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A Comparative Study between Green's Function Method and Fourier Transform Method in Determining Thermal Wave Characteristics

열전도파 특성을 위한 Green's 함수법과 Fourier 변환법의 비교 연구

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Key Words : Thermal Wave(열전도파), Resonance(공진), Thermal Relaxation Time(열이완시간), Phase Lag(위상지연)

요 약 : 고체내의 열에너지의 전달을 분석하기 위하여 고전적인 Fourier 열전도 법칙과 에너지 보존식에서 유도되는 열전도 방정식을 사용해 왔다. 이러한 열전도 방정식은 열전도가 무한한 속도로 진행된다는 것을 의미하고 있다. 그러나 극저온상태에서나 매우 급속한 열전도과정 중 매우 짧은 시간의 상태에서 non-Fourier 모델에 기초를 둔 쌍곡선형 열전도 방정식이 도입되었다. 최근의 이에 관한 연구에서 열전도가 파장의 형태로 유한한 전파속도를 갖는다는 것이 실험적으로 증명되었고 이로부터 여러 가지 실험적인 해석과 이론 해석이 전개되었다. 본 논문에서는 열전파 속도의 유한한 성질을 나타내는 수정된 열전도 법칙을 이용하여 1차원 평판에 대하여 공간에 대한 finite Fourier 변환 방법과 Green 함수 방법으로 해석하여 열전도파의 파동 성질, 공진 현상 및 위상차를 고찰하고자 한다. 열전도파가 갖는 모달 주파수에 대해 임계값을 갖으며 이 임계값을 초과할 때 공진 현상과 위상차를 고찰할 수 있었다.

1. INTRODUCTION

The Fourier's law of heat conduction conventionally used assumes that any thermal disturbance on a body propagates at an infinite speed. In practice, thermal wave speed is very high; that is, for metals under room temperature the thermal wave speed would be in the order of 10^2 to 10^3 m/s. Therefore, the analysis of the temperature distribution resulting from the Fourier's law of heat conduction agrees well with most actual phenomena. However, it is not easy for the Fourier's law to be applied to the analysis at cryogenic temperatures in extremely short periods of time due to a finite speed of heat propagation. The experiment to detect a thermal wave with a finite speed of heat propagation was carried out by Peshkov[1] and the thermal wave speed for liquid Helium II at a temperature of 1.4 K is 19 m/s in his work. This phenomenon was called "second sound", by reason of an analogy between thermal waves and ordinary acoustic waves. Kaminski[2] showed that for the sand specimen measured by traditional thermocouples, the thermal wave speed (C) and the thermal relaxation time (τ) are 0.143 mm/s and 20 second, respectively.

In order to explain the non-Fourier's phenomena with a finite speed of heat propagation, Cattaneo[3] and Vernotte[4] postulated a modified heat flux model:

$$\vec{q}(\vec{r},t)+\tau\frac{\partial\vec{q}}{\partial t}(\vec{r},t)=-k\nabla T(\vec{r},t) \qquad (1)$$

where $q(\vec{r},t)$ is the heat flux vector and k is

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the thermal conductivity. The constant τ represents a time lag, which exists between the heat flux built up at a later time due to the insufficient time of response and the temperature gradient established in a material volume. If the relaxation time approaches zero, Eq. (1) becomes the classical Fourier's heat conduction model with an infinite speed of heat propagation.

The modified heat conduction law that involves rapid temperature changes plays an important role for a better understanding of new phenomena, such as laser annealing, laser surface hardening by high-power short-pulse lasers, rapid melting, rapid thermal processing of thin films and the heat-conduction problems at extreme temperature gradients. Since the phenomena of thermal wave model possess a behavior like a damped wave, recent works[5,6,7] focus attention on the research about the thermal resonance and damping.

In this paper, to analyze the frequency characteristics of thermal waves in a finite medium, the thermal wave model resulting from combining the modified heat conduction equation with the energy conservation equation is applied to the Green's function method together with the integral transform pair and also transformed by using a finite Fourier sine transform. The transformed temperature distributions, therefore, are obtained from the thermal wave equation by the two different methods. To investigate the resonance characteristics and the phase lags, boundary surfaces which are being periodically heated on one side and insulated on the other are considered.

2. PROBLEM FORMULATION

In a one-dimensional slab, the modified heat flux law, Eq. (1), involving the relaxation time degenerates into the following form:

$$\tau\frac{\partial q}{\partial t}(x,t)+q(x,t)=-k\frac{\partial T}{\partial x}(x,t). \qquad (2)$$

The energy conservation equation in a one-dimensional situation can be written as

$$-\frac{\partial q}{\partial x}(x,t)+g(x,t)=\rho C_p\frac{\partial T}{\partial x}(x,t), \qquad (3)$$

where g is an inner heat source function, ρ is the mass density and C_p is the specific heat of the solid medium. The heat flux representation is obtained by eliminating the temperature T from Eqs. (2) and (3). The resulting equation is called the thermal wave equation with no heat source term and may be expressed as

$$\frac{\partial^2 q}{\partial x^2}(x,t)=\frac{1}{C^2}\frac{\partial^2 q}{\partial t^2}(x,t)+\frac{1}{\alpha}\frac{\partial q}{\partial t}(x,t),$$
$$0<x<l, \qquad t>0. \qquad (4)$$

where α is the thermal diffusivity. C is the thermal wave speed that represents the rate of the thermal diffusivity to the thermal relaxation time as follows.

$$C^2=\frac{\alpha}{\tau} \qquad (5)$$

For simplicity in this analysis, thermal properties used here are assumed to be constant and also no volumetric heat source is in the finite slab. This situation can be found out in many engineering applications by the use of

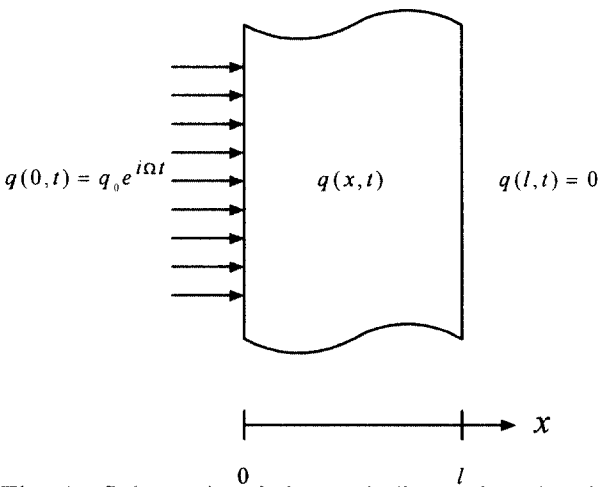

Fig. 1 Schematic of the periodic surface heating problem

very short pulse laser heating. The boundary and initial conditions are expressed as

$$q(0,t)=q_o e^{(i\Omega t)}, \tag{6a}$$

$$q(l,t)=0, \quad t>0, \tag{6b}$$

$$q(x,0)=0, \tag{6c}$$

$$\frac{\partial q}{\partial t}(x,0)=0, \qquad 0<x<l. \tag{6d}$$

where q_o is a factor corresponding to the incident amplitude at the surface $x=0$.

For convenience in the subsequent analysis, dimensionless variables are defined as

$$\eta = \frac{Cx}{2\alpha}, \quad \eta_l = \frac{Cl}{2\alpha}, \tag{7a}$$

$$\xi = \frac{C^2 t}{2\alpha}, \tag{7b}$$

$$Q(\eta,\xi) = \frac{q(x,t)}{T_{ref}(Ck/\alpha)}, \tag{7c}$$

where T_{ref} is the reference temperature. It is chosen as $T_{ref}=q_o/(Ck/\alpha)$ in this work for convenience. In the case of no surface heating, the reference temperature could be taken into account as $T_{ref}=T_o$. Under these dimensionless forms, the dimensionless surface heat flux frequency, ω_o, can be expressed as $\omega_o=2\alpha\Omega/C^2$. Introducing the dimensionless quantities defined in Eqs. (7a-7c) into Eq. (4), we can express the following governing equation and its boundary and initial conditions in terms of the above dimensionless variables as

$$\frac{\partial^2 Q}{\partial \eta^2}(\eta,\xi)=\frac{\partial^2 Q}{\partial \xi^2}(\eta,\xi)+2\frac{\partial Q}{\partial \xi}(\eta,\xi), \qquad \eta>0, \quad \xi>0 \tag{8}$$

$$Q(0,\xi) = \exp(i\omega_o \xi), \tag{9a}$$

$$Q(\eta_l,\xi) = 0, \qquad \xi>0, \tag{9b}$$

$$Q(\eta,0) = 0, \tag{9c}$$

$$\frac{\partial Q}{\partial \xi}(\eta,0) = 0, \qquad 0<\eta<\eta_l. \tag{9d}$$

Since this dimensionless thermal wave equation, Eq. (8), with heat flux is the same formulation as a damped wave equation, we can see that the thermal wave yields resonance characteristics and phase lag. The solution of this system is developed in the following section.

3. ANALYSIS

3.1 Green's Function Method

The thermal wave in a finite medium with a periodically heated surface has phase lag and resonance characteristics which depend on the heating frequency. In order to investigate these phenomena, we applied the Green's function to the dimensionless thermal wave equation, Eq. (8), as follows:

$$\begin{aligned}&\int_{\xi_o=0}^{\xi+\varepsilon}\int_{\eta_o=0}^{\eta_l} G(\eta,\xi\mid\eta_o,\xi_o)L_o[Q]\,d\eta_o d\xi_o \\ &= \int_{\xi_o=0}^{\xi+\varepsilon}\int_{\eta_o=0}^{\eta_l} QL_o^*[G(\eta,\xi\mid\eta_o,\xi_o)]\,d\eta_o d\xi_o \\ &+ \int_{\xi_o=0}^{\xi+\varepsilon}\left[G\frac{\partial Q}{\partial \eta_o}-Q\frac{\partial G}{\partial \eta_o}\right]_0^{\eta_l} d\xi_o \\ &+ \int_{\eta_o=0}^{\eta_l}\left[-G\frac{\partial Q}{\partial \xi_o}+Q\left(\frac{\partial G}{\partial \xi_o}-2G\right)\right]_0^{\xi+\varepsilon} d\eta_o \end{aligned}, \tag{10}$$

where L_o is chosen as the modified heat flux linear operator and has the form

$$L_o=\frac{\partial^2}{\partial \eta_o^2}-\frac{\partial^2}{\partial \xi_o^2}-2\frac{\partial}{\partial \xi_o}. \tag{11}$$

Also, the formal adjoint operator L_o^* is given as

$$L_o^*=\frac{\partial^2}{\partial \eta_o^2}-\frac{\partial^2}{\partial \xi_o^2}+2\frac{\partial}{\partial \xi_o}. \tag{12}$$

The solution to the periodic surface heat flux problem is obtainable by replacing the boundary and the initial conditions defined by Eqs. (9(a)-9(d)) and no heat source:

$$Q(\eta,\xi)=-\int_{\xi_o=0}^{\xi}\left[Q(0,\xi_0)\frac{\partial G}{\partial \eta_0}(\eta,\xi\mid 0,\xi_o)\right]d\xi_o \tag{13}$$

To determine an appropriate Green's function, we take the finite integral transform pair through the aid of the orthogonality relation:

Integral Transform

$$\overline{G}(\omega_m, \xi_o) = \int_0^{\eta_l} \psi_m(\omega_m, \eta_o) G(\eta, \xi \mid \eta_o, \xi_o)\, d\eta_o \ ; \tag{14}$$

Inversion Formula

$$G(\eta, \xi \mid \eta_o, \xi_o) = \sum_{m=1}^{\infty} \frac{\psi_m(\omega_m, \eta_o)\ \overline{G}(\omega_m, \xi_o)}{N(\omega_m)}, \tag{15}$$

where $N(\omega_m)$ is the normalization integral given by $\frac{\eta_l}{2}$. Here, the eigenfunction is given as

$$\psi(\omega_m, \eta) = \sin \omega_m \eta, \tag{16}$$

where the eigenvalues ω_m are defined as

$$\omega_m = \frac{m\pi}{\eta_l}, \quad m = 1, 2, 3 \cdots \tag{17}$$

Through the application of the above integral transform pair, the Green's function $G(\eta, \xi \mid \eta_o, \xi_o)$ satisfying Eq. (12) is determined as

$$G(\eta, \xi \mid \eta_o, \xi_o) = \sum_{m=1}^{\infty} \frac{\psi_m(\omega_m, \eta)\, \psi_m(\omega_m, \eta_o)\, e^{(\xi_o - \xi)}}{2\sqrt{\omega_m^2 - 1}\, N(\omega_m)\, i} \times \left(e^{\sqrt{\omega_m^2 - 1}(\xi_o - \xi) i} - e^{-\sqrt{\omega_m^2 - 1}(\xi_o - \xi) i}\right), \ \xi > \xi_o. \tag{18}$$

Since the Green's function is known, the heat flux distributions $Q(\eta, \xi)$ can be obtained by substituting Eq. (18) into Eq. (13):

$$Q(\eta, \xi)_{\text{Green}} = Q_{\text{steady}} + Q_{\text{transient}}, \tag{19}$$

$$Q_{\text{steady}} = \sum_{m=1}^{\infty} \frac{2\omega_m \sin \omega_m \eta}{\eta_l(\omega_m^2 - \omega_o^2 + 2\omega_o i)}\, e^{i\omega_o \xi}, \tag{20}$$

$$Q_{\text{transient}} = \sum_{m=1}^{\infty} \frac{\omega_m \sin \omega_m \eta}{i\eta_l \sqrt{\omega_m^2 - 1}} \times \left[\frac{e^{-i\sqrt{\omega_m^2 - 1}\,\xi}}{1 + i\sqrt{\omega_m^2 - 1} + i\omega_o} - \frac{e^{i\sqrt{\omega_m^2 - 1}\,\xi}}{1 - i\sqrt{\omega_m^2 - 1} + i\omega_o}\right]. \tag{21}$$

By concentrating on the heat flux response in time, we may express the heat flux wave governed by Eq. (20) in terms of its modal representation:

$$Q(\eta, \xi)_{\text{Green}} = \sum_{m=1}^{\infty} \sin \omega_m \eta \mid A_{Q,\text{Green}} \mid e^{(\omega_o \xi - \phi) i}. \tag{22}$$

By substituting Eq. (22) into Eq. (20), we obtain the heat flux amplitude and the phase, respectively, as

$$\mid A_{Q,\text{Green}} \mid = \frac{2\omega_m}{\eta_l [(\omega_m^2 - \omega_o^2)^2 + 4\omega_o^2]^{1/2}}, \tag{23}$$

$$\phi = \tan^{-1}\left[\frac{2\omega_o}{(\omega_m^2 - \omega_o^2)}\right]. \tag{24}$$

At a given value of ω_m, therefore, the value of the exciting frequency ω_o at which the heat flux amplitude $\mid A_{Q,\text{Green}} \mid$ possesses a maximum is determined by

$$\frac{d \mid A_{Q,\text{Green}} \mid}{d\omega_o} = 0. \tag{25}$$

If only the positive frequency of ω_o for the applied periodic surface heating is considered in Eq. (25), the resonance frequency at which the amplitude reaches a maximum is given as

$$\omega_o^{\max} = \sqrt{\omega_m^2 - 2}, \quad \text{subject to} \quad \omega_m > 1.414. \tag{26}$$

For a specified value of the modal frequency ω_m, by substituting Eq. (26) into Eq. (23), the maximum heat flux amplitude $\mid A_{Q,\text{Green}} \mid^{\max}$ under the action of the external heating oscillating at the resonance frequency $\omega_o^{\max}$ becomes,

$$\mid A_{Q,\text{Green}} \mid^{\max} = \frac{\omega_m}{\eta_l \sqrt{\omega_m^2 - 1}} \quad \text{for} \quad \omega_m > 1.414. \tag{27}$$

3.2 Finite Fourier transform

To investigate thermal wave characteristics as a second case, the finite Fourier transform is applied to the dimensionless thermal wave Eq. (8). The finite Fourier sine transform[8] and the inversion formula are defined as, respectively,

$$f_s[Q(\eta)] \equiv Q_s(\omega_m) = \int_0^{\eta_l} Q(\eta)\sin(\omega_m \eta)d\eta, \quad (28)$$

$$Q(\eta) = \frac{2}{\eta_l}\sum_{m=1}^{\infty} Q_s(\omega_m)\sin(\omega_m \eta). \quad (29)$$

where the modal frequency is given as

$$\omega_m = \frac{m\pi}{\eta_l}, \qquad m = 1,2,3\ldots \quad (30)$$

By taking the finite Fourier sine transform, the properties governed by the boundary and initial conditions (9) are as follows,

$$\frac{\partial^2 Q(\eta)}{\partial \eta^2} = \omega_m\, e^{i\omega_o \xi} - \omega_m^2 Q_s(m), \quad (31a)$$

$$\frac{\partial^2 Q(\eta)}{\partial \xi^2} = \frac{d^2 Q_s}{d\xi^2}(\xi), \quad (31b)$$

$$2\frac{\partial Q(\eta)}{\partial \xi} = 2\frac{dQ_s}{d\xi}(\xi). \quad (31c)$$

The heat flux thermal wave Eq. (8) having the two boundary conditions becomes

$$\frac{d^2 Q_s}{d\xi^2} + 2\frac{dQ_s}{d\xi} + \omega_m^2 Q_s = \omega_m\, e^{i\omega_o \xi}, \quad (32)$$

$$Q_s = 0, \qquad 0 < \eta < \eta_l, \quad (33a)$$

$$\frac{dQ_s}{d\xi} = 0, \quad 0 < \eta < \eta_l. \quad (33b)$$

The solution of the transformed equation has the following form:

$$Q_s(\xi) = Ae^{\lambda_1 \xi} + Be^{\lambda_2 \xi} + Q_o e^{i\omega_o \xi}. \quad (34)$$

where $\lambda_1 = -1+\sqrt{1-\omega_m^2}$, $\lambda_2 = -1-\sqrt{1-\omega_m^2}$

Hence a general solution of the nonhomogeneous equation is

$$Q_s(\xi)_{\text{FFT}} = Q_s(\xi)_{\text{homogeneous}} + Q_s(\xi)_{\text{particular}}, \quad (35)$$

$$Q_s(\xi)_{\text{homogeneous}} = \frac{\omega_m}{\omega_m^2 - \omega_o^2 + i\,2\omega_o} \cdot \frac{e^{-\xi}}{i\,2\sqrt{\omega_m^2 - 1}} \times \left\{ \left[-1 - i(\sqrt{\omega_m^2 - 1} + \omega_o) e^{i\sqrt{\omega_m^2 - 1}\,\xi} \right] + \left[1 - i(\sqrt{\omega_m^2 - 1} - \omega_o) e^{-i\sqrt{\omega_m^2 - 1}\,\xi} \right] \right\} \quad (36)$$

$$Q_s(\xi)_{\text{particular}} = \frac{\omega_m}{\omega_m^2 - \omega_o^2 + i\,2\omega_o} e^{i\omega_o \xi}. \quad (37)$$

In order to study the resonance phenomenon and the phase lag here, the homogeneous solution, Eq. (36), will be neglected because it is reduced exponentially in time. Accordingly, it does not affect the frequency response of the thermal waves as time goes by. The second term, $Q_s(\xi)_{\text{particular}}$, is sufficient to be considered as the resonance phenomenon. This solution depends on both the modal frequency ω_m and the oscillatory frequency ω_o of the periodic pulse. This also can be expressed as an amplitude and a phase lag which are due to the heating frequency and the heat flux wave in the system.

$$Q_s(\xi)_{\text{particular}} = \sqrt{\frac{\omega_m^2}{(\omega_m^2 - \omega_o^2)^2 + (2\omega_o)^2}}\, e^{i(\omega_o \xi - \phi_{Q,\text{Fourier}})}, \quad (38)$$

where $\phi_{Q,\text{FFT}} = \tan^{-1}\left(\frac{2\omega_o}{\omega_m^2 - \omega_o^2}\right)$. (39)

For the heat flux mode oscillating in the solid with a specific modal frequency ω_m, the thermal resonance is obviously proposed by the following expression:

$$|A_{Q,\text{FFT}}| = \sqrt{\frac{\omega_m^2}{(\omega_m^2 - \omega_o^2)^2 + 4\omega_o^2}}. \quad (40)$$

When the value of ω_m is given, the value of the exciting frequency ω_o at which the amplitude of the heat flux wave possesses a maximum is determined by the following

stationary condition:

$$\frac{d\,|A_{Q,\mathrm{FFT}}|}{d\omega_o}=0, \tag{41}$$

which yields the following equation for ω_o:

$$\omega_o=\pm\sqrt{\omega_m^2-2}. \tag{42}$$

By taking the positive root for frequency in (42), the resonance frequency at which the amplitude of the heat flux wave amplifies and reaches a maximum becomes

$$\omega_o^{\max}=\sqrt{\omega_m^2-2} \quad \text{subject to} \quad \omega_m>\sqrt{2}\,. \tag{43}$$

The resonance frequency $\omega_o^{\max}$ is the externally applied frequency which is rendering a maximum temperature amplitude of $|A_{Q,\mathrm{FFT}}|$. For large values of ω_m, $\omega_o^{\max}$ approaches the modal frequency ω_m. The corresponding maximum amplitude of the temperature wave, denoted by $|A_{Q,\mathrm{FFT}}|^{\max}$, can be obtained by substituting Eq. (43) into (40). The result is

$$|A_{Q,\mathrm{FFT}}|^{\max}=\frac{\omega_m}{2\sqrt{\omega_m^2-1}}, \quad \text{for} \quad \omega_m>\sqrt{2}\,. \tag{44}$$

Finally, when the particular solution (37) from the finite Fourier sine transform is applied to the inversion formula, Eq. (29), the resultant is the same as that of Eq. (20). Consequently, the heat flux characteristics such as the resonance amplitude, the phase lag, and the maximum amplitude are completely equal between the two mathematical methods.

4. RESULTS AND DISCUSSIONS

4.1 RESONANCE

The thermal wave equations derived from combining the modified heat flux law with the energy conservation law are analyzed by the finite Fourier sine transform and the Green's function technique, respectively. For the resonance to occur, one boundary condition at $x=0$ is periodically heated and the other surface at $x=l$ is insulated.

The thermal resonance of heat flux wave propagating in solids with a finite speed of heat propagation is studied. The emphasis is placed on the externally applied exciting frequency at which the amplitude of the thermal waves reaches a maximum. The amplitude of thermal waves excited is related to the exciting frequencies as well as the modal frequencies relating to the wave modes. Since the resonance phenomenon is a special behavior in time, a one-dimensional slab is considered for the illustration purpose. The emphasis will be on studying a comparative analysis among the finite Fourier sine transform method and the Green's function method under the same boundary conditions. The use of the q-representation is concerned with the investigation of heat flux resonance due to the problems involving flux-specified boundary conditions.

Figs. 2 and 3 show the amplitude of the heat flux wave derived from the Green's function method, while Figs. 4 and 5 show from the finite Fourier sine transform under the same boundary conditions. However, the heat flux amplitude obtained from the Green's function method is $2/\eta_l$ times larger than that of the finite Fourier sine transform, as shown in Figs. 2 to 5. It is the reason that the amplitude from the finite Fourier transform is regarded as the only $V_s(m)$ of the inversion formula, Eq. (30). Therefore, the results from the two different methods have the same magnitude of the amplitude and the same physical characteristics, although the heat flux amplitude from the Green's function method is twice as large as those by the finite Fourier sine transform in those figures. When the modal frequency ω_m increases from 1.4142 to 4, the amplitude of $|A_{Q,\mathrm{FFT}}|^{\max}$ decreases. Figs. 3 and 5 show the heat flux amplitude varying with the frequency ratio, which is the ratio of the heating

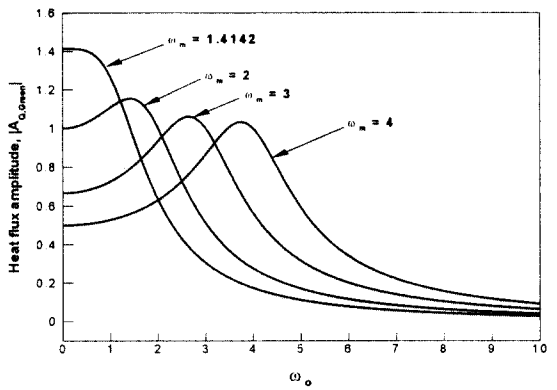

Fig. 2 Amplitudes of heat flux waves derived from the Green's function method varying with the heating frequency

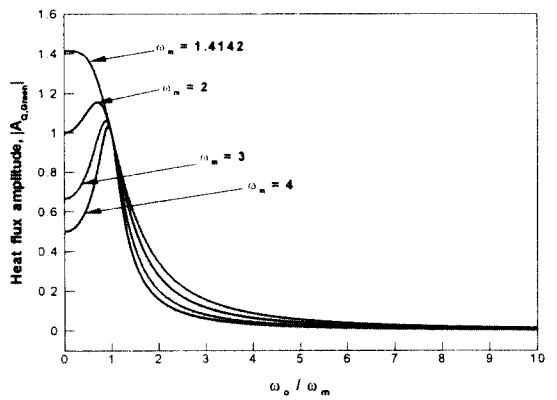

Fig. 3 Amplitude of heat flux waves derived from the Green's function method varying with the frequency ratio.

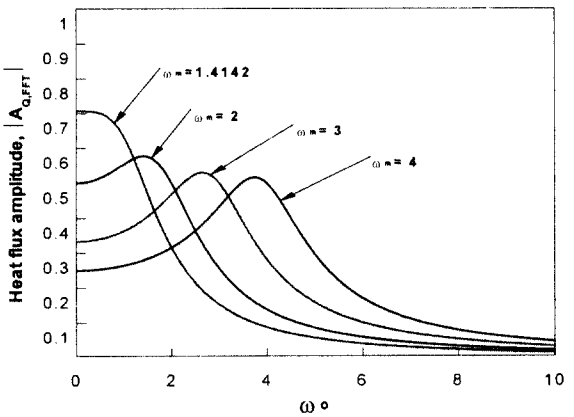

Fig. 4 Amplitudes of heat flux waves derived from the finite Fourier sine transform varying with the heating frequency

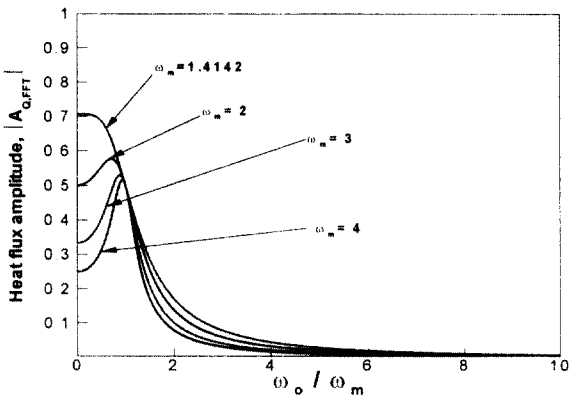

Fig. 5 Amplitude of heat flux waves derived from the finite Fourier sine transform varying with the frequency ratio

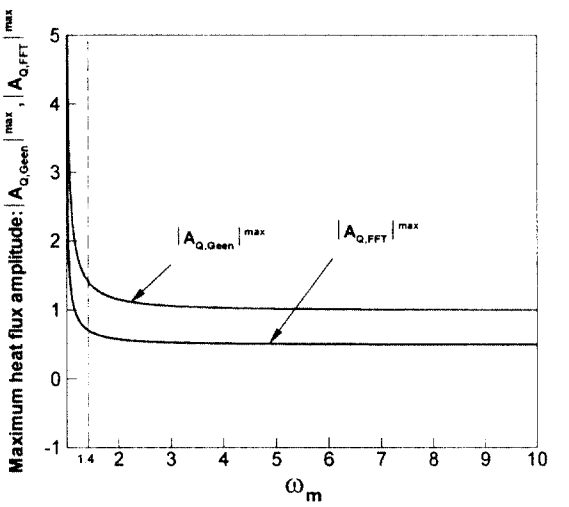

Fig. 6 Comparison of the maximum heat flux amplitude between the Green's function methods and the finite Fourier transform

frequency to the modal frequency. The maximum amplitude occurs in the neighborhood of the frequency ratio $\omega_o/\omega_m=1$. In other words, the resonance frequency at which the amplitude possesses a maximum value approaches the modal frequencies ω_m of the corresponding modes. In the case that the modal frequency is close to the exciting frequency, the flux resonance occurs actively.

In Figure 6, it is shown that the maximum heat flux amplitude obtained from the two different methods are agreed well. The peak

values in each curve in Figs. 2 and 4 are shown and analytically presented from Eqs. (27) and (44), respectively. As the modal frequencies increase, the maximum heat flux amplitude decreases.

4.2 PHASE LAG

The physical concept of time-lag between the heat flux vector and the temperature gradient is used to describe the relaxation behavior in thermal wave propagation. Equation, $q(\vec{r}, t+\tau) = -k\nabla T(\vec{r}, t)$, depicts that the temperature gradient established at time t results in a delayed heat flux occurring at a later time $t+\tau$. In addition to revealing the delayed response explicitly, Eq. (1) also has an important characteristic, namely the path-dependency, in the thermal loading history. From this physical points of view, the phase lag in the thermal wave theory has been examined. The particular solutions of the governing thermal wave equations are expected to be a time-lag phenomenon.

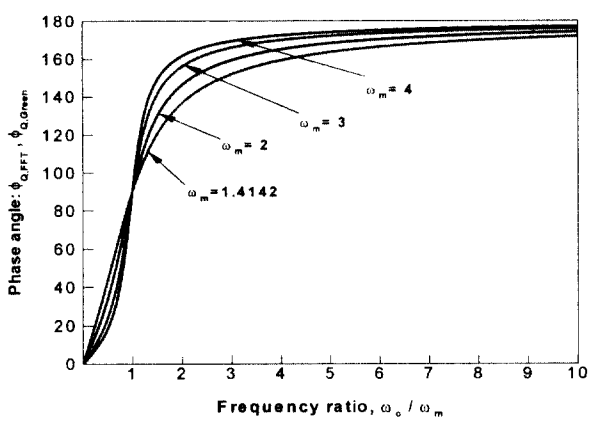

Fig. 7 Comparisons of the phase angles in the heat flux model calculated by the finite Fourier transform and the Green's function method.

The comparison of the heat flux wave systems calculated by the finite Fourier sine transform and the Green's function method concerning the phase angle is shown in Figure 7. The variation of phase angle here is completely the same. The phase angle depends on the heating frequency ω_o, regardless of the amplitude. The phase angle also is very small for small values of the frequency ratio. For very large values of the frequency ratio, the phase angle approaches 180° asymptotically. Thus the amplitude is in phase for $\omega_o/\omega_m \ll 1$ and out of phase for $\omega_o/\omega_m \gg 1$. In the vicinity of $\omega_o/\omega_m = 1$ with the thermal resonance occurring actively, the phase angle of thermal waves will be 90 degree.

5. CONCLUSION

The phase lag and resonance characteristics of the heat flux wave under the same excitation are analyzed by two different mathematical methods: the Green's function method and the finite Fourier sine transform. The thermal wave equation derived from the modified heat flux law and the energy conservation law was used and yielded the heat flux wave distributions.

For the resonance to occur, the side of the left boundary surface was periodically heated and the right side was insulated. It has shown that the modal frequency of the thermal wave must be larger than a minimum value of $\omega_m = 1.4142$. The desired frequencies to excite the thermal waves for the resonance phenomenon have been related to the modal frequencies. The resonance occurred at the modal frequency in the vicinity of the heating frequency and the amplitude of heat flux wave was markedly reduced in the domain of other modal frequencies. The wave modes were excited and resonated only when the modal frequency exceeded 1.4142. The results presented above in this paper agreed well with those given by Tzou[5,6,7] and Juhng et al.[9,10] in the cases of the same boundary conditions. The phase differed by 90 degree in the vicinity of the frequency domain in which the thermal resonance was activated. These resonance characteristics may be applied to the measurement of the properties such as thermal wave speed and thermal diffusivity.

As a result of the comparison of the Green's function method and the finite Fourier transform, they agree well with the above viewpoint. Thus, in order to find out the tendency of thermal characteristics the finite Fourier transform technique is mathematically easier than the Green's function method.

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