\{y \in D : \exists w \in T(y) \quad \textit{such that} \quad \langle Aw, x - y\rangle \notin -\mathit{int}\, C(x), \quad \textit{for all} \quad x \in D\} \subset L.$$

Then the following generalized multivalued vector variational inequality (**GMVVI**) is solvable;

(GMVVI) Find $x_0 \in K$ such that for each $x \in K$, there exists $s_0 \in T(x_0)$ such that

$$\langle As_0, x - x_0\rangle \notin -\text{int}\, C(x_0).$$

In this note, we point out some mistakes of the proof and correct them.

Mistake 1. Since the following two $x_0 s$ in the inequality

$$\langle As_0, x - x_0\rangle \notin -\text{int}\, C(x_0)$$

of the (**GMVVI**) are same each other, to solve the (**GMVVI**), the inequality

$$\langle Aw, x - y\rangle \notin -\text{int}\, C(x)$$

in the conditions 4^0, 5^0 and the definition of $F(x)$ in the Proof of Theorem 2 must be changed into the inequality

$$\langle Aw, x - y\rangle \notin -\text{int}\, C(y).$$

Mistake 2. In the Proof of Theorem 2, the authors explained "Since $y_n \in F(x)$ for all n, there exists $t_n \in T(y)$ such that $\cdots$".

And then they used the compactness of $T(y)$ for the fixed y to obtain the limit of the sequence $\{t_n\}$. They obviously mistook y instead of y_n. They should have taken y_n for all n, and maybe they should have used the compactness of $T(y_n)$ for each n according to the definition of $F(y)$.

Mistake 3. In the Proof of Theorem 2, the authors did not explain what the sequence $\{x_n\}$ and the point x_0 are. Maybe they mistook the sequence $\{x_n\}$ and the point x_0 instead of $\{y_n\}$ and y_0, respectively.

Now we correct the Proof of Theorem 2.1 in [3] by using the upper semicontinuity instead of the continuity of the multivalued mapping $T : X \to 2^{L(X,Y)}$ in the hypothesis of Theorem 2.1.

For our proof, we need the following lemma.

LEMMA. *Let X, Y be topological spaces and $W : X \to 2^Y$ a multivalued mapping. If Y is regular, and W is closed valued and upper semicontinuous, then the graph $G_r(W)$ of W is closed.*

PROOF Let $\{x_\alpha\}$ and $\{y_\alpha\}$ be nets in X and Y, respectively such that $x_\alpha \to x_0$, $y_\alpha \in W(x_\alpha)$ and $y_\alpha \to y_0$. Assume that $y_0 \notin W(x_0)$, then by the regularity of Y, there exist neighborhoods U and V of y_0 and $W(x_0)$ respectively such that $U \cap V = \emptyset$. Since W is upper semicontinuous, there exists a neighborhood M of x_0 such that for $x_\alpha \in M$, $W(x_\alpha) \subset V$. Hence for $x_\alpha \in M$, $W(x_\alpha) \cap U = \emptyset$, which contradicts the fact that $y_\alpha \to y_0$.

Now we show that $F(x)$ is closed for all $x \in X$ by using the upper semicontinuity of the multivalued mapping $T : X \to 2^{L(X,Y)}$ as the following Correction.

Correction. Define a set-valued mapping $F : X \to 2^X$ by, for $x \in X$

$$F(x) = \{y \in X \mid \text{there exists } w \in T(y) \text{ such that } \langle Aw, x - y\rangle \notin -\text{int } C(y)\}.$$

Let $\{y_\alpha\}$ be a net in $F(x)$ such that $y_\alpha \to y_0 \in X$. Then for each $\alpha \in I$, there exists $w_\alpha \in T(y_\alpha)$ such that

$$\langle Aw_\alpha, x - y_\alpha\rangle \notin -\text{int } C(y_\alpha).$$

Hence the net $\{w_\alpha\}$ clusters at some point $w_0 \in T(y_0)$. In fact, suppose that $\{w_\alpha\}_{\alpha\in I}$ does not cluster in $L(X, Y)$. Then $t \in T(y_0)$ is not a cluster point of the net $\{w_\alpha\}$, so that there exists an open neighborhood $U(t)$ of t such that $U(t) \cap \{w_\alpha | \alpha \in I\}$ is finite. Since $\{U(t) : t \in T(y_0)\}$ is an open cover of a compact set $T(y_0)$, there exists a finite subcover $\{U(t_i) | i = 1, 2, \cdots, m\}$ of $T(y_0)$. Since $U := \bigcup_{i=1}^{m} U(t_i)$ is a neighborhood of $T(y_0)$, by the upper semicontinuity of T at y_0, there is a neighborhood V of y_0 such that $T(V) \subset U$.

On the other hand, since $y_\alpha \to y_0$, there exists $\alpha_0 \in I$ such that $y_\alpha \in V$ for $\alpha \geq \alpha_0$. Therefore $w_\alpha \in T(y_\alpha) \subset T(V) \subset U = \bigcup_{i=1}^{m} U(t_i)$ for $\alpha \geq \alpha_0$, and so $\{w_\alpha | \alpha \in I\} \cap U(t_j)$ is infinite for some $j \in \{1, 2, \cdots, m\}$. That is a contradiction to the choice of $U(t_j)$. Thus we can choose a convergent subnet $\{w_\beta\}$ of the net $\{w_\alpha\}$, say $w_\beta \to w_0$. Without loss of generality, we can assume that $w_\alpha \to w_0$. Since $w_\alpha \in T(y_\alpha)$ and $y_\alpha \to y_0$, from the upper semicontinuity of T, $w_0 \in T(y_0)$, and from the continuity of A,

$$\langle Aw_\alpha, x - y_\alpha\rangle \to \langle Aw_0, x - y_\alpha\rangle.$$

Since Aw_0 is continuous from the weak topology of X to the weak topology of Y

$$\langle Aw_0, x - y_\alpha\rangle \to \langle Aw_0, x - y_0\rangle \text{ weakly in } Y.$$

Since

$$\langle Aw_\alpha, x - y_\alpha \rangle \notin -\mathrm{int}\, C(y_\alpha),$$

or

$$\langle Aw_\alpha, x - y_\alpha \rangle \in W(y_\alpha),$$

and W is closed-valued and upper semicontinuous, by Lemma

$$\langle Aw_0, x - y_0 \rangle \in W(y_0),$$

that is,

$$\langle Aw_0, x - y_0 \rangle \notin -\mathrm{int}\, C(y_0).$$

Therefore $y_0 \in F(x)$ and so $F(x)$ is closed for every $x \in X$.

REFERENCES

[1] C. Bardaro and R Ceppitelli, *Some further generalizations of Kanaster-Kuratowski-Mazurkiewicz theorem and minimax inequalities*, J. Math Anal Appl **132** (1988), 484–490

[2] K Fan, *A generalization of Tychonoff's fixed point theorem*, Math Ann **142** (1961), 305–310

[3] A H Siddiqi, R Ahmad and M F Khan, *Existence of solution for generalized multivalued vector variational inequalities without convexity*, Indian J. Pure Appl Math **28(8)** (1997), 1057–1060

Department of Mathematics
Dongeui University
Pusan 614-714, Korea
E-mail: mee@hyomin.dongeui.ac.kr