논문 00-02-01

Input conductance of neuron for Hopfield Neural Networks

Hopfield 신경회로망에서 뉴론의 입력단 컨덕턴스

Min-Je Kang*
(康 珉 齊*)

ABSTRACT

This paper discusses the influence of the input conductance on system stability for the continuous type Hopfield Neural Networks.. The input conductance is connected from the neuron input to ground. The input conductance has been proved to effect on stability in input space. Transient analysis is used to test the stability in input space. Also, it has been studied how to adjust the input conductance for improving the system's performance.

요 약

이 논문은 연속형 Hopfield 신경회로망에서 뉴론의 입력단에 연결하는 컨덕턴스가 시스템의 안정도에 미치는 영향에 대해 논의 하고자 한다. 이 컨덕턴스는 뉴론의 입력단과 ground사이에 캐패시터와 병렬로 연결되어 있는 데, 시스템의 안정도에 영향을 미치는 것으로 알려져 있으나, 이 것에 대해 알려진 것이 별로 없어, 여기서 그 것에 자세히 논의 한다. 그리고 또한 이 컨덕턴스를 조절하여 시스템의 안정도와 더불어 시스템의 performance를 개선하는 방법에 대해 다룬다.

Keywords: Input Conductance, Hopfield, Neural Networks, Stability, Transient Analysis

Ⅰ. Introduction

It has been shown that transients in a network of connected processors with sigmoidal voltage transfer characteristics converge to a stable minimum of its computational energy, i.e. the Lyapunov function for the network[1,2]. Since then, Hopfield type neural networks have been applied in solving problems in many areas such as associative memory, pattern recognition, and optimization problems. In implementing such application circuits, there often happens some problems; one of them is to select values for input conductance and capacitance of neuron. The input conductance of neuron is represented in the circuit by a resistor of value ri = 1/gi and an input capacitor ci connected in parallel from the neuron input to ground[3]. These components have

* 濟州大學校 電氣.電子工學部
(Dept. of Electronic Engineering, Cheju National University)

接受日: 2000年6月26日 修正完了日: 2000年11月16日

been known partially to define the time constant of the neuron and to provide for the integrative analog summation of the synaptic input currents from other neurons in the networks. However, they have not been reported how to choose the value of them.

In this paper, we investigate the influence of the input conductance on system stability. It has been shown that the stability of input space depends on the value of input conductance. Transient analysis is used to test the stability in input space. Also, it has been studied how to adjust the input conductance for improving the system's performance. To solve the minimization problem using Hopfield type neural networks, the objective function which is to be minimized should be matched to the first two term of energy function. The usual method for ignoring the third term of energy function is to use a high sigmoid gain. However, as a sigmoid gain increases, the output of neuron becomes discrete. Therefore, we discuss new method for ignoring the third term of energy function by adjusting the input conductances, with this method any sigmoid gain can be used.

Ⅱ. Hopfield Model for Neural Networks

In this paper, we are going to discuss about Hopfield type neural networks as shown in Fig. 1. Using KCL(Kirchoff Current Law) at the input of the neuron, the following equation can be obtained[3]

$$c_i \frac{du_i}{dt} = \sum_{j=1}^{n} w_{ij} v_j + i_i - \left(\sum_{j=1}^{n} w_{ij} + g_i \right) u_i, \quad i = 1, \ldots, n \tag{1}$$

c_i represents the nonzero input capacitance of the ith neuron. Similarly, gi represents the input conductance between the ith neuron input and ground. Conductance wij connects the output of the jth neuron to the input of ith neuron. Current ii is the bias comming current into the ith neuron input. Also, each neuron maps its input voltage ui into the output voltage vi through the activaton function $f(u_i)$. Usually the following sigmoid function is used for the activation function as

$$f(u_i) = \frac{1}{1 + e^{-\lambda u_i}} \tag{2}$$

Denoting the total conductance connected to the ith neuron input node as Gi, where

$$G_i = \sum_{j=1}^{n} w_{ij} + g_i \tag{3}$$

Equation(1) can be simplified to the form of a single state equation as

$$c_i \frac{du_i}{dt} = \sum_{j=1}^{n} w_{ij} v_j + i_i - G_i u_i, \quad i = 1, \ldots, n \tag{4}$$

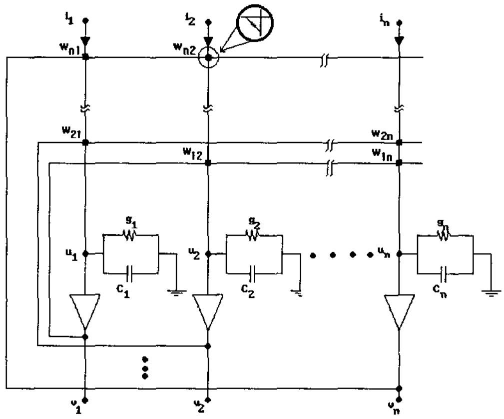


Fig. 1. The Electrical model of Continuous type Hopfield Neural networks

Let us introduce the matrices c and G defined as

$c = diag[c_1, c_2, \ldots, c_n]$ and

$G = diag[G_1, G_2, \ldots, G_n]$ and arrange ui(t),vi(t), and ii in n-dimensional column state vector u(t), output vector v(t), and bias current vector i. The final equations of the entire model network consisting of the state equation written in matrix form can now be expressed as

$$c \frac{du}{dt} = wv(t) + i - Gu(t) \tag{5}$$

Hopfield(1984) has introduced the energy function

E(v) which is defined by[3]

$$E(v) = -\frac{1}{2} v^t w v - i^t v + G\int_{0.5}^{v} f^{-1}(z)\,dz \quad (6)$$

And, the negative differentiate of equation (6), that is, decreasing energy function, can be obtained ad follows

$$-\frac{dE}{dv} = wv + i - Gu \quad (7)$$

The interesting thing found here by comparing equation (5) and (7) is that the negative differentiate of energy function E(v) is ratio to the velocity of input u. And capacitance c is the constant of ratio as follows

$$-\frac{dE}{dv} = c\frac{du}{dt} \quad (8)$$

Also, if the weight matrix w is symmetric and $\sigma(u)$ is continuously increasing function, then this energy function E(v) is a Liapunov function. This is clear from the following[5]:

$$\frac{dE}{dt} = \frac{dE}{dv}\frac{dv}{dt} = -C\frac{du}{dt}\frac{dv}{dt} = C\left(\frac{du}{dt}\right)^2 f'(u) \leq 0 \quad (9)$$

III. Stability Analysis of Input space

Our objective in this section is to analyze how the input conductance gi affects the stability of the neuron input. The input conductance gi has been known partially to define time constant and also to contribute to the system stability. However, it has not been known how to effect on time constant and stability. Here, we discuss the influence of the input conductances on stability of input space.

Proposition: In the continuous Hopfield neural networks, the following statements about the stability of input space are true:

(1) If the total conductance G_i is great than zero, then the input space is exponentially stble.

(2) If the total conductance G_i is equal to zero, then the input space is linearly unstable.

(3) If the total conductance G_i is less than zero, then the input space is exponentially unstable.

Proof (1):

Denoting the total current as I_i, where

$$I_i = i_i + \sum_{j=1}^{n} w_{ij} v_j \quad (10)$$

As seen Fig. 2, Eq. (4) can be simplified to the form of a single state equation as

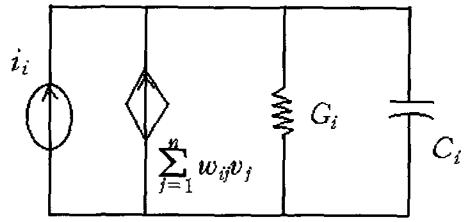


Fig.2.The Equivalent circuit seen from input node i.

$$C_i\frac{du_i}{dt} = -G_i u_i + I_i \quad (11)$$

As seen in previous section, the output of system becomes asymptotically stabilized in time decreasing the energy function. Therefore, after some time in which the output is stabilized, the current $\sum_{j=1}^{n} w_{ij}v_j$ can be regarded as a constant. This means that after the output is stabilized, the total current I_i also can be treated as a constant because the bias current i_i is also constant. By using the matrix notation defined in the previous section, the final equation in matrix form can be expressed as

$$c\frac{du}{dt} = -Gu + I \quad (12)$$

The solution of this equation[10] can be obtained by taking the Laplace transform

$$U(s)=[sI+\frac{G}{c}]^{-1}u(0)+[sI+\frac{G}{c}]^{-1}\frac{I}{cs}$$

$$=\begin{bmatrix} s+\frac{G_1}{c_1} & 0 & \cdots & 0 \\ 0 & s+\frac{G_2}{c_2} & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & s+\frac{G_n}{c_n} \end{bmatrix}^{-1} \left\{ \begin{bmatrix} u_1(0) \\ u_2(0) \\ \vdots \\ u_n(0) \end{bmatrix} + \begin{bmatrix} \frac{I_1}{c_1 S} \\ \frac{I_2}{c_2 S} \\ \vdots \\ \frac{I_n}{c_n S} \end{bmatrix} \right\} \quad (13)$$

where u(0) is input voltage at the time when the output space is regarded being stabilized.

Here, we consider the stability of input space after the output space is stabilized.

Eq. (13) can be rearranged as

$$U_i(s)=(u_i(0)-\frac{I_i}{G_i})\frac{1}{(s+\frac{G_i}{c_i})}+\frac{I_i}{G_i}\frac{1}{s} \quad (14)$$

The inverse Laplace transform of Eq. (14) gives

$$u_i(t)=(u_i(0)-\frac{I_i}{G_i})e^{-\frac{G_i t}{c_i}}+\frac{I_i}{G_i} \quad (15)$$

If the total conductance G_i is greater than zero, the first term of Eq. (15) is disappearing exponentially and finally $u_i(t)$ becomes as

$$\lim_{t\to\infty} u_i(t)=\frac{I_i}{G_i} \quad (16)$$

Therefore, If the total conductance G_i is great than zero, then the input space is exponentially stable.

Case Study

The 2-bit A/D converter[2] is selected for a case study. The connecting weights and the bias currents for 2bit A/D converter are as follow, where x is analog input.

$$w=\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}, \qquad i=\begin{bmatrix} x-\frac{1}{2} \\ 2x-2 \end{bmatrix} \quad (17)$$

The state equation of inputs for 2-bit A/D converter for x=1.6, c1=c2=1u is as below[9], because v1 and v2 are established near to 0 and 1 respectively after output space is stabilized.

$$\begin{bmatrix} \frac{du_1}{dt} \\ \frac{du_2}{dt} \end{bmatrix}=\begin{bmatrix} 10^6 & 0 \\ 0 & 10^6 \end{bmatrix}\left(\begin{bmatrix} 2-g_1 & 0 \\ 0 & 2-g_2 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}+\begin{bmatrix} I_1 \\ I_2 \end{bmatrix}\right) \quad (18)$$

where, $I=i+wv=\begin{bmatrix} 1.1 \\ 1.2 \end{bmatrix}+\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$

The transient simulations are performed for c1=c2=1u, x=1.6, α=2 and different valuses of input conductances. Microsim's Pspice version 8.0 is used to simulate transient analysis. Fig. 3 shows the schematic diagram for c1=c2=1u, x=1.6, α=2 and g1=g2=2.5.

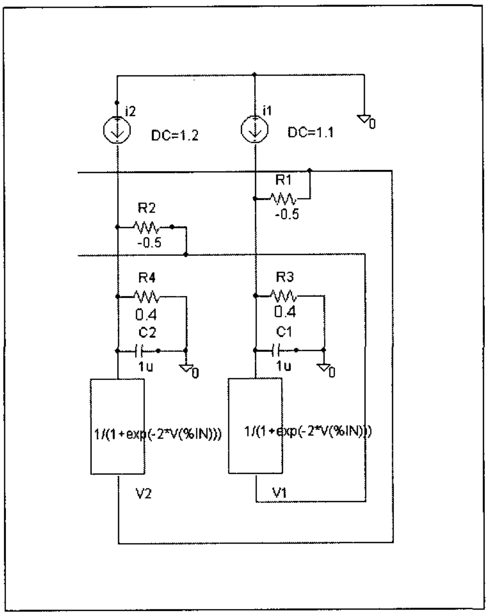
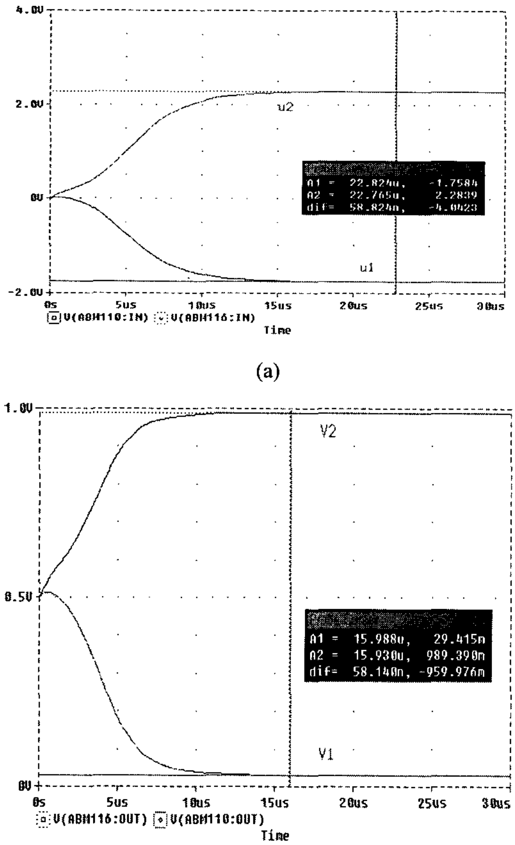


Fig. 3. Pspice schematic diagram for 2bit A/D converter in case x=1.6, g1=g2=2.5, c1=c2=1u and α=2.

As shown in Fig. 4b, outputs v1 and v2 converge nearly to 0.029 and 0.989 respectively as the time elapses. So, using Eq.(16), the inputs for this case are obtained as below

$$\lim_{t\to\infty}\begin{bmatrix} u_1(t) \\ u_2(t)\end{bmatrix} = \begin{bmatrix} \dfrac{1.1-2\times 0.989}{0.5} \\ \dfrac{1.2-2\times 0.029}{0.5}\end{bmatrix} = \begin{bmatrix} -1.756 \\ 2.284\end{bmatrix} \tag{19}$$

And we can realize that this analytic result is same as the simulation result shown in Fig. 4a



(a)

(b)

Fig. 4. Transient simulation for 2bit A/D converter in case x=1.6, g1=g2=2.5, c1=c2=1u andα=2: (a) input, (b) output.

Proof (2):

If $G_i=0$, Eq. (13) can be rearranged as

$$U(s)=[sI]^{-1}u(0)+[sI]^{-1}\frac{I}{cs}$$

$$=\begin{bmatrix} s & 0 & \dots & 0 \\ 0 & s & \dots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \dots & s\end{bmatrix}^{-1}\left\{\begin{bmatrix} u_1(0) \\ u_2(0) \\ \vdots \\ u_n(0)\end{bmatrix} + \begin{bmatrix} \frac{I_1}{c_1 s} \\ \frac{I_2}{c_2 s} \\ \vdots \\ \frac{I_n}{c_n s}\end{bmatrix}\right\} \tag{20}$$

Eq. (20) can be rearranged as

$$U_i(s)=u_i(0)\frac{1}{s}+\frac{I_i}{c_i}\frac{1}{s^2} \tag{21}$$

The inverse Laplace transform of Eq. (21) gives

$$u_i(t)=u_i(0)+\frac{I_i}{c_i}t \tag{22}$$

Same as before, by regarding I_i as a constant after the output space being stabilized, the second term in Eq. (22) changes linearly in time, so the input space is linearly unstable.

Case Study

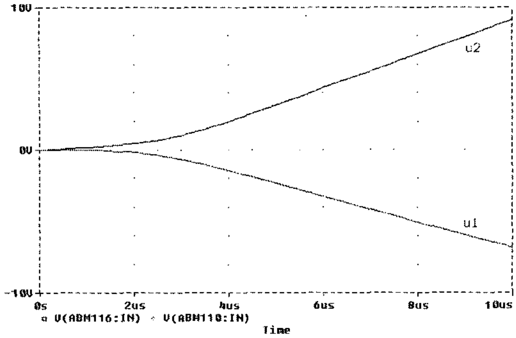
The 2-bit A/D converter[2] is also selected for this case study.

Fig. 5 shows the transient simulations of input and output spaces for g1=g2= 2 ($g_i = -\sum_{j=1}^{n} w_{ij}$, thus, $G_i=0$) and other conditions are same as before. As seen in output space simulation, the output can be regarded being stabilized after the time of 6us. Therefore, after the time of 6us, Eq. (22) for this case can be expressed as

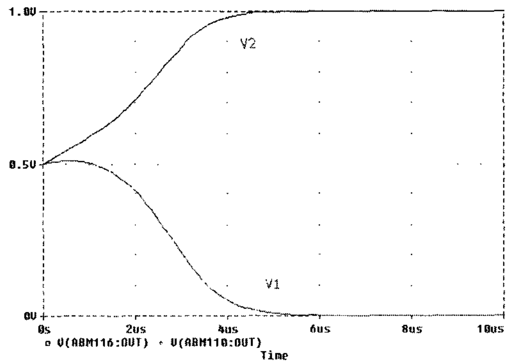
$$\begin{bmatrix} u_1(t-6us) \\ u_2(t-6us)\end{bmatrix} = \begin{bmatrix} u_1(6us) \\ u_2(6us)\end{bmatrix} + \begin{bmatrix} -0.9\times 10^6(t-6us) \\ 1.2\times 10^6(t-6us)\end{bmatrix}$$

단, $t \geq 6us$ (23)

As seen in Fig. 5, the slope of u1(t), u2(t) are $-0.9\times 10^6 t$, $1.2\times 10^6 t$ respectively after the $6us$. This fact is agreed with Eq. (23).



(a)



(b)

Fig. 5.Transient simulation for 2bit A/D converter in case x=1.6,g1=g2=2,c1=c2=1u and α=2:(a)input,(b) output.

Proof (3):

As seen in Eq. (15), the total conductance G_i is less than zero, the first term of Eq. (15) is changing exponentially. Therefore, If the total conductance G_i is less than zero, then the input space is exponentially unstable.

Case Study

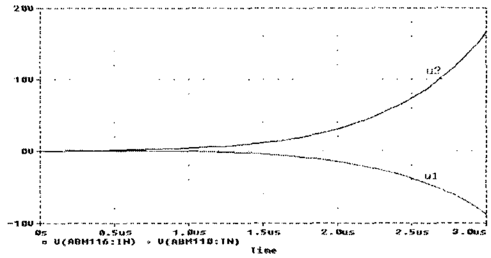
The 2-bit A/D converter is selected for this case study also.

Fig. 6 shows the transient simulations for g1=g2= 0.5 ($g_i < -\sum_{j=1}^{n} w_{ij}$, thus, $G_i<0$) and other conditions are same as before. As seen in output output space simulation, the output can be regarded being stabilized after the time of 2.5us. Therefore, After the time of 2.5us, Eq. (15) for this case is as

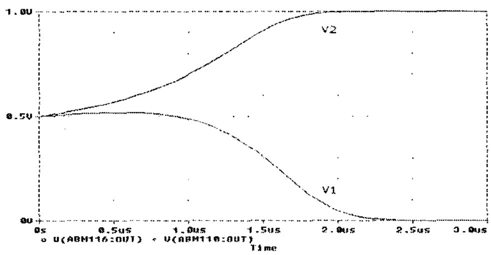
$$\begin{bmatrix} u_1(t-2.5us) \\ u_2(t-2.5us) \end{bmatrix} + \begin{bmatrix} \frac{0.9}{1.5} \\ -\frac{1.2}{1.5} \end{bmatrix} = \begin{bmatrix} u_1(2.5us)-\frac{0.9}{1.5} & 0 \\ 0 & u_2(2.5us)+\frac{1.2}{1.5} \end{bmatrix} \begin{bmatrix} e^{1.5\times 10^6(t-2.5us)} \\ e^{1.5\times 10^6(t-2.5us)} \end{bmatrix}$$

$$단, \quad t \geq 2.5us \qquad (24)$$

As seen in Fig. 6, inputs changes to diverge exponentially even after outputs are asymptotically stabilized.



(a)



(b)

Fig. 6 Transient simulation for 2bit A/D converter in case x=1.6, g1=g2=0.5, c1=c2=1u andα=2: (a) input, (b) output.

The last two simulations have shown that the input conductances, the values of which are not bigger than $-\sum_{j=1}^{n} w_{ij}$, make system inputs unstable. There is one thing kept in mind that input space is unstable under the certain circumstance such as in the last two cases, however output space always converges asymptotically to a stable point in any cases.

IV. Relation Between the third term Energy and the Input Conductance

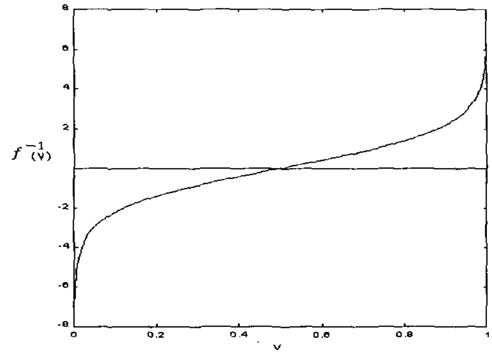
The first two terms in Eq. (6) are only used for mapping to the objective function which is to be minimized. The third term exists in the continuous type Hopfield networks satisfying energy function as a Lyapunov function. Therefore, it is recommendable to reduce this term as small as possible for forcing this energy function matched to the objective function. The common method for ignoring the third term is to use high sigmoid gain. However, as a sigmoid gain α goes infinite, the output of neuron becomes discrete rather than continuous.

In this section, we discuss new method for ignoring the third term of energy function with a finite sigmoid gain. With this new method, the system can keep the continuous output of neuron as well as ignore the third term of energy function.

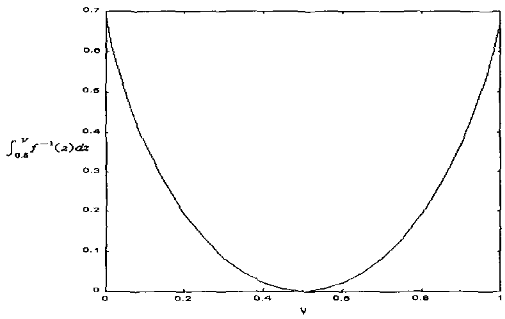
Using the inverse function $f^{-1}(z)$, the third term in Eq. (6) can be expressed as

$$E_3=\frac{1}{\alpha}\sum_{i=1}^{n} G_i \int_{\frac{1}{2}}^{v_i} \ln\left(\frac{z}{1-z}\right)dz \qquad (25)$$

The integral is zero for $v_i=1/2$ and positive otherwise, getting large as v_i approaches 0 or 1 because of slowness with which $f(v)$ approaches its asymptotes (Fig. 7a). As mentioned early in this section, it can be known from Eq. (25) that the common method for neglecting the third term of energy function E_3 is to select the sigmoid gain α very high.



(a)



(b)

Fig. 7. (a) The output-input transfer characteristic, $u=f^{-1}(V)$.
(b) The contribution of f^{-1} to the third term of energy function.

Consider now new method of ignoring the third term of energy function with the finite sigmoid gain. As known in Eq. (25), E_3 is a function of v_i, the sigmoid gain α, and the total conductances G_i. Here, we introduce a new method to eliminate this term by adjusting the total conductances G_i. To eliminate the third term energy, the total conductances G_i can be adjusted to zero.

$$G_i = g_i + \sum_{j=1}^{n} w_{ij} = 0 \qquad i = 1, 2, \ldots, n. \qquad (26)$$

Thus, the Input conduces g_i can be adjusted as follows

$$g_i = -\sum_{j=1}^{n} w_{ij} \qquad i = 1, 2, \ldots, n. \qquad (27)$$

This choice of g_i in the system would ensure convergence exactly to the objective function. However, it has been shown in the previous section that in case of the total conductances G_i equal to zero, the input space is unstable as it changes linearly in time. Therefore, the total conductances G_i should be selected as small as possible to neglect the third term energy, but greater than zero to make the input space stable.

Case Study

The 2-bit A/D converter is also selected for a case study. The energy function for 2-bit A/D converter including weights and bias currents can be expressed as follows [3], where x is analog input.

$$E = -\frac{1}{2}[v_1 \; v_2]\begin{bmatrix} 0 & -2 \\ -2 & 0 \end{bmatrix}\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} - [v_1 \; v_2]\begin{bmatrix} x - \frac{1}{2} \\ 2x - 2 \end{bmatrix} + \begin{bmatrix} \frac{G_1}{\alpha}\int_{0.5}^{v_1} \ln\left(\frac{z}{1-z}\right)dz \\ \frac{G_2}{\alpha}\int_{0.5}^{v_2} \ln\left(\frac{z}{1-z}\right)dz \end{bmatrix} \qquad (28)$$

Fig. 8 shows the energy map and transient result in case of x=1.6, α=2, and $g_1 = g_2 = 2.8$. We can see that the system converges near to the answer (the digital outputs should be v_1=0 and v_2=1 for analog input x=1.6). However the system would converge farther to the correct answer as the total conductances G_i grows, because this minima moves toward the center of the energy map as the third term energy grows. Fig. 9 shows the energy map and transient result in case of x=1.6, α=2, and $g_1 = g_2 = 2.1$. To neglect the third term energy, $g_1 = g_2 = 2.1$ are selected. As seen in Fig. 9, the minima is located more closely at the correct answer compared as in Fig. 8, and so converges the system in transient simulation.

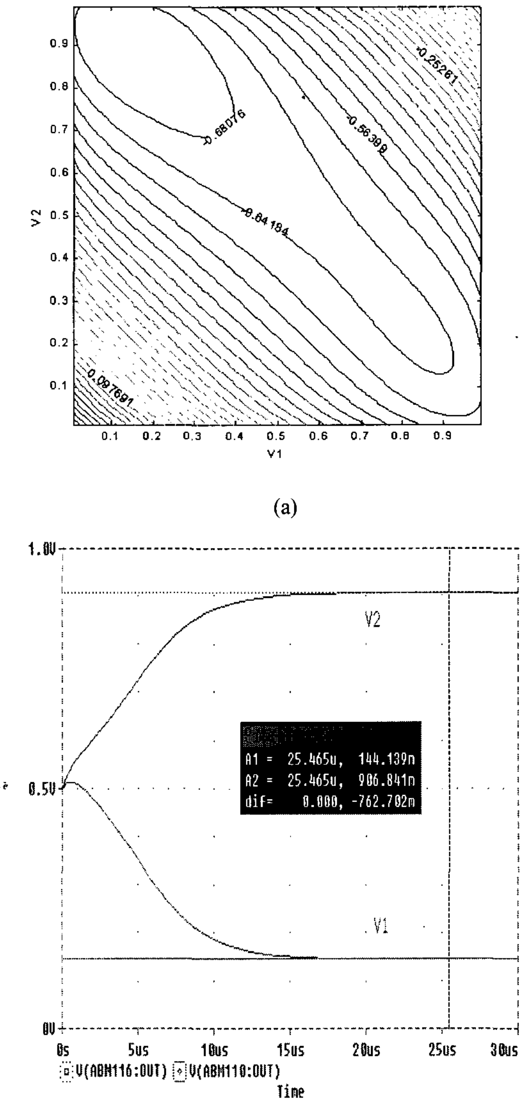
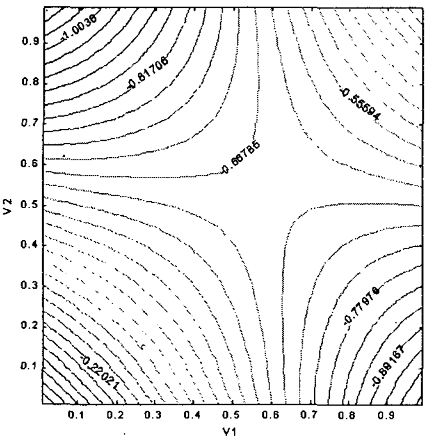
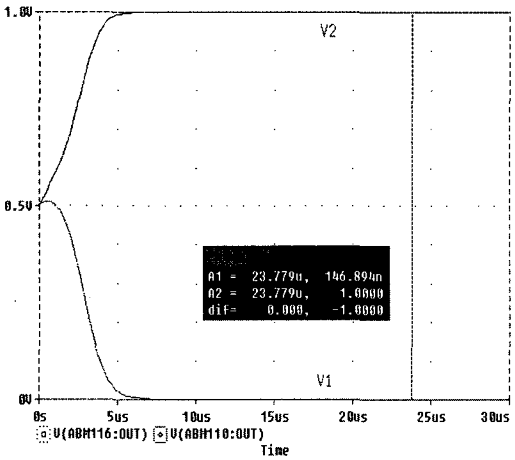


Fig. 8. 2bit A/D converter in case x=1.6, g1=g2=2.8, and α=2:

(a) The energy map, (b) Transient simulation of output.



(a)



(b)

Fig. 9. 2bit A/D converter in case x=1.6, gl=g2=2.1, andα=2: (a) The energy map (b) Transient simulation of output.

V. Conclusion

The conductance and capacitance at a neuron's input leading to ground have been known partially to define the system's time constant and to effect the system's stability. However, not much things about these components have been published, therefore choosing the adequate value of these components would be one of the problems occurred during implementing hardwares or simulating circuit level systems.

In this paper, the input conductance connected parallel with capacitance has been analyzed for the effects on stability of systems. The input conductance has been proved to be able to effect much on stability in input space and, on improving the performance of the system. That is, in case of the total conductance G_i >0, the input of system converges to a stable point decreasing exponentially. However, in this case, the output converges to other point rather than the minima of objective function, because the third term of energy function becomes large as the total conductance G_i gets grow. In case of G_i=0, the third term of energy function is eliminated, but the input of system is analyzed unstable, because the input changes linearly in time. In case of G_i<0, the input of system changes exponentially in time, and results in unstable.

Therefore, the input conduces g_i have to be adjusted greater than zero for proving the stability and but near to zero for improving the performance of the system.

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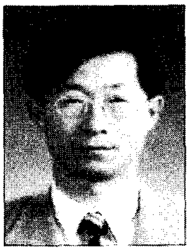
저 자 소 개

康 珉 齊 (正會員)



1958년 4월 27일생. 1982년 서울대 전기공학과 졸업. 1991년 Univ. of Louisville 전기공학과 졸업(공박). 1992년~현재 제주대 전기. 전자공학부 부교수, 주관심 분야: 신경회로망, 신호처리.

Tel : 064-754-3666 Fax : 064-756-1745

E-mail : minjk@cheju.cheju.ac.kr