Two-parameter 탄성지반위에 놓인 고차전단변형 적층판의 해석

Higher-order Shear Deformable Analysis of Laminated Plates on Two-parameter Elastic Foundations

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요　　약 : 본 연구의 주된 목적은 전단 층을 갖는 two-parameter 탄성지반 위에 놓인 복합적층판의 처짐에 관한 규명이다. 본 논문은 탄성지반에 놓인 비등방성 구조의 변형거동과 2중 조화함수를 이용한 3차 전단변형이론의 확장에 초점을 두고있다. 유도된 식들을 검증하기 위해 Timoshenko의 탄성지반 위에 놓인 단순지지 된 등방성판과 LUSAS프로그램에 의한 이방성판의 처짐과 비교하였으며 본 연구의 결과들은 등방성판과 이방성판의 결과와 매우 정확히 일치함을 알 수 있었다. 처짐에 관한 수치해석결과들은 폭-두께 비, 형상 비, 재료 비등방성과 전단지반계수 등에 따른 효과를 보여준다.

ABSTRACT : The main purpose of this paper is to present deflections of laminated composite plates on the two-parameter foundations, that is, an elastic foundation with shear layer. This paper focuses on the deformation behaviour of anisotropic structures on elastic foundations. The third-order shear deformation theory is applied by using the double-fourier series. To validate the derived equations, the obtained displacements for simply supported isotropic and orthotropic plates on elastic foundations are compared with those of Timoshenko and LUSAS program. The results show an excellent agreement for the isotropic and orthotropic plates on the elastic foundations. Numerical results for displacements are presented to show the effects of side-to-thickness ratio, aspect ratio, material anisotropy, and shear modulus of foundations.

핵 심 용 어 : Two-parameter 탄성지반, 복합적층판, 전단지반계수, 고차전단변형 이론, Winkler 모형.

KEYWORDS : Two-parameter elastic foundation, Laminated composite plates, Shear modulus of foundations, Higher-order shear deformation theory, Winkler model.

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1. Introduction

Laterally loaded elastic laminated composite plates on an elastic foundation constitute an adequate idealization for many practical problems. For example, footings, mat foundations, foundation slabs of spillway dam, steel bearing plates on concrete, floor systems for buildings, ships and bridges, etc.

Plates support directly other structures on the foundations or are used for connection structures with underground structures. Static and dynamic analyses of plates on the elastic foundation are an important field of study in structural engineering, foundation engineering, and the part of system control.

Generally, the analysis of bending of beams on an elastic foundation is developed on the assumption that the reaction forces of the foundation are proportional at every point to the displacement of the beam at that point. This assumption was first introduced by Winkler [(1)] and formed the basis of Zimmermann's classical work [(2)] on the analysis of railroad track. It has been shown by Föppl's classical experiment [(3)] and Hentenyi's analytical work [(4)] that Winkler's assumption, in spite of its simplicity, leads to satisfactory results in stress analysis of beams on an elastic foundation.

On the other hand, by means of the hypothesis of isotropic, linearly elastic semi-infinite space, the physical properties of a natural foundation can be correctly described.

To bridge the gap between these two extreme cases, interactions between Winkler's springs were considered by several authors. Hetenyi [(4,5)] treated the problems of beams or plates on an elastic foundation by assuming a continuous beam or plate imbedded in the material of foundation which is itself without any continuity. Pasternak [(6)] assumed that the shear interactions exist between the springs. He solved a large number of problems involving beam and plates on elastic foundations.

Establishing more realistic foundation models, and developing simplified methods for analyzing complex structures which taking into account the elasticity of the foundations are among the modern trends in the theory of structures on elastic foundations.

The present investigation is concerned about the application of analytical method using double-Fourier series and investigate higher-order shear deformation theory to foundation effects of layered anisotropic laminated composite plates. Our models are presented for equations governing simply supported, rectangular, anti-symmetric angle-ply, cross-ply plates under uniform loadings on multi-layers laminated plates of a fiber-reinforced material.

In the present paper, an analytical method is introduced to analyze the problems of laminated composite plates on an elastic foundation. The foundation is based on a two-parameter elastic foundation model proposed by Pasternak [6]. Numerical results for displacements are presented to show the effect of side-to-thickness ratio, material anisotropy. Comparison of the result by first-order shear deformation theory and by the classical theory are made.

2. Foundation Model

2-1 One-parameter Foundation Model

It is assumed that the intensity of the reaction F_e at any point of the bottom plate is proportional to the displacement u_3 at that point by Winkler's one-parameter foundation model so that $F_e = ku_3$, in which k is the modulus of the foundation. [1]

2-2 Two-parameter Foundation Model

Laminated Plates resting on two-parameter elastic foundation are shown in Fig. 1. It is assumed that the intensity of the reaction F_e at any point of the bottom plate is proportional to the displacement u_3 at that point with effect of shear layer by Pasternak's two-parameter foundation model.
The intensity of the reaction F_e becomes.

$$F_e = ku_3 - G(\nabla^2 u_3) \quad (1)$$

where G is shear modulus of the foundation and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$.

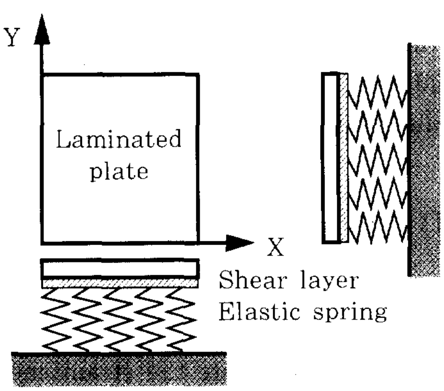

Fig. 1 Laminated plates on two-parameter elastic foundations

3. Higher-order Shear Deformable Laminated Plate Theory

The laminated plate with constant thickness h is composed of orthotropic laminae stacking symmetrically or antisymmetrically about the middle surface of plate. Rectangular cartesian coordinates(X,Y,Z) are used for the plate coordinates where the X-Y plane coincides with the middle surface of plate, as shown in Fig. 2.

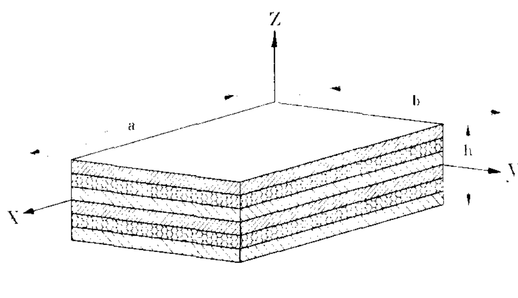

Fig. 2 Coordinate system for laminated plate

A higher-order shear deformation theory [8] is employed in this study. The displacement field is assumed to be of the form:

$$u_1 = u_o + z\left\{\phi_x - \frac{4}{3}\left(\frac{z}{h}\right)^2\left[\phi_x + \frac{\partial w_o}{\partial x}\right]\right\}$$

$$u_2 = v_o + z\left\{\phi_y - \frac{4}{3}\left(\frac{z}{h}\right)^2\left[\phi_y + \frac{\partial w_o}{\partial y}\right]\right\}$$

$$u_3 = w_o \qquad (2)$$

where (u_1, u_2, u_3) are the displacement along the (X, Y, Z) coordinates, (u_0, v_0, w_0) are the displacement of a point on the middle surface and ϕ_x and ϕ_y are the rotation at $Z=0$ of normals to the midsurface with respect to the X and Y axes, respectively. The particular choice of the displacement field in Eq. (2) is dictated by the desire to represent the transverse shear strains by quadratic functions of the thickness coordinate, Z, and by the requirement that the transverse normal strain be zero.

The strain components are related to the midplane displacements of the laminated plate as

$$\varepsilon_x = u_{1,x} = \varepsilon_x^o + z\cdot(\chi_x^o + z^2\cdot\chi_x^2)$$

$$\varepsilon_y = u_{2,y} = \varepsilon_y^o + z\cdot(\chi_y^o + z^2\cdot\chi_y^2)$$

$$\varepsilon_z = u_{3,z} = 0$$

$$\varepsilon_{xy} = u_{1,y} + u_{2,x} = \varepsilon_{xy}^o + z\cdot(\chi_{xy}^o + z^2\cdot\chi_{xy}^2)$$

$$\varepsilon_{xz} = u_{1,z} + u_{3,x} = \varepsilon_{xz}^o + z^2\cdot\chi_{xz}^2$$

$$\varepsilon_{yz} = u_{2,z} + u_{3,y} = \varepsilon_{yz}^o + z^2\cdot\chi_{yz}^2 \qquad (3)$$

where, $\varepsilon_x^o = u_{o,x}$, $\varepsilon_y^o = v_{o,y}$,

$$\chi_x^o = \phi_{x,x}\ ,\ \chi_x^2 = -c_2(w_{o,xx} + \phi_{x,x}),$$

$$\chi_y^o = \phi_{y,y}\ ,\ \chi_y^2 = -c_2(w_{o,yy} + \phi_{y,y}),$$

$$\chi_{xy}^o = \phi_{y,x} + \phi_{x,y}\ ,\ \varepsilon_{xy}^o = u_{o,y} + v_{o,x},$$

$$\chi_{xy}^2 = -c_2(2w_{o,xy} + \phi_{x,y} + \phi_{y,x}),$$

$$\chi_{xz}^2 = -c_1(w_{o,x} + \phi_x),\ \varepsilon_{xz}^o = \phi_x + w_{o,x},$$

$$\chi_{yz}^2 = -c_1(w_{o,y} + \phi_y),\ \varepsilon_{yz}^o = \phi_y + w_{o,y},$$

$$c_1 = 4/h^2,\ c_2 = c_1/3$$

The equilibrium equations can be obtained from the principle of minimum total potential energy

$$\delta\Pi = 0$$

where Π is the total potential energy,

$$\Pi = \frac{1}{2}\int_V[\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_{yz}\varepsilon_{yz} + \sigma_{xz}\varepsilon_{xz} + \sigma_{xy}\varepsilon_{xy}]\,dxdydz - \int_R q\,w_o\,dxdy + \frac{1}{2}\int_R\left\{kw_o^2 - G\left[\left(\frac{\partial w_o}{\partial x}\right)^2 + \left(\frac{\partial w_o}{\partial y}\right)^2\right]\right\}dxdy \quad (4)$$

where V is the volume of the plate and R is the midplane of the plate.

To derive the equations of motion which solved dynamic and static problems, we use Hamilton′s principle:

$$\delta\int_0^t[(T-(U+V)]\,dt = \delta\int_0^t(T-\Pi)\,dt = 0 \qquad (5)$$

where U, V and T are the strain energy, work done by external forces, total kinetic energy, respectively.

The stress resultants are defined by

$$(N_i,\ M_i,\ P_i) = \int_{-h/2}^{h/2} \sigma_i\ (1,\ z,\ z^3)\ dz$$

$$(i = x,\ y,\ xy)$$

$$(Q_x,\ R_x) = \int_{-h/2}^{h/2} \sigma_{xz}\ (1,\ z^2)\ dz \qquad (6)$$

$$(Q_y,\ R_y) = \int_{-h/2}^{h/2} \sigma_{yz}\ (1,\ z^2)\ dz$$

Substituting Eq. (6) into Eq. (5) and integrating the expression in Eq. (5) by parts and collecting the coefficients of $\delta u, \delta v, \delta w, \delta\phi_x$ and $\delta\phi_y$ we can obtain the equilibrium equations of the plate under finite deformation with the presence of applied transverse forces and the intensity of the reaction of the bottom plate can be written as [18]

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_o \ddot{u}_o + J_1 \ddot{\phi}_x - c_2 I_3 \frac{\partial \ddot{w}_o}{\partial x}$$

$$\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_o \ddot{v}_o + J_1 \ddot{\phi}_y - c_2 I_3 \frac{\partial \ddot{w}_o}{\partial y}$$

$$\begin{aligned} &\frac{\partial \overline{Q}_x}{\partial x} + \frac{\partial \overline{Q}_y}{\partial y} \\ &+ c_2\left(\frac{\partial^2 \hat{P}_x}{\partial x^2} + 2\frac{\partial^2 \hat{P}_{xy}}{\partial x \partial y} + \frac{\partial^2 \hat{P}_y}{\partial y^2}\right) \\ &+ (q - F_e) = I_o \ddot{w}_o - c_2^2 I_6 \left(\frac{\partial^2 \ddot{w}_o}{\partial x^2} + \frac{\partial^2 \ddot{w}_o}{\partial x^2}\right) \\ &+ c_2\left[I_3\left(\frac{\partial^2 \ddot{u}_o}{\partial x^2} + \frac{\partial^2 \ddot{v}_o}{\partial x^2}\right) \right. \\ &\left. + J_4\left(\frac{\partial^2 \ddot{\phi}_x}{\partial x^2} + \frac{\partial^2 \ddot{\phi}_y}{\partial x^2}\right)\right] \end{aligned} \qquad (7)$$

$$\frac{\partial \hat{M}_x}{\partial x} + \frac{\partial \hat{M}_{xy}}{\partial y} - \hat{Q}_x = J_1 \ddot{u}_o + K_2 \ddot{\phi}_x - c_2 J_4 \frac{\partial \ddot{w}_o}{\partial x}$$

$$\frac{\partial \hat{M}_{xy}}{\partial x} + \frac{\partial \hat{M}_y}{\partial y} - \hat{Q}_y = J_1 \ddot{v}_o + K_2 \ddot{\phi}_y - c_2 J_4 \frac{\partial \ddot{w}_o}{\partial y}$$

where, $F_e = k w_o - G\left(\frac{\partial^2 w_o}{\partial x^2} + \frac{\partial^2 w_o}{\partial y^2}\right)$,

$$\hat{M}_i = \beta M_i - \gamma c_2 P_i,$$

$$\hat{P}_i = \gamma c_2 P_i + \alpha M_i\ ,\quad \hat{Q}_i = \beta Q_i - \gamma c_1 R_i$$

$$\overline{Q}_i = \beta(1-\alpha)\, Q_i - \gamma c_1 R_i,$$

$$I_i = \sum_{k=1}^{N} \int_{z_k}^{z_{k+1}} \rho^k z^i\, dz \quad (i = 0, 1, 2, \cdots, 6),$$

$$J_i = I_i - c_2 I_{i+2}\ ,\ K_2 = I_2 - 2c_2 I_4 + c_2^2 I_6 .$$

The equilibrium equations of the classical laminate theory are obtained by setting $\alpha = 1$, $\beta = 0$ and $\gamma = 0$. The equilibrium equations of the first-order shear deformation theory can be deduced by setting $\alpha = 0$, $\beta = 1$ and $\gamma = 0$. The equilibrium equations of the higher-order laminate theory can be obtained by setting $\alpha = 0$, $\beta = 1$ and $\gamma = 1$.

The stress resultants can also be written as a function of strains and curvatures in the following form

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \\ P_x \\ P_y \\ P_{xy} \end{Bmatrix} = \begin{bmatrix} A_{ij} & B_{ij} & E_{ij} \\ B_{ij} & D_{ij} & E_{ij} \\ E_{ij} & F_{ij} & H_{ij} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_{xy}^o \\ \chi_x^o \\ \chi_y^o \\ \chi_{xy}^o \\ \chi_x^2 \\ \chi_y^2 \\ \chi_{xy}^2 \end{Bmatrix}$$

$$\begin{Bmatrix} Q_x \\ Q_y \\ R_x \\ R_y \end{Bmatrix} = \begin{bmatrix} A_{55} & A_{45} & D_{55} & D_{45} \\ A_{45} & A_{44} & D_{45} & D_{44} \\ D_{55} & D_{45} & F_{55} & F_{45} \\ D_{45} & D_{44} & F_{45} & F_{44} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \chi_x^2 \\ \chi_y^2 \end{Bmatrix} \quad (8)$$

where A_{ij}, B_{ij}, D_{ij}, and etc., are the plate stiffnesses. They are defined in term of transformed reduced stiffness coefficients $(\overline{Q_{ij}})_k$ for the individual k-th layer by

$$(A_{ij},\ B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}) = \int_{-h/2}^{h/2} \overline{Q_{ij}}(1, z, z^2, z^3, z^4, z^6)\,dz \quad (i, j = 1, 2, 6, 5, 4)$$

4. Boundary Condition, Displacement Variations and Plate Formulation

The following simply supported boundary conditions are assumed

For cross-ply :

$$u_1(x,0) = u_1(x,b) = u_2(0,y) = u_2(a,y) = 0$$

$$N_y(x,0) = N_y(x,b) = N_x(0,y) = N_x(a,y) = 0$$

For angle-ply :

$$u_1(0,y) = u_1(a,y) = u_2(x,0) = u_2(x,b) = 0$$

$$N_y(0,y) = N_y(a,y) = N_x(x,0) = N_x(x,b) = 0$$

$$u_3(x,0) = u_3(x,b) = u_3(0,y) = u_3(a,y) = 0$$

$$\phi_x(x,0) = \phi_x(x,b) = \phi_y(0,y) = \phi_y(a,y) = 0$$

$$P_y(x,0) = P_y(x,b) = P_x(0,y) = P_x(a,y) = 0$$

$$M_y(x,0) = M_y(x,b) = M_x(0,y) = M_x(a,y) = 0 \quad (9)$$

We assumes the following form of spatial variation of (u_3, ϕ_x, ϕ_y) that satisfies the boundary conditions in Eq. (9).

$$\begin{aligned} u_3 &= \sum_{m,n=1}^{\infty} W_{mn} f_3(X, Y) \\ \phi_x &= \sum_{m,n=1}^{\infty} X_{mn}\, f_2(X, Y) \\ \phi_y &= \sum_{m,n=1}^{\infty} Y_{mn}\, f_1(X, Y) \end{aligned} \quad (10)$$

The variation of u_1 and u_2 is different for cross-ply and antisymmetric angle-ply laminates. For cross-ply laminates,

$$\begin{aligned} u_1 &= \sum_{m,n=1}^{\infty} U_{mn}\, f_2(X, Y) \\ u_2 &= \sum_{m,n=1}^{\infty} V_{mn}\, f_1(X, Y) \end{aligned} \quad (11)$$

for antisymmetric angle-ply laminates,

$$\begin{aligned} u_1 &= \sum_{m,n=1}^{\infty} U_{mn}\, f_1(X, Y) \\ u_2 &= \sum_{m,n=1}^{\infty} V_{mn}\, f_2(X, Y) \end{aligned} \quad (12)$$

where,

$f_1(X, Y) = \sin\alpha x \ \cos\beta y$,

$f_2(X, Y) = \cos\alpha x \ \sin\beta y$,

$f_3(X, Y) = \sin\alpha x \ \sin\beta y$ and

$\alpha = m\pi\ /\ a$, $\beta = n\pi\ /\ b$.

Substituting Eqs. (8),(10)-(12) into Eq. (7) and collecting the coefficients, one obtains [18]

$$[M]\{\ddot{\triangle}\}+[K]\{\triangle\}=\{F\}-\{F_e\} \quad (13)$$

where $[M]$ is the mass matrix , $[K]$ is the coefficient matrix , $\{F\}$ is the force vector, $\{\triangle\}=\{U_{mn}\,V_{mn}\,W_{mn}\,X_{mn}\,Y_{mn}\}^T$ and $\{F_e\}$ is the intensity of the reaction of foundations.

The static solution can be obtained by setting the time derivative terms in Eq. (13) to zero:

$$[K]\{\triangle\}=\{F\}-\{F_e\} \quad (14)$$

5. Numerical Results

For illustrative purposes, the displacement has been calculated for cross-ply and various angle-ply laminates. The Young's moduli, shear moduli and Poisson's ratio for each lamina and the modulus of the foundations are arbitrarily taken to be, respectively.

For isotropic case :

$a=b=10\,cm$, $\nu=0.25$, $q=10N/cm^2$,

$E=25.0\times10^6 N/cm^2$, $h=0.1\,cm$.

For orthotropic case :

$a=b=10\,cm$, $\nu=0.25$, $q=10N/cm^2$,

$E_1/E_2=5$, $G_{12}/E_2=0.5$, $E_2=10^6 N/cm^2$,

$h=0.1\,cm$.

To validate the derived equations, the obtained displacements of simply supported isotropic and orthotropic plates subjected to a uniform loadings are compared with those of Timoshenko and LUSAS program in Table 1 and Table 2. They show the excellent agreement.

Table 1. Center displacement of the isotropic plates on elastic foundation with various modulus of foundations

k(N/cm^3)		Displacement (cm)	
		Timoshenko [7]	Present
CH	30	0.13469442	0.13473884
SF	50	0.11437665	0.11440656
SP	80	0.093065416	0.093082686
SW	100	0.082675523	0.082687606
GW	200	0.052554239	0.052555058

(CH : Clay High compressibility , SF : Sand Fines , SP : Sand Poorly graded , SW : Sand Well graded ,GW : Gravel Well Graded)

Table 2. Center displacement of the orthotropic plates on elastic foundation with various modulus of foundations

k(N/cm^3)	Displacement (cm)	
	LUSAS [17]	Present
30	0.383235	0.386683
50	0.238211	0.239687
80	0.148537	0.149014
100	0.117681	0.117889
200	0.055944	0.055829

Unless otherwise stated, the following property of the materials have been used throughout the calculations :

$$E_1/E_2=25,\ G_{12}/E_2=G_{13}/E_2=0.5,$$
$$G_{23}/E_2=0.2,\ \nu_{12}=0.25,\ a=b=10\,cm,$$
$$q=10N/cm^2,\ E_2=1.0\times10^6 N/cm^2.$$

The effect of side to thickness ratio on the displacement for square laminates with lamination scheme(0/90/90/0) is illustrated in Fig. 3. Considering the cases, it is evident that the displacement predicted by the classical plate theory is smaller than that calculated by the higher-order shear deformation theory. The difference becomes particularly significant for a/h<10.

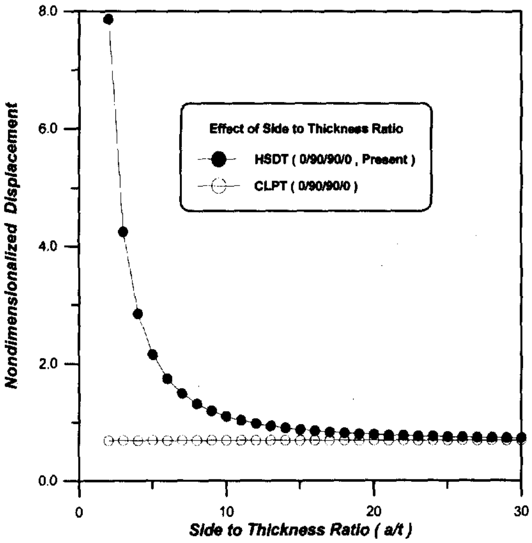

Fig. 3 Effect of side to thickness ratio on the displacement

To show the effect of the modulus of foundation on the transverse displacement, cross-ply laminated plate with a/h=100, subjected to applied uniform loading, were analyzed and the results are shown in Fig. 4. From this figure one can see that the modulus of foundation has a noticeable influence on the displacement of the laminated plates.

Fig. 5 shows the displacements by different theories and layer numbers in lamination angle (0/90) and (0/90/90/0). From the figure, it can be observed that the dis-

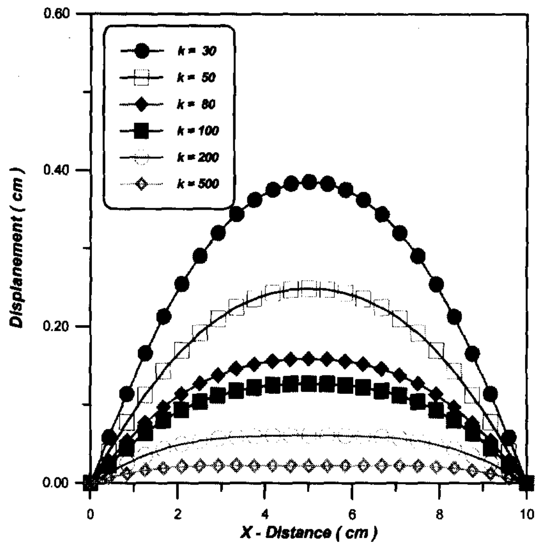

Fig. 4 Effect of modulus of elastic foundation

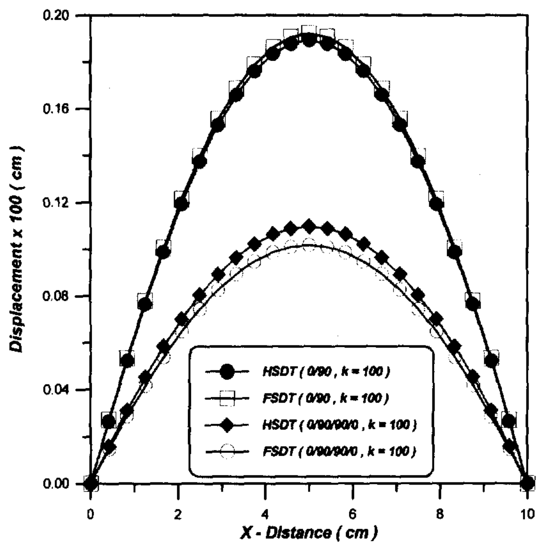

Fig. 5 Comparison of displacement for two-layer laminated composite plates

placement is predicted by the first-order shear deformation theory is smaller than that calculated by the higher-order shear deformation theory. Fig. 6 shows the displacement of a two-layer lamination angle

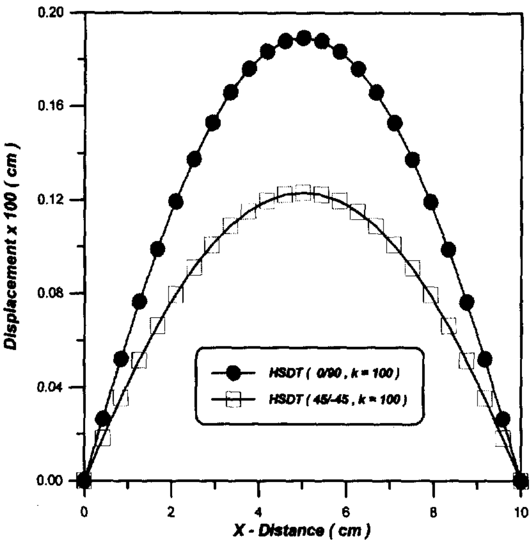

Fig. 6 Displacement of two-layer plate with different lamination angle

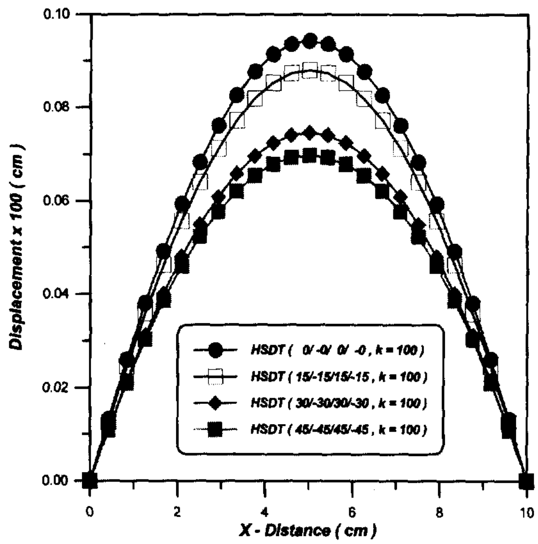

Fig. 7 Effect of ply angle on the displacement

(0/90) and (45/-45), all edge simple supported laminated plate with $k = 100 N/cm^3$. It can be seen that the maximum displacement for (0/90) lamination is higher by 58 % than that of (45/-45) lamination angle plate.

Therefore, it is apparent that angle of lamination has a significant effect on maximum transverse displacement.

Fig. 7 shows the influence of ply angle θ on the displacement for square, antisymmetric angle-ply laminated plates ($\theta / -\theta / \theta / -\theta$). The minimum value of displacement is seen to occur at $\theta = 45^o$ for these laminated plates.

It is shown in Fig. 8 that the bending and extension coupling has a significant effect on the displacement in two layer laminated plates and the bending-extension coupling effect decreases out rapidly as the number of layers increases.

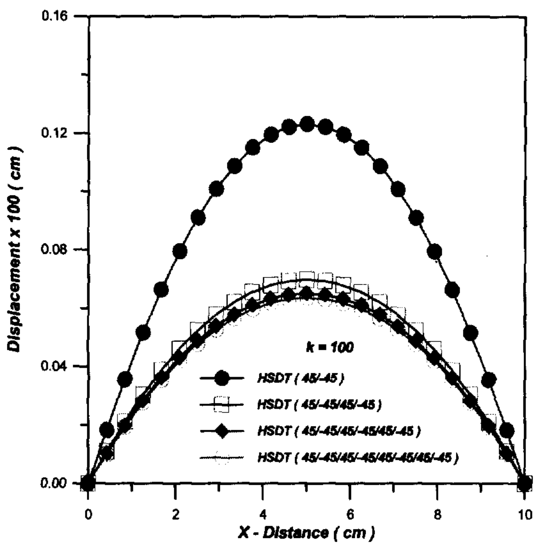

Fig. 8 Displacement of laminated plates by various layer numbers

Fig. 9 shows the influence of aspect ratio b/a upon the displacement. It is observed that the displacement of center line(Y=b/2) decreases rapidly as the aspect ratio decreases.

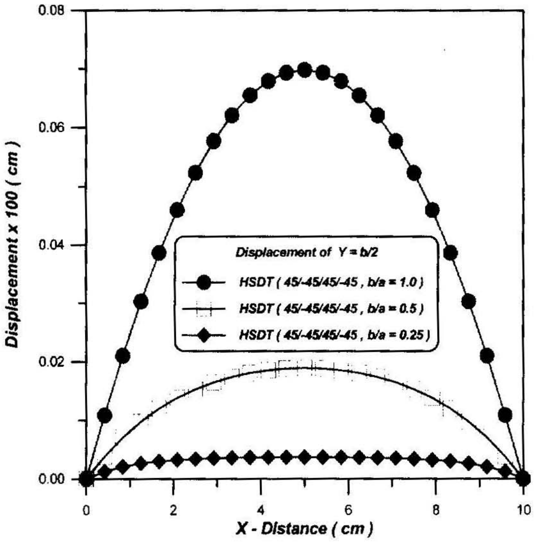

Fig. 9 Effect of aspect ratio on the displacement

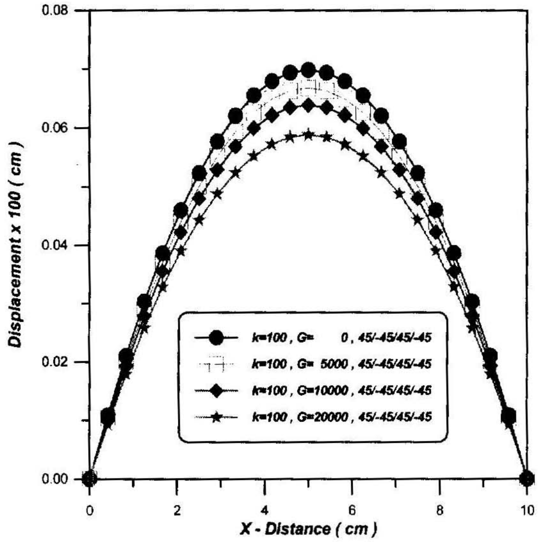

Fig. 11 Effect of shear modulus of elastic foundation on displacement

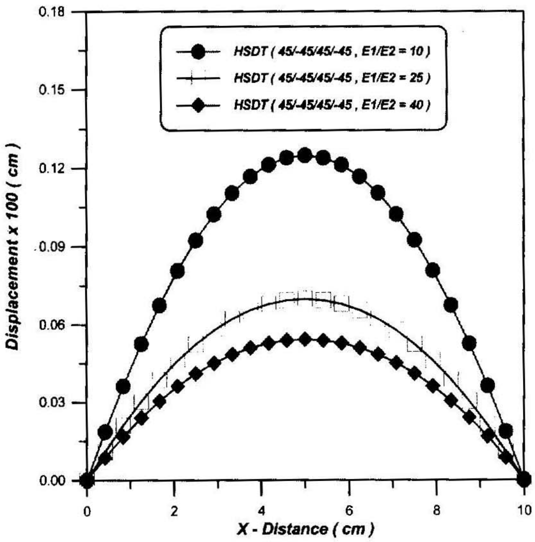

Fig. 10 Effect of elastic modulus ratio on the displacement

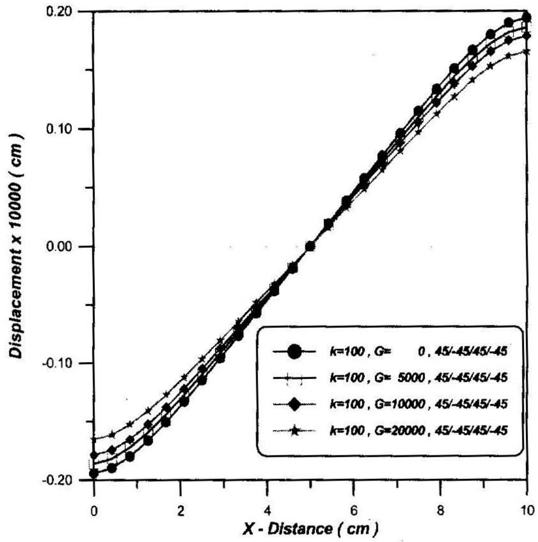

Fig. 12 Effect of shear modulus of elastic foundation on displacement

Fig. 10 shows the influence of modulus ratio E_1/E_2 upon the displacement. It is observed that the displacement decreases rapidly as the modulus ratio increases.

To show the effect of the shear modulus of elastic foundations on the transverse displacement, antisymmetric angle-ply laminated plates with a/h=10, subjected to applied uniform loading, were analyzed and the results are shown in Figs. 11-12.

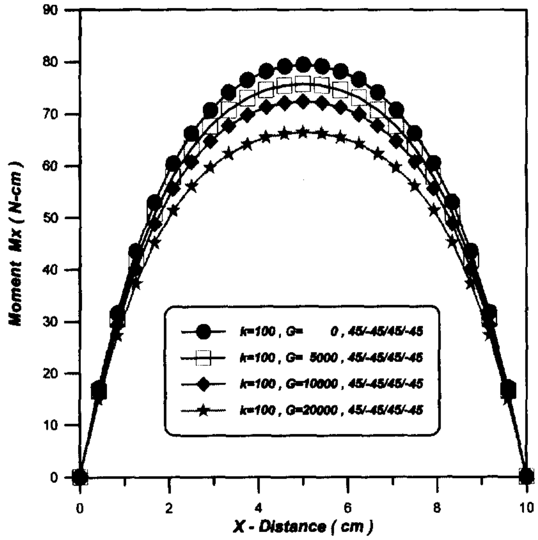

Fig. 13 Effect of shear modulus of elastic foundation on moments

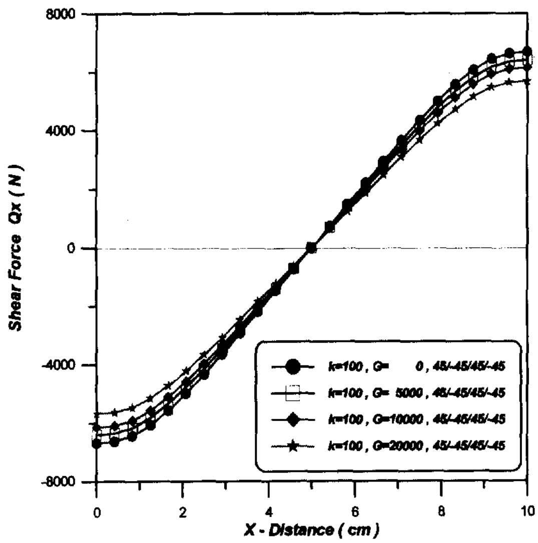

Fig. 14 Effect of shear modulus of elastic foundation on shear force

Fig. 12 shows the displacement in the Y-direction. From these figures one can see that the shear modulus of elastic foundations has a noticeable influence on the displacement of the laminated plates as the modulus of foundations cases.

Figs. 13-14 show that the effect of shear modulus of elastic foundations in stress resultants. From these figures, one can see that the shear modulus of elastic foundations has a noticeable influence on the displacement of the laminated plates as was seen in the case of the modulus of foundations cases.

6. Concluding Remarks

Most of civil structures bear the character of connected with the foundations, so the effect which is affected by various modulus of foundations is important to structural engineers. Especially, if the utility of composite structures is widely spread, also concern with modulus of foundations will be very important. The bending analysisof symmetrically or anti-symmetrically laminated simply supported square plates on two-parameter elastic foundations is performed by the higher-order shear deformation theory. From the numerical results considered, following conclusions are drawn.

(1) Though the modulus of the foundations are constant, it is observed that the displacement decreases as the shear modulus of the foundations increases. Therefore the modulus of the foundations and the shear modulus of the foundations are should considered simultaneously for exact analysis of structures on elastic foundations.

(2) When the modulus of the foundations increase, the shape of displacement is gradually decreased and flatted. Therefore

the modulus of the foundations are the indispensable conditions for prediction of correct shape of displacement.

(3) The displacement of laminated composite plates on GW(Gravel Well graded) foundations is smaller than on CH(Clay High compressibility) foundations. Therefore, the case of CH foundations, the displacement is considered evidently for safety of structures.

(4) The result predicted by FSDT and HSDT are very close ; it is interesting to note that the FSDT underestimates displacements for the two-layer case and overestimates for the four-layer case when compared to HSDT. Because it is that bending-streching coupling is negligible for laminates with four or more layers. The FSDT requires the use of shear correction factors that depend on the detailed lamination scheme. In the FSDT, the shear correction factors play a crucial role in the determination of displacement. No such corrections are required in the present HSDT.

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