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The Effect of Structural Factors on the Torsional Rigidity of Yarns

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Abstract: In this paper, in order to examine the torsional behaviour of twisted yarn closely, the torsional rigidity would be derived in terms of physical and mechanical characteristics of its constituent fibers and yarn structural parameters by energy-method. And the propriety of the theory will be discussed by comparing with experimental results. The torsional rigidity of yarn in both experimental and theoretical results decreases with surface helix angle increases. But the experimental values are more higher than those of the theoretical ones.

Key words: torsional rigidity, structural parameters, energy-method, strain energy

INTRODUCTION

The torsional rigidity of yarn is dependent upon a combination of inherent fiber's properties, i.e., torsional and bending properties as well as upon the geometrical arrangement of fibers in the yarn. And this torsional rigidity of yarns is the property which affects largely to stiffness and drape of textile products.

The practical problems of torque in yarns can hardly be questioned if it be considered that the following undoubtedly incomplete list of yarn and fabric characteristics depends upon yarn torque in whole or in part: single yarn wildness; plied yarn balance; knitted goods curling tendencies; fabric drape; fabrics crease resistance; the production of novelty effects in fabrics.

Torsional stress during twisting process is related to the mechanical properties and torsional stability of yarns. When tension, after twisting and setting process, is applied to the yarn with torque free-state, the torque is occurred, and this takes place snarling and buckling problems.

Therefore, in order to solve these problems, many researches had been proceeded about single yarn's torsional rigidity for a long time.

Platt (1958) had derived the theoretical torsional stress of yarn from the structure of yarn is an ideal helix structure and elastic. But he neglected the frictional force between fibers in the yarn.

Freeston (1966) studied about torque balance of the plied yarn. He derived the theoretical equation on the torsional moment and torsional recovery under assumption that yarns are regarded as viscoelastic materials, but he neglected contribution of tensile and bending occurring during twisting yarn.

Zurek (1980) studied on the theoretical torsional rigidity of cotton-polyester blended yarn , and he discussed the propriety of the theory by comparing with experimental results.

Postle (1976, 1979) analyzed the yarn torque developed by fiber tensile stress in the plied yarn during twisting process and he studied about the yarn torque and its dependence on distribution of fiber tensile stress in the yarn. But most of researches were about the torsional rigidity of single yarns during spinning process. And the most of theoretical analysis method had been proceeded by the force-method (Postle, 1964; Kim, 1988).

In this paper, in order to examine the torsional behaviour of twisted yarn closely, the torsional rigidity was derived in the terms of physical and mechanical characteristics of its constituent fibers and yarn structural parameters by energy-method. And then, the propriety of the theory was discussed by comparing with experimental results.

THEORETICAL

Assumptions

In order to derive the theoretical torsional rigidity of twisted yarn, the assumptions were established as follows

(1) The cross-sections of twisted yarns and its constituent fibers are circular.

(2) The diameter of fiber is negligible small as compared with that of yarn.

(3) There is no interaction between fibers in the yarn.

(4) The yarn and its constituent fiber are elastic.

(5) The yarn structure is an ideal helix structure before and after deformation.

According to the above assumptions, the model of

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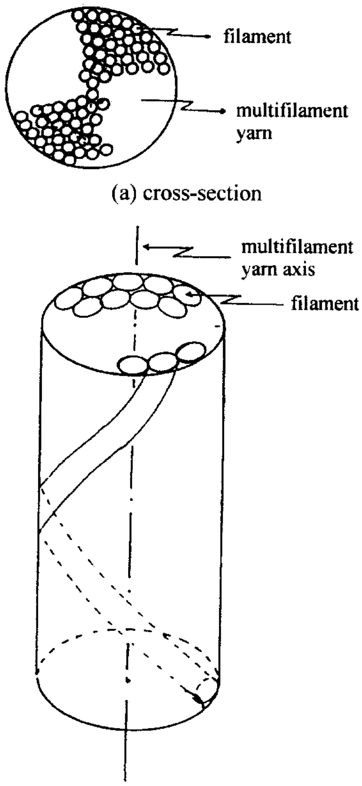

(a) cross-section

(b) lateral view

Fig. 1. The configuration of yarn.

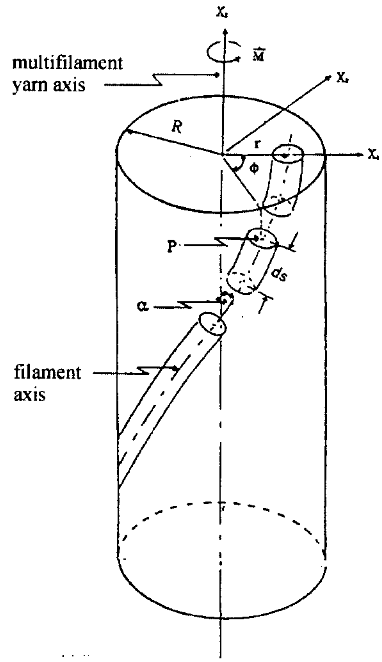

Fig. 2. The path of single filament in multifilament yarn.

twisted yarn's structure was shown in Fig. 1.

The torsional rigidity of constituent fiber

In order to derive the torsional rigidity of fiber, the geometry path of fibers in the yarn was shown in Fig. 2.

At Fig. 2, R is the radius of yarn, p is the point on fiber axis with distance r from the yarn axis. And ϕ is the rotation angle of p around yarn axis, ds is differential fiber length and α is helix angle of fiber with distance r from yarn axis. In order to find out occurring moment at p, when the torsional moment is applied to the yarn, part of ds in Fig. 2 is enlarged at Fig. 3.

The torsional moment, M which is applied to yarn axis, can be resolved into the torsional moment, M_t, with tangential direction to fiber cross-section and the bending moment, M_b, with normal direction to that.

$$M_t = M \cos \alpha \tag{1}$$

$$M_b = M \sin \alpha \tag{2}$$

The torsion, τ_f, due to the torsional moment M_t at p on fiber becomes as;

$$\tau_f = \frac{M\cos\alpha}{GI_p} \tag{3}$$

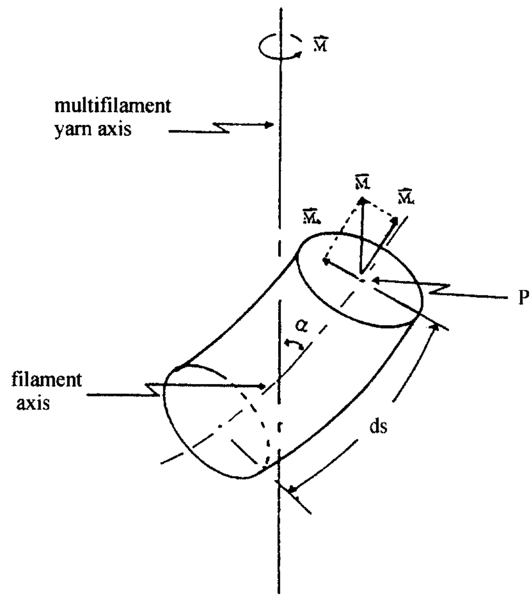

Fig. 3. Infinitesimal element of fiber.

where, GI_p is the torsional rigidity of straight-state fiber.

The strain energy by torsion per unit length of fiber can be rewritten as Equation (4).

$$\frac{1}{2}M_t\tau_f = \frac{1}{2}\frac{M^2\cos^2\alpha}{GI_p} \tag{4}$$

The curvature κ_f at p, due to the bending moment, M_b, is

$$\kappa_f = \frac{M\sin\alpha}{EI} \quad (5)$$

where, EI is the bending rigidity of straight-state fiber. And the strain energy by bending per unit length of fiber can be rewritten as Equation (6).

$$\frac{1}{2}M_b\kappa_f = \frac{1}{2}\frac{M^2\sin^2\alpha}{EI} \quad (6)$$

Therefore, the total strain energy due to κ_f and τ_f at differential length of fiber, ds can be expressed as follow Equation (7).

$$du = \frac{1}{2}M^2\left(\frac{\cos^2\alpha}{GI_p} + \frac{\sin^2\alpha}{EI}\right)ds \quad (7)$$

The calculated component position vectors at point, p are as follows

$$X_1 = r\cos\phi$$
$$X_2 = r\sin\phi$$
$$X_3 = r\phi \quad (8)$$

From Equation (8), ds can be obtained as Equation (9) by using of differential geometry.

$$\frac{ds}{d\phi} = \sqrt{\sum_{i=1}^{i=3}\left(\frac{dX_i}{d\phi}\right)\left(\frac{dX_i}{d\phi}\right)}$$
$$ds = \gamma \operatorname{cosec}\alpha \, d\phi \quad (9)$$

From Equations (7), (9), and (10), the strain energy, du of fiber can be rewritten as Equation (11).

$$\psi = \frac{GI_p}{EI} \quad (10)$$

$$du = \frac{1}{2}M^2 r\operatorname{cosec}\alpha\left(\frac{\cos^2\alpha + \psi\sin^2\alpha}{GI_p}\right)d\phi \quad (11)$$

Here, when the fiber has one turn around the yarn axis, the strain energy due to bending and torsion is as follows;

$$U = \int_{\phi=0}^{\phi=2\pi}\frac{1}{2}M^2 r\operatorname{cosec}\alpha\left(\frac{\cos^2\alpha+\psi\sin^2\alpha}{GI_p}\right)d\phi$$

$$U = \pi r M^2 r\operatorname{cosec}\alpha\left(\frac{\cos^2\alpha+\psi\sin^2\alpha}{GI_p}\right) \quad (12)$$

The torsion, θ of center of helix fiber might be occurred due to the torsional moment applied to yarn axis. The work done, dW at the differential length dX_3 of helical axis which was developed due to torsion, θ is as follows;

$$dW = \frac{1}{2}M\times\theta\times dX_3 \quad (13)$$

The term, dX_3 in Equation (13) can be expressed as Equation (14), and then, we can substitute Equation (14) into Equation (13). The dW can be derived as equation (15).

$$dX_3 = \frac{r\,d\phi}{\tan\alpha} \quad (14)$$

$$dW = \frac{1}{2}M\theta\frac{r\,d\phi}{\tan\alpha} \quad (15)$$

When the fiber has one turn around the yarn axis, the work done can be expressed as follow Equation (16).

$$W = \frac{1}{2}\int_{\phi=0}^{\phi=2\pi}M\theta\frac{r}{\tan\alpha}d\phi = M\theta\frac{\pi r}{\tan\alpha} \quad (16)$$

On the other hand, according to the assumption that there is no interaction between fibers in the yarn, the strain energy of one fiber is as same as the work done of helical axis. Therefore, the torsional rigidity, H_f of the helical fiber can be derived from Equations (12), (16) and (17). Here, the equation (17) shows the relation between the torsional moment and torsion.

$$H_f = \frac{M}{\theta} \quad (17)$$

$$H_f = \frac{GI_p}{\cos\alpha + \frac{\sin^2\alpha}{\cos\alpha}\psi} \quad (18)$$

The torsional rigidity, H_f of the helical fiber is derived in the terms of the ratio, ψ, between the torsional and bending rigidity of fiber with straight-state and the structural parameter helix angle, α.

The torsional rigidity of yarns

If there is no interaction between fibers in the yarn, the theoretical minimum torsional rigidity of yarn can be expressed as equation (19).

$$H_y = N\times H_f = \frac{m_y}{m_f}\times H_f \quad (19)$$

Where,
H_y: the theoretical minimum torsional rigidity of yarn
H_f: the torsional rigidity of helical fiber
m_f: linear density of fiber
m_y: linear density of yarn
N : the number of fibers in the yarn

On the other hand, according to the assumption that the yarn structure is an ideal helix structure before and after deformation, the helical structure may be maintained after applying torsional moment to the yarn axis.

Here, let's consider the differential element dA in the yarn cross-section at distance r from the yarn axis as Fig. 4.

$$dA = 2\pi r dr \quad (20)$$

The number of fiber per unit area perpendicular to yarn

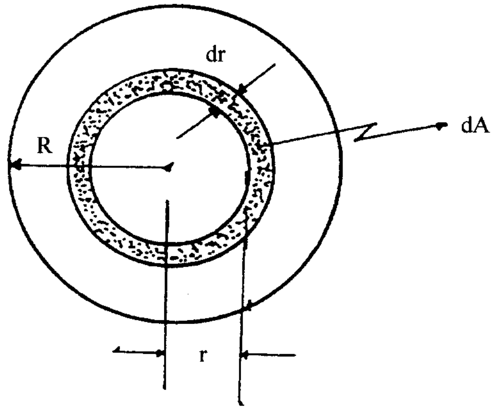

Fig. 4. Infinitesimal element of cross-section.

cross-section before twisting is;

$$n_0 = \frac{N}{\pi R^2} \tag{21}$$

After twisting, the number of fiber per unit area of yarn will vary as following Equation (22).

$$n_1 = n_o \cos\alpha \tag{22}$$

Therefore, the number of fiber in differential area, *dA* can be expressed as follows;

$$dn_1 = n_o \cos\alpha \; 2\pi r \, dr \tag{23}$$

The Equation (23) can be replaced as the yarn and fiber linear density.

$$dn_1 = \frac{m_y}{\pi R^2 m_f} \cos\alpha \; 2\pi r \, dr \tag{24}$$

From the assumption that the diameter of fiber is negligible small as compared with that of yarn, the torsional rigidity of yarn can be derived from Equation (18) and Equation (24).

$$H_y = \int H_f \, dn_1 = \frac{2\pi G I_p m_y}{\pi R^2 m_f} \int_{r=0}^{r=R} \frac{r\cos\alpha}{\cos\alpha + \frac{\sin^2\alpha}{\cos\alpha}\psi} dr$$

$$= \frac{2 G I_p m_y}{R^2 m_f} \int_{r=0}^{r=R} \frac{r}{1+\psi\tan^2\alpha} dr \tag{25}$$

The fiber helix angle, α which located at distance *r* from the yarn axis is given as the function of *r*. By using Equation (26), let's integrate Equation (25).

$$\tan\alpha = 2\pi r t \tag{26}$$

Where, t is turns per unit length of yarn.

$$H_y = \frac{2\pi G I_p m_y}{R^2 m_f} \int_{r=0}^{r=R} \frac{r}{1+\psi(2\pi r t^2)} dr$$

$$= \frac{2 G I_p m_y}{R^2 m_f} \left\{ \frac{1}{2\psi(2\pi t)^2} \ln[\psi(2\pi R t^2)+1] \right\} \tag{27}$$

Let's substitute the term *2t* and *2πRt* in Equation (27) into the surface helix angle, Q and then, the torsional rigidity of yarn can be derived in the terms of constituent fiber properties, i.e., bending and torsional rigidity, yarn structure parameter, and surface helix angle as following Equation (28).

$$H_y = \frac{G I_p m_y}{m_f \psi \tan^2 Q} \ln[\psi \tan^2 Q + 1] \tag{28}$$

EXPERIMENTAL

Samples

Polyester multi-filament yarns which were five different types were employed. In order to prevent from yarn snarling, the yarns were setted by heat.

The measurement of bending and torsional rigidity of fibers

The Young's modulus was measured by Instron (Kato-Tech), and the bending rigidity was calculated from Young's modulus and fibers diameter. The torsional rigidity was measured by KES-YN tester.

Surface helix angle and linear density of yarn

The surface helix angle was measured by microscope, and the linear density was measured by the method of KSK-0415.

The torsional rigidity of yarn

Torsional rigidity of samples were measured by KES-

Table 2. Yarn characteristics

Sample code	Turns per unit length (t.p.m.)	Linear density (tex)	Helix angle (degree)
A-1	200	97.6	10.8
A-2	240	98.3	12.8
A-3	300	99.6	16
A-4	340	100.6	18
A-5	400	102.7	21.2

Table 1. Fiber characteristic

Properties of fiber	Diameter μm	Linear density (denier)	E gf/cm^2 $\times 10^6$	G gf/cm^2 $\times 10^6$	EI gf/cm^2 $\times 10^5$	GI_p gf/cm^2 $\times 10^5$	ψ GI_p/EI	Density g/cc
polyester	25.38	6.3	125.9	24.1	25.64	9.82	0.38	1.38

YN tester.

RESULTS AND DISCUSSION

In order to investigate the relationship between surface helix angle of yarn and the torsional rigidity of one for various ψ, the theoretical Equation (28) was modified as follows;

$$\frac{H_y m_f}{m_y G I_p} = \frac{1}{\psi \tan^2 Q} \ln[\psi \tan^2 Q + 1] \quad (29)$$

And Fig. 5 shows the relationship between surface helix angle and the torsional rigidity of yarn for various ψ from Equation (29). When the paralleled fiber bundle are twisted, each constituent fibers may be bent and twisted. Therefore, the torsional rigidity of yarn may be influenced by fiber's bending and torsional rigidity.

Fig. 5 shows that the torsional rigidity of yarn increases with the ψ decreases. This means that the bending rigidity of fiber contributes to the yarn's torsional rigidity more than torsional rigidity of that.

In this paper, because the theoretical Equation (29) was derived under the assumption that there is no interaction between fibers, each constituent fibers in the yarn may be easily bent and twisted, when applied torsional moment to the yarn axis. The larger the resistance of fiber to bend is than the resistance of fiber to twist, the more the torsional rigidity of yarn increase.

These theoretical results are in accordance with Zurek's research (1980).

On the other hand, Fig. 5 shows that the torsional rigidity of yarn decreases with surface helix angle increases.

Table 3. Theoretical and experimental torsional rigidity of yarns

Sample code	Specific torsional rigidity (gf·cm²/tex× 10⁻⁶)	
	Theoretical value	Experimental value
A-1	13.93	14.06
A-2	13.89	14.06
A-3	13.81	13.89
A-4	13.75	13.86
A-5	13.63	13.81

In practice, when the yarn would be twisted, the lateral force in the yarn may be occurred. Frictional force between fibers to resist fiber slippage is taken place by this lateral force. And this lateral forces will increase the torsional rigidity of yarns. But, in this paper, because of the assumption that there is no interaction between fibers, the torsional rigidity of yarn decreases with surface helix angle increases, because of the obliquity of fiber to yarn axis. This results are in accord with Zurek's (1980) and Platt's (1958) researches.

To examine the propriety of the theory, the theoretical results were compared with experimental results. In experimental torsional rigidity is the rigidity that exclude the tension effect to yarn torsional rigidity, when the yarn was twisted. The theoretical and experimental results were shown in Table 3.

The relationship between surface helix angle and the specific torsional rigidity of yarn was shown as Fig. 6.

Here, the specific torsional rigidity of yarn is the rigidity that was neglected the increasing effect of linear density with twist increasing.

In order to discuss the relationship between surface helix angle and the specific torsional rigidity of yarn, the the-

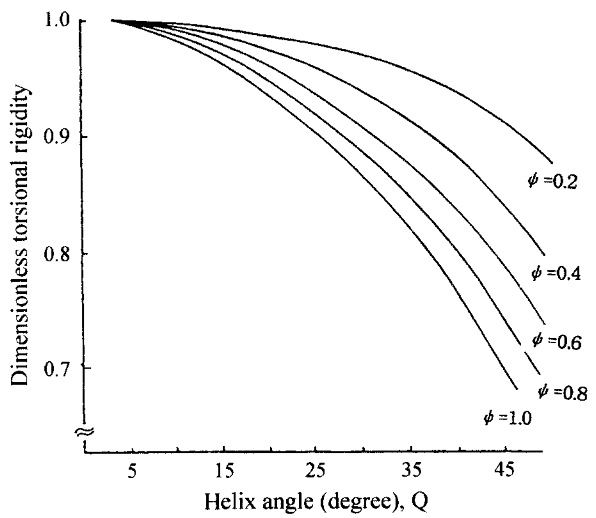

Fig. 5. Dimensionaless multifilament yarn torsional rigidity to helix angle for various ψ.

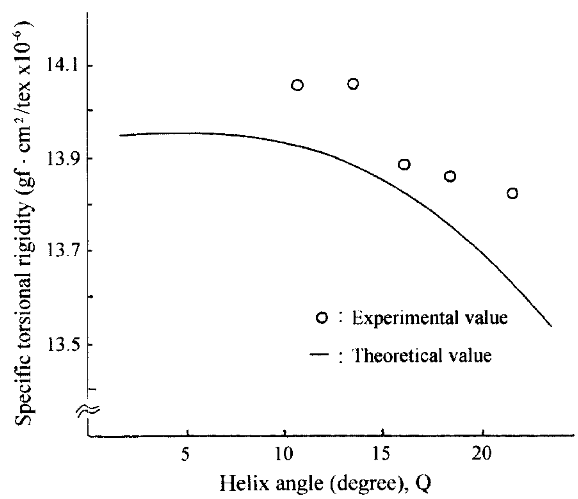

Fig. 6. Theo retical and experimental specific torsional rigidity to helix angle for multifilament yarn.

oretical Equation (27) was modified as follows

$$\frac{H_y}{m_y} = \frac{GI_p}{m_f \psi \tan^2 Q} \ln[\psi \tan^2 Q + 1] \qquad (30)$$

In Fig. 6, the torsional rigidity of yarn in both experimental and theoretical results decreases with surface helix angle increases. This reason was thought as the effect of the increasing fiber's obliquity to yarn axis with twist increasing.

Both experimental and theoretical results are same tendency, but the experimental values are more higher than the theoretical ones. Because the lateral force to prevent fiber slippage was neglected in the theoretical Equation (30), the experimental values is much higher than the theoretical ones.

CONCLUSIONS

The torsional rigidity of twisted yarn was derived in the terms of physical and mechanical characteristics of its constituent fibers and yarn structural parameters by energy-method. And the propriety of the theory was examined by comparing with experimental results.

The obtained results were as follows.

1. The theoretical torsional rigidity of yarn was derived in the terms of physical and mechanical characteristic of fiber and yarn structural parameters as follows.

$$H_y = \frac{GI_p m_y}{m_f \psi \tan^2 Q} \ln[\psi \tan^2 Q + 1]$$

2. Theoretical analysis was shown that the torsional rigidity of yarn increases with ψ decrease.

3. The torsional rigidity of yarn in both experimental and theoretical results decreases with surface helix angle increases. But the experimental values are more higher than those of the theoretical ones.

REFERENCES

Platt M. M., Klein. W. G. and Hamburger.W. J. (1958) Mechanics of elastic performance of textile materials Part XIII : Torque development in yarn system : single yarn, *Textile, Res. J.*, **28**, 1-14.

Freestone W. D., Platt M. M. and Butter Worth (1966) Mechanics of elastic performance of textile materials Part XVII : Torsional recovery of filaments and single yarn and plied yarn balance, *Textile. Res. J.*, **36**, 12-30.

Postle R. and Dhingra R. C. (1976) Analysis of the torque developed by fiber tensile stress in plied yarn structure, *J. Textile. Inst.*, **67**, 409-412.

Postle R. and Bennet J. M. (1979) A study of yarn torque and its dependence on distribution of fiber tensile stress in the yarn Part I : Theoretical analysis, *J. Textile. Inst.*, **70**, 121-132.

Postle R. and Bennet J. M (1979) A study of yarn torque and its dependence on distribution of fiber tensile stress in the yarn Part II : Experimental, *J. Textile. Inst.*, **70**, 133-141.

Postle R., Burton P. and Chaikin M (1964) The torque in twisted singles yarns, *J. Textile. Inst.*, **55**, 448-461.

Zurek W. and Durska I. (1980) The torsional rigidity of blend yarns, *Textile. Res. J.*, **50**, 555-567.

Kim S. J. (1988) Thesis, effects of fibre characteristics and yarn structure on the torsional rigidity of spun yarns, Seoul National University (1988).

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