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Electric Charge and Magnetic Flux on Astrophysical Black Hole

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ABSTRACT

We suggest a possible scenario of an astrophysical black hole with non-vanishing electric charge and magnetic flux. The equilibrium charge on a rotating black hole in a force-free magnetosphere is calculated to be $Q \sim BJ$ with a horizon flux of $\sim BM^2$, which is not large enough to disturb the background Kerr geometry. Being similar to the electric charge of a magnetar, in sign and order of magnitude, both electric charge and magnetic flux are supposed to be continuous onto a black hole.

Key Words : black hole physics –magnetic fields

Neutrality of an astrophysical objects including black holes is commonly accepted without any serious objection. However theoretically it is also well-known that rotating black holes in an aligned vacuum magnetic field assume a horizon charge $Q = 2BJ$ in their lowest energy state (Wald 1974, Dokuchaev 1987). It indicates a possibility of charged rotating black hole in an astrophysical environment. Here, J denotes the angular momentum and B the asymptotic magnetic field-strength.

In the Blandford-Znajek process(Blandford and Znajek 1977), a rotating black hole interacts with the surrounding magnetosphere in the force-free limit. The electric field on the horizon has a non-vanishing normal component, which may be interpreted in terms of a surface density of electric charge in accord with the membrane paradigm developed by Thorne, Price and Macdonald (1986). The integral of the surface charge defines a net electric charge Q. All "three hairs" of mass, M, angular momentum, J, and Q, therefore, appear in black hole-magnetic field interactions. Since the horizon charge corotates with the horizon, it induces a magnetic flux (van Putten, 2000)

$$\phi_H = 4\pi Q M \Omega_H = 8\pi B M^2 \sin^2(\lambda/2) \tag{1}$$

where $\sin\lambda = a/M$ and Ω_H parametrizes the angular velocity of the black hole (van Putten 1999). Together with the charge-free flux, this recovers a net horizon flux (Wald 1974, Dokuchaev 1987, van Putten 2000 and Kim, Lee and Lee 2001).

This observation suggests the possibility that a magnetic flux on a rapidly rotating black hole is similarly maintained by a finite horizon charge also when the magnetosphere is force-free. It may be noted that the electric horizon charge has been calculated for inflow in the approximation of ideal magnetohydrodynamics (Ruffini & Wilson, 1975). The force-free limit considered here is different, as a singular limit defined by neglecting Reynolds stresses. The presented calculation is of particular interest to the high-angular momentum black hole-disk or torus model of gamma-ray bursts with magnetic fields $B \sim 10^{15}$G (see, e.g., van Putten & Ostriker (2001) for a recent discussion),

Coulomb's law of electrodynamics suggests that the net electric charge Q can be obtained by the integral

$$Q = \frac{1}{4\pi}\oint d\mathbf{S}\cdot\mathbf{E} \tag{2}$$

over a closed surface surrounding the system at hand. These equations hold not only in flat space-time but can also be shown to hold in curved space-time, provided that the dot product is interpreted covariantly with appropriate definitions of the electric field $\mathbf{E}$ and the charge density ρ_e. This has been made explicit by Thorne and Macdonald (1982) in the 3+1 formulation of electrodynamics in curved space-time. At each space-time point, a fiducial reference frame is chosen by splitting the 4-dimensional space-time into three space directions and one "universal time" direction. Thus, Coulomb's law assumes similar expressions in curved and flat space-time.

By Eq.(2), we can determine the charge on a black hole, given the electromagnetic field surrounding the black hole. The force-free magnetosphere in the Kerr metric is potentially a realistic example, as in, e.g., Okamoto (1992). This applies when the magnetic energy density is not large enough to affect the background metric significantly: $B^2M^2 << 1$.

The axially symmetric force-free magnetosphere surrounding a Kerr black hole is described by surfaces of continuous magnetic flux $\Psi(r,\theta)$ in Boyer-Lindquist coordinates (r,θ,ϕ). These surfaces are in rigid rotation (Thorne, Price and Mcdonald 1986) with angular velocity $\Omega_F = \Omega_F(\Psi)$. The the total electric charge on the stretched horizon is given by

$$Q = -\int_{horizon} (\Omega_F - \Omega_H)\frac{\Sigma^2}{4\pi\rho^2}\partial_r\Psi \sin\theta\, d\theta, \tag{3}$$

and the total magnetic flux through upper hemisphere by

$$\Phi_B = \int_0^{\pi/2} B_n 2\pi\varpi\rho\, d\theta = \Psi(\theta=\pi/2) - \Psi(\theta=0). \tag{4}$$

Here, we use the Boyer-Lindquist expressions $\Delta = r^2 + a^2 - 2Mr$, $\rho^2 = r^2 + a^2\cos^2\theta$, $\Sigma^2 = (r^2+a^2)^2 - a^2\Delta\sin^2\theta$. $\Omega_F(\Psi)$, the electric current flowing into the black hole $I(\Psi)$, and Ψ are determined by the Maxwell equation for the force-free magnetosphere, which leads to the equation(Blandford and Znajek 1977, Macdonald and Thorne 1982, Beskin 1997) given by the Grad-Shavranov (GS) equation:

$$\nabla \cdot \left\{ \frac{\alpha}{\varpi^2}\left[1 - \frac{(\Omega_F - \omega)^2\varpi^2}{\alpha^2}\right]\nabla\Psi \right\} + \frac{\Omega_F - \omega}{\alpha}\frac{d\Omega_F}{d\Psi}(\nabla\psi)^2 + \frac{16\pi^2}{\alpha\varpi^2}I\frac{dI}{d\Psi} = 0. \qquad (5)$$

Recently Beskin and Kuznetsova (2000) showed by solving the GS equation that for transonic flow onto a black hole the poloidal magnetic field in the supersonic domain near horizon remains actually the same as one can find from the force-free solution. It implies that in calculating the electric charge, Eq.(3), one can use the force-free poloidal magnetic field obtained from Eq.(5). This stream equation is quite complicated and does not appear to allow exact analytical solutions.

In the restricted case of a non-rotating black hole with $\Omega_F = 0$, Eq. (5) reduces to the following simple form;

$$\nabla \cdot \left\{ \frac{\alpha}{\varpi^2}\nabla\Psi \right\} = 0. \qquad (6)$$

General properties of Eq. (6) are discussed by Ghosh (2000). Although we do not have the solution Ψ in the general rotating black hole case, a numerical study by Macdonald (1984) demonstrates that the effect of the black hole rotation to the poloidal magnetic field structure is small even at high specific angular momentum $a \le 0.75M$. Based on these observations we will assume in this work that the poloidal structure of the magnetosphere surrounding a black hole in the rotation case can be inferred from the nonrotating case.

Adopting a form of solutions suggested by Ghosh and Abramowicz (1997) which are linear combinations of the specific solutions discussed by Macdonald (1984) and Ghosh (2000),

$$\Psi_\pm = \Psi_0[(r-2M)(1\mp\cos\theta) \mp r_0\cos\theta - 2M(1\pm\cos\theta)\ln(1\pm\cos\theta)] \qquad (7)$$

for the upper $(+)$ and lower$(-)$ part, the electric charge on the black hole is now given by Eqs. (3), (7), and assuming an optimal case where $\Omega_F \sim \Omega_H/2$, we have

$$Q \sim \frac{1}{8\pi}\frac{J\Phi_B}{Mr_H} = BJ \qquad (8)$$

where for an order of magnitude estimate, we consider $\Psi_0 \sim \frac{\Phi_B}{4M}$ and $\Phi_B \sim 8\pi M r_H B$. This result is similar to the Wald charge in the ground state of the black hole in a vacuum magnetic field. For a rapidly rotating black hole with strong magnetic fields ($B = 10^{15}$G), in the context of GRBs, the charge can be estimated as

$$Q \sim 10^{15}C\left(\frac{B}{10^{15}G}\right)\left(\frac{M}{M_\odot}\right)^2. \qquad (9)$$

Compared to the extremal charge on the black hole, one can see that the above charge is much smaller

$$Q \ll Q_{extremal} = 4.2\times10^{20}C\left(\frac{M}{M_\odot}\right) \qquad (10)$$

and therefore the Kerr metric can be treated safely as a background for the force-free magnetosphere.

Pulsars are known to be rapidly rotating neutron stars with strong magnetic fields (Shapiro & Tuekolsky 1983). Recently, several magnetars with even stronger magnetic fields (Kouveliotou et al. 1999) have been found. The vacuum surrounding these compact objects is unstable which evolves into a magnetoshpere with charged particles. Goldreich-Julian (Goldreich & Julian 1969) suggested a model of a degenerate pulsar magnetosphere in the force-free limit. Assuming the poloidal field structure to be a magnetic dipole

$$\mathbf{B}^P = \frac{B_0R^3}{r^3}(\cos\theta\,\mathbf{e}_{\hat r} + \sin\theta\,\mathbf{e}_{\hat\theta}) \qquad (11)$$

the electric charge on the rotating neutron star in a degenerate magnetosphere (Goldreich and Julian 1969, Ruderman and Sutherland 1975) can be calculated as

$$Q_{NS} = \frac{B_0\Omega R^3}{3} \qquad (12)$$

Using the total magnetic flux (the stream function on the neutron star surface reads $\Psi(\theta) = \Phi_B(1-\cos^2\theta)$) through upper hemisphere, $\Phi_B^{NS} = \pi R^2 B_0$, and assuming the neutron star as a rigid sphere, $J_{NS} = (2/5)MR^2\Omega$, we get

$$Q_{NS} = \frac{5}{6\pi}\left(\frac{J\Phi_B}{MR}\right)_{NS}. \qquad (13)$$

Thus, the same structure as in Eqn. (8) is obtained, both in sign and order of magnitude. Taking $M \sim 1.5M_\odot$, $R \sim 10^6$cm, $\Omega \sim 10^4$/s for a magnetar with $B \sim 10^{14}$G, we conclude $Q_{NS} = 10^{15}$C - the same sign and order of magnitude as that on the rotating black hole in a force-free magnetosphere obtained above.

These results give an indication of the charge during accretion-induced or prompt collapse into a black hole. During collapse, the specific angular momentum $a = J/M$ will change continuously. Since the time-scale of accretion of a charge BJ is of order M, the notion that both the neutron star and the final black hole state have charges of order $\sim BJ$ indicates that the charge Q should remain about this value. This would hold in particular when the accreting matter is magnetized at the level of the collapsing neutron star, in which case the initial charge on the neutron star and the final charge on the black hole are similar within a factor of unity. In this event, with $r_H/R_{NS} \sim 10^{-1}$ we get from Eq. (8) and Eq. (13)

$$\Phi_{BH} \sim \Phi_{NS}. \tag{14}$$

The continuity of magnetic flux provides the necessary horizon flux on the black hole for tapping the rotational energy, for example, via the Blandford-Znajek process. The relationship (16) allows the power in Poynting flux to be alternatively written as

$$P_{BZ} \sim 10^{50}\mathrm{erg/s}\left(\frac{aB}{10^{-4}}\right)^2 = 10^{50}\mathrm{erg/s}\left(\frac{Q/M}{10^{-4}}\right)^2. \tag{15}$$

The expression on the right brings about the relationship between P_{BZ} and the associated charge-to-mass ratio. The charge-to-mass ratio remains small for stellar mass black holes in astrophysical environments, which is consistent with the assumed Kerr geometry.

We interpret these results to indicate that the central charge in the collapsar/hypernova scenario of GRBs remains essentially continuous and, with it, the associated magnetic flux. Provided that the magnetic field continues to be supported by surrounding magnetized matter, e.g., fall back matter stalled against an angular momentum barrier, extraction of rotational energy from the newly formed black hole may continue by interaction with the magnetic field. Wherever the magnetosphere is force-free, the interaction with the magnetic field is notably so in the form of a Poynting flux (Blandford & Znajek, 1977; Lee, Wijers & Brown, 2000). The horizon charge is important in preserving magnetic flux, especially when the black hole spins rapidly, but will not provide a source of significant energy for a stellar mass black hole. The magnetic flux is essential in extracting out the rotational energy of the black hole (Blandford and Znajek 1977). Hence we can expect Blandford-Znajek mechanism can work for a rotating black hole emerging in a final stage of a stellar collapse.

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