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Boundary Controllability of Delay Integrodifferential Systems in Banach Spaces

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Abstract

Sufficient conditions for boundary controllability of time varying delay integrodifferential systems in Banach spaces are established. The results are obtained by using the strongly continuous semigroup theory and the Banach contraction principle.

1. Introduction

Controllability of nonlinear systems represented by ordinary differential equations in Banach spaces has been extensively studied by several authors. Balachandran et al. [1] studied the controllability of nonlinear integrodifferential systems in Banach spaces whereas in [2] they have investigated the local null controllability of nonlinear functional differential systems. Controllability of nonlinear functional integrodifferential systems in Banach spaces has been studied by Park and Han [11]. Kwun et al. [9] discussed the approximate controllability for delay Volterra systems while Balachandran and Sakthivel [3] established a set of sufficient conditions for the controllability of delay integrodifferential systems in Banach spaces.

Several abstract settings have been developed to describe the distributed control systems on a domain Ω in which the control is acted through the boundary Γ. But in these approaches one can encounter the difficulty for the existence of sufficiently regular solution to state space system, the control must be taken in a space of sufficiently smooth functions. A semigroup approach to boundary input problems for linear differential equations was first presented by Fattorini[7]. This approach was extended by Balakrishnan [4] where he showed that the solution of a parabolic boundary control equation with L^2 controls can be expressed as a mild solution to an operator equation. Barbu and Precupanu [5] studied a class of convex control problems governed by linear evolution systems covering the principal boundary control systems of parabolic type. In [6] Barbu investigated a class of boundary-distributed linear control systems in Banach spaces. Lasiecka [10] established the regularity of optimal boundary controls for

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parabolic equations with quadratic cost criterion. Recently Han and Park [8] derived a set of sufficient conditions for the boundary controllability of a semilinear system with a nonlocal condition. The purpose of this paper is to study the boundary controllability of time varying delay integrodifferential systems in Banach spaces by using the Banach fixed point theorem.

2. Preliminaries

Let E and U be a pair of real Banach spaces with $\|.\|$ and $|.|$, respectively. Let σ be a linear closed and densely defined operator with $D(\sigma) \subseteq E$ and $R(\sigma) \subseteq E$ and let θ be a linear operator with $D(\theta) \subseteq E$ and $R(\theta) \subseteq X$, a Banach space.

Consider the boundary control delay system of the form

$$\begin{aligned} \dot{x}(t) &= \sigma x(t) + f(t, x(\gamma_1(t)), x(\gamma_2(t)), \ldots, x(\gamma_n(t))), \quad t \in J = [0, b], \\ \theta x(t) &= B_1 u(t), \\ x(0) &= x_0, \end{aligned} \tag{1}$$

where $\gamma_i(t), i = 1, 2, \ldots, n$ are continuous functions, the state $x(.)$ takes values in the Banach space E, $B_1 : U \to X$ is a linear continuous operator, the control function $u \in L^1(J, U)$, a Banach space of admissible control functions and the nonlinear operator $f : J \times E^n \to E$ is continuous.

Let $A : E \to E$ be a linear operator defined by

$$D(A) = \{x \in D(\sigma); \theta x = 0\}, \quad Ax = \sigma x, \quad \text{for} \ \ x \in D(A).$$

Let $B_r = \{y \in E : \|y\| \le r\}$, for some $r > 0$. We shall make the following hypotheses:

(A_1) $D(\sigma) \subset D(\theta)$ and the restriction of θ to $D(\sigma)$ is continuous relative to graph norm of $D(\sigma)$.

(A_2) The operator A is the infinitesimal generator of a C_0 semigroup $T(t)$ and there exists a constant $M > 0$ such that $\|T(t)\| \le M$.

(A_3) There exists a linear continuous operator $B : U \to E$ such that $\sigma B \in L(U, E)$, $\theta(Bu) = B_1 u$, for all $u \in U$. Also $Bu(t)$ is continuously differentiable and $\|Bu\| \le C\|B_1 u\|$ for all $u \in U$, where C is a constant.

(A_4) For all $t \in (0, b]$ and $u \in U$, $T(t)Bu \in D(A)$. Moreover, there exists a positive function $\nu \in L^1(0, b)$ such that $\|AT(t)B\| \le \nu(t)$, a.e. $t \in (0, b)$ and choose a constant $K > 0$ such that $\int_0^b \nu(t)dt \le K$.

If $x(t)$ is the solution of (1), then we can define a function $z(t) = x(t) - Bu(t)$ and from our assumption we see that $z(t) \in D(A)$. Hence (1) can be written in terms of A and B as

$$\begin{aligned} \dot{x}(t) &= Az(t) + \sigma Bu(t) + f(t, x(\gamma_1(t)), x(\gamma_2(t)), \ldots, x(\gamma_n(t))), \quad t \in J \qquad (2) \\ x(t) &= z(t) + Bu(t), \\ x(0) &= x_0, \end{aligned}$$

If u is continuously differentiable on $[0, b]$ then z can be defined as a mild solution to the Cauchy problem

$$\begin{aligned} \dot{z}(t) &= Az(t) + \sigma Bu(t) - B\dot{u}(t) + f(t, x(\gamma_1(t)), x(\gamma_2(t)), \ldots, x(\gamma_n(t))), \\ z(0) &= x_0 - Bu(0), \end{aligned}$$

and the solution of (1) is given by

$$\begin{aligned} x(t) &= T(t)[x_0 - Bu(0)] + Bu(t) + \int_0^t T(t-s)[\sigma Bu(s) - B\dot{u}(s) \\ &\quad + f(s, x(\gamma_1(s)), x(\gamma_2(s)), \ldots, x(\gamma_n(s)))]ds. \qquad (3) \end{aligned}$$

Since the differentiability of the control u represents an unrealistic and severe requirement, it is necessary to extend the concept of the solution for the general inputs $u \in L^1(J, U)$. Integrating (3) by parts, we get

$$\begin{aligned} x(t) &= T(t)x_0 + \int_0^t [T(t-s)\sigma - AT(t-s)]Bu(s)ds \\ &\quad + \int_0^t T(t-s)f(s, x(\gamma_1(s)), x(\gamma_2(s)), \ldots, x(\gamma_n(s)))ds. \qquad (4) \end{aligned}$$

Thus (4) is well defined and it is called a mild solution of the system(1).

Definition: The system (1) is said to be controllable on the interval J if for every $x_0, x_1 \in E$, there exists a control $u \in L^2(J, U)$ such that the solution $x(.)$ of (1) satisfies $x(b) = x_1$.

We further assume the following conditions:

(A_5) The linear operator W from $L^2(J, U)$ into E defined by

$$Wu = \int_0^b [T(b-s)\sigma - AT(b-s)]Bu(s)ds$$

induces an invertible operator $\tilde{W}$ defined on $L^2(J, U)/kerW$ and there exists a positive constant $K_1 > 0$ such that $\|\tilde{W}^{-1}\| \leq K_1$.

(i) $f : J \times E^n \to E$ is continuous and there exist constants M_1 and M_2 such that for all $v_i, w_i \in B_r$, $i = 1, 2, \ldots, n$ we have

$$\|f(t, v_1, v_2, \ldots, v_n) - f(t, w_1, w_2, \ldots, w_n)\| \le M_1 \sum_{i=1}^{n} \|v_i - w_i\|$$

and

$$M_2 = \max_{t \in J} \|f(t, 0, \ldots, 0)\|.$$

(ii) There exists a constant q such that for all x_1, $x_2 \in E$

$$\|x_1(\gamma_i(t)) - x_2(\gamma_i(t))\| \le q\|x_1(t) - x_2(t)\|, \quad \text{for} \quad i = 1, 2, \ldots n.$$

(iii) $M\|x_0\| + K_1[bM\|\sigma B\| + K][\|x_1\| + M\|x_0\| + L] + L \le r$, where $L = bM(M_1 nr + M_2)$.

(iv) Let $p = nqbMM_1[1 + (bM\|\sigma B\| + K)K_1]$ be such that $0 \le p < 1$.

3. Controllability of Delay System

Theorem:3.1 If the hypotheses (A_1)-(A_5) and (i) - (iv) are satisfied , then the boundary control delay system (1) is controllable on J.

Proof: Let $Y = C(J, B_r)$. Using the hypothesis (A_5), for an arbitrary function $x(.)$ define the control

$$u(t) = \tilde{W}^{-1}[x_1 - T(b)x_0 - \int_0^b T(b-s)f(s, x(\gamma_1(s)), x(\gamma_2(s)), \ldots, x(\gamma_n(s)))ds](t). \quad (5)$$

We shall show that, when using this control, the operator Ψ defined on Y by

$$\begin{aligned}\Psi x(t) \;=\; & T(t)x_0 + \int_0^t [T(t-s)\sigma - AT(t-s)]B\tilde{W}^{-1}[x_1 - T(b)x_0 \\ & - \int_0^b T(b-\tau)f(\tau, x(\gamma_1(\tau)), x(\gamma_2(\tau)), \ldots, x(\gamma_n(\tau)))d\tau](s)ds \\ & + \int_0^t T(t-s)f(s, x(\gamma_1(s)), x(\gamma_2(s)), \ldots, x(\gamma_n(s)))ds\end{aligned}$$

has a fixed point. This fixed point is then a solution of (1). Clearly $\Psi x(b) = x_1$, which means that the control u steers the delay system (1) from the initial state x_0 to x_1 in time b provided we can obtain a fixed point of the operator Ψ.
First we show that Ψ maps Y into itself. For $x \in Y$,

$$\|\Psi x(t)\| \;\le\; \|T(t)x_0\| + \|\int_0^t [T(t-s)\sigma - AT(t-s)]B\tilde{W}^{-1}[x_1 - T(b)x_0$$

$$
\begin{aligned}
&\qquad - \int_0^b T(b-\tau) f(\tau, x(\gamma_1(\tau)), x(\gamma_2(\tau)), \ldots, x(\gamma_n(\tau))) d\tau](s) ds\| \\
&\qquad + \| \int_0^t T(t-s) f(s, x(\gamma_1(s)), x(\gamma_2(s)), \ldots, x(\gamma_n(s))) ds\| \\
\leq\ & \|T(t)x_0\| + \int_0^t \|T(t-s)\| \|\sigma B\| \|\tilde{W}^{-1}\| [\|x_1\| + \|T(b)x_0\| \\
&\qquad + \int_0^b \|T(b-\tau)\| [\|f(\tau, x(\gamma_1(\tau)), x(\gamma_2(\tau)), \ldots, x(\gamma_n(\tau))) \\
&\qquad\qquad - f(\tau, 0, \ldots, 0)\| + \|f(\tau, 0, \ldots, 0)\|] d\tau] ds \\
&+ \int_0^t \|AT(t-s)B\| \|\tilde{W}^{-1}\| [\|x_1\| + \|T(b)x_0\| \\
&\qquad + \int_0^b \|T(b-\tau)\| [\|f(\tau, x(\gamma_1(\tau)), x(\gamma_2(\tau)), \ldots, x(\gamma_n(\tau))) \\
&\qquad\qquad - f(\tau, 0, \ldots, 0)\| + \|f(\tau, 0, \ldots, 0)\|] d\tau] ds \\
&+ \int_0^t \|T(t-s)\| [\|f(s, x(\gamma_1(s)), x(\gamma_2(s)), \ldots, x(\gamma_n(s))) \\
&\qquad\qquad - f(s, 0, \ldots, 0)\| + \|f(s, 0, \ldots, 0)\|] ds \\
\leq\ & M\|x_0\| + bM\|\sigma B\| K_1 [\|x_1\| + M\|x_0\| + bM(M_1 nr + M_2)] \\
&+ KK_1 [\|x_1\| + M\|x_0\| + bM(M_1 nr + M_2)] \\
&+ bM(M_1 nr + M_2) \\
\leq\ & M\|x_0\| + K_1[bM\|\sigma B\| + K][\|x_1\| + M\|x_0\| + L] + L \\
\leq\ & r.
\end{aligned}
$$

Thus Ψ maps Y into itself. Now, for $x_1, x_2 \in Y$ we have

$$
\begin{aligned}
&\|\Psi x_1(t) - \Psi x_2(t)\| \\
&\leq \int_0^t [\|T(t-s)\| \|\sigma B\| + \|AT(t-s)B\|] \|\tilde{W}^{-1}\| [\int_0^b \|T(b-\tau)\| \\
&\qquad \|f(\tau, x_1(\gamma_1(\tau)), x_1(\gamma_2(\tau)), \ldots, x_1(\gamma_n(\tau))) \\
&\qquad - f(\tau, x_2(\gamma_1(\tau)), x_2(\gamma_2(\tau)), \ldots, x_2(\gamma_n(\tau)))\| d\tau] ds \\
&\quad + \int_0^t \|T(t-s)\| \|f(s, x_1(\gamma_1(s)), x_1(\gamma_2(s)), \ldots, x_1(\gamma_n(s))) \\
&\qquad - f(s, x_2(\gamma_1(s)), x_2(\gamma_2(s)), \ldots, x_2(\gamma_n(s)))\| ds \\
&\leq b[M\|\sigma B\| + K] K_1 bMM_1 [\|x_1(\gamma_1(\tau)) - x_2(\gamma_1(\tau))\| \\
&\qquad + \|x_1(\gamma_2(\tau)) - x_2(\gamma_2(\tau))\| + \ldots + \|x_1(\gamma_n(\tau)) - x_1(\gamma_n(\tau))\|] \\
&\quad + bMM_1 [\|x_1(\gamma_1(s)) - x_2(\gamma_1(s))\| \\
&\qquad + \|x_1(\gamma_2(s)) - x_2(\gamma_2(s))\| + \ldots + \|x_1(\gamma_n(s)) - x_1(\gamma_n(s))\|] \\
&\leq bMM_1 [1 + (bM\|\sigma B\| + K) K_1] \sup_{t \in J} [\|x_1(\gamma_1(t)) - x_2(\gamma_1(t))\|
\end{aligned}
$$

$$
\begin{aligned}
& + \|x_1(\gamma_2(t)) - x_2(\gamma_2(t))\| + \ldots + \|x_1(\gamma_n(t)) - x_1(\gamma_n(t))\|] \\
\leq \; & nqbMM_1[1 + (bM\|\sigma B\| + K)K_1]\|x_1(t) - x_2(t)\| \\
\leq \; & p\|x_1(t) - x_2(t)\|.
\end{aligned}
$$

Therefore, Ψ is a contraction mapping and hence there exists a unique fixed point $x \in Y$ such that $\Psi x(t) = x(t)$. Any fixed point of Ψ is a mild solution of (1) on J which satisfies $x(b) = x_1$. Thus the system (1) is controllable on J.

4. Controllability of Delay Integrodifferential System

Consider the boundary control delay integrodifferential system of the form

$$
\begin{aligned}
\dot{x}(t) &= \sigma x(t) + f(t, x(\gamma_1(t)), \int_0^t k(t,s) g(s, x(\gamma_2(s)))ds), \quad t \in J = [0, b], \\
\tau x(t) &= B_1 u(t), \\
x(0) &= x_0,
\end{aligned} \tag{6}
$$

where the nonlinear operators $f : J \times E \times E \to E$, $g : J \times E \to E$ and $k : J \times J \to R$ are given.

To establish the results we shall assume the following conditions:

(a) $f : J \times E \times E \to E$ is continuous and there exist constants N_1 and N_2 such that for all $v_1, v_2 \in B_r$ and $w_1, w_2 \in E$ we have

$$\|f(t, v_1, w_1) - f(t, v_2, w_2)\| \leq N_1[\|v_1 - v_2\| + \|w_1 - w_2\|]$$

and

$$N_2 = \max_{t \in J} \|f(t, 0, 0)\|.$$

(b) $g : J \times E \to E$ is continuous and there exist constants L_1 and L_2 such that for all $v_1, v_2 \in B_r$ we have

$$\|g(t, v_1) - g(t, v_2)\| \leq L_1 \|v_1 - v_2\|$$

and

$$L_2 = \max_{t \in J} \|g(t, 0)\|.$$

(c) There exists a constant L such that

$$\|k(t,s)\| \leq L, \quad \text{for} \quad (t,s) \in J \times J.$$

(d) There exists a constant q such that for all x_1, $x_2 \in E$

$$\|x_1(\gamma_i(t)) - x_2(\gamma_i(t))\| \leq q\|x_1(t) - x_2(t)\|, \quad \text{for} \;\; i = 1, 2.$$

(e)$M\|x_0\| + K_1[bM\|\sigma B\| + K][\|x_1\| + M\|x_0\| + N] + N \leq r$
where $N = bM[N_1(r + bL(L_1 r + L_2)) + N_2]$.

(f) Let $a = [(bM\|\sigma B\| + K)K_1 + 1]bqMN_1[1 + bLL_1]$ be such that $0 \leq a < 1$.

Using the similar argument as in the previous section we can obtain a mild solution of (6) and it can be written as

$$\begin{aligned} x(t) &= T(t)x_0 + \int_0^t [T(t-s)\sigma - AT(t-s)]Bu(s)ds \\ &\quad + \int_0^t T(t-s)f(s, x(\gamma_1(s)), \int_0^s k(s,\tau)g(\tau, x(\gamma_2(\tau)))d\tau)ds. \end{aligned} \tag{7}$$

Theorem:4.1 If the hypotheses (A_1)-(A_5) and (a) - (f) are satisfied , then the boundary control delay integrodifferential system (6) is controllable on J.

Proof: Using the hypothesis (A_5), for an arbitrary function $x(.)$ define the control

$$u(t) = \tilde{W}^{-1}[x_1 - T(b)x_0 - \int_0^b T(b-s)f(s, x(\gamma_1(s)), \int_0^s k(s,\tau)g(\tau, x(\gamma_2(\tau)))d\tau)ds](t).$$

We shall show that, when using this control, the operator Φ defined on Y by

$$\begin{aligned} \Phi x(t) &= T(t)x_0 + \int_0^t [T(t-s)\sigma - AT(t-s)]B\tilde{W}^{-1}[x_1 - T(b)x_0 \\ &\qquad + \int_0^b T(b-\tau)f(\tau, x(\gamma_1(\tau)), \int_0^\tau k(\tau,\eta)g(\eta, x(\gamma_2(\eta)))d\eta)](s)ds \\ &\quad + \int_0^t T(t-s)f(s, x(\gamma_1(s)), \int_0^s k(s,\tau)g(\tau, x(\gamma_2(\tau)))d\tau)ds \end{aligned}$$

has a fixed point. This fixed point is then a solution of (6). Clearly $\Phi x(b) = x_1$, which means that the control u steers the delay integrodifferential system (6) from the initial state x_0 to x_1 in time b provided we can obtain a fixed point of the nonlinear operator Φ.

First we show that Φ maps Y into itself. For $x \in Y$,

$$\begin{aligned} \|\Phi x(t)\| &\leq \|T(t)x_0\| + \|\int_0^t [T(t-s)\sigma - AT(t-s)]B\tilde{W}^{-1}[x_1 - T(b)x_0 \\ &\qquad - \int_0^b T(b-\tau)f(\tau, x(\gamma_1(\tau)), \int_0^\tau k(\tau,\eta)g(\eta, x(\gamma_2(\eta)))d\eta)d\tau](s)ds\| \\ &\quad + \|\int_0^t T(t-s)f(s, x(\gamma_1(s)), \int_0^s k(s,\tau)g(\tau, x(\gamma_2(\tau)))d\tau)ds\| \\ &\leq \|T(t)x_0\| + \int_0^t \|T(t-s)\|\|\sigma B\|\|\tilde{W}^{-1}\|[\|x_1\| + \|T(b)x_0\| \end{aligned}$$

$$
\begin{aligned}
&\qquad + \int_0^b \|T(b-\tau)\|[\|f(\tau, x(\gamma_1(\tau)), \int_0^\tau k(\tau,\eta)g(\eta, x(\gamma_2(\eta)))d\eta) \\
&\qquad\qquad - f(\tau,0,0)\| + \|f(\tau,0,0)\|]d\tau]ds \\
&\quad + \int_0^t \|AT(t-s)B\|]\|\tilde{W}^{-1}\|[\|x_1\| + \|T(b)x_0\| \\
&\qquad + \int_0^b \|T(b-\tau)\|[\|f(\tau, x(\gamma_1(\tau)), \int_0^\tau k(\tau,\eta)g(\eta, x(\gamma_2(\eta)))d\eta) \\
&\qquad\qquad - f(\tau,0,0)\| + \|f(\tau,0,0)\|]d\tau]ds \\
&\quad + \int_0^t \|T(t-s)\|[\|f(s, x(\gamma_1(s)), \int_0^s k(s,\tau)g(\tau, x(\gamma_2(\tau)))d\tau) \\
&\qquad\qquad - f(s,0,0)\| + \|f(s,0,0)\|]ds \\
\leq\ & M\|x_0\| + [bM\|\sigma B\|K_1[\|x_1\| + M\|x_0\| + bM[N_1(r + bL(L_1 r + L_2)) \\
&\quad + N_2]] + KK_1[\|x_1\| + M\|x_0\| + bM[N_1(r + bL(L_1 r + L_2)) + N_2]] \\
&\qquad + bM[N_1(r + bL(L_1 r + L_2)) + N_2] \\
\leq\ & M\|x_0\| + K_1[bM\|\sigma B\| + K][\|x_1\| + M\|x_0\| + N] + N \\
\leq\ & r.
\end{aligned}
$$

Thus Φ maps Y into itself. Now, for $x_1, x_2 \in Y$ we have

$$
\begin{aligned}
&\|\Phi x_1(t) - \Phi x_2(t)\| \\
\leq\ & \int_0^t [\|T(t-s)\|\|\sigma B\| + \|AT(t-s)B\|]\|\tilde{W}^{-1}\|[\int_0^b \|T(b-\tau)\| \\
&\qquad \|f(\tau, x_1(\gamma_1(\tau)), \int_0^\tau k(\tau,\eta)g(\eta, x_1(\gamma_2(\eta)))d\eta) \\
&\qquad - f(\tau, x_2(\gamma_1(\tau)), \int_0^\tau k(\tau,\eta)g(\eta, x_2(\gamma_2(\eta)))d\eta)\|d\tau]ds \\
&\quad + \int_0^t \|T(t-s)\|\|f(s, x_1(\gamma_1(s)), \int_0^s k(s,\tau)g(\tau, x_1(\gamma_2(\tau)))d\tau) \\
&\qquad - f(s, x_2(\gamma_1(s)), \int_0^s k(s,\tau)g(\tau, x_2(\gamma_2(\tau)))d\tau)\|ds \\
\leq\ & b[M\|\sigma B\| + K]K_1 bMN_1[\|x_1(\gamma_1(\tau)) - x_2(\gamma_1(\tau))\| \\
&\qquad + bLL_1\|x_1(\gamma_2(\eta)) - x_2(\gamma_2(\eta))\|] \\
&\quad + bMN_1[\|x_1(\gamma_1(\tau)) - x_2(\gamma_1(\tau))\| \\
&\qquad + bLL_1\|x_1(\gamma_2(s)) - x_2(\gamma_2(s))\|] \\
\leq\ & [(bM\|\sigma B\| + K)K_1 + 1]bMN_1[\sup_{t\in J}\|x_1(\gamma_1(t)) - x_2(\gamma_1(t))\| \\
&\qquad + bLL_I \sup_{t\in J}\|x_1(\gamma_2(t)) - x_2(\gamma_2(t))\|] \\
\leq\ & [(bM\|\sigma B\| + K)K_1 + 1]bMN_1[1 + bLL_1]q\|x_1(t) - x_2(t)\| \\
\leq\ & a\|x_1(t) - x_2(t)\|.
\end{aligned}
$$

Therefore, Φ is a contraction mapping and hence there exists a unique fixed point $x \in Y$ such that $\Phi x(t) = x(t)$. Any fixed point of Φ is a mild solution of (6) on J which satisfies $x(b) = x_1$. Thus the system (6) is controllable on J.

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