Control chart pattern recognition using average of cumulative sum

*Chuen-Sheng Cheng, *Shin-Jia Chen
*Department of Industrial Engineering and Management,
Yuan-Ze University,
135 Yuan-Tung Road, Taoyuan, Taiwan, R.O.C.

Key Words : Shewhart charts; Unnatural patterns; Average of cumulative sum.

1. Introduction

The statistical process control chart is a monitoring technique useful in determining whether a process is behaving as intended or if there are some unnatural causes of variation. A process is considered to be out of control if a point falls outside the control limits or a series of points exhibit an unnatural pattern (also known as nonrandom variation). Analysis of unnatural patterns is an important aspect of control charting. It is well known that a particular unnatural pattern on a control chart is often associated with a specific set of assignable causes [5]. Identification of unnatural patterns can greatly narrow the set of possible causes that must be investigated, hence facilitating the rapid diagnosis and corrective action.

Over the years, numerous supplementary rules known as zone rules or run rules [2~4] have been proposed to improve the control chart performances. However, supplementary rules are not very effective in control chart pattern recognition.

It often relies on the expertise and experience of the analyst to determine if an unnatural pattern exists.

This paper presents a graphical approach that is capable of detecting unnatural patterns as well as characterizing some important details of the patterns, such as change point, magnitude of shift, period of cycle, etc., which is critical for effective assignable cause analysis. We will demonstrate by examples the use of the proposed method.

2. Control Chart Pattern Recognition

Recognizing patterns of samples as symptomatic of process problems is a critical task in control charting. These unnatural patterns can reveal potential quality problem at early stage. Control chart pattern recognition is usually done manually by examining sequential points and looking for unnatural patterns. Numerous practitioners and educators have identified several types of out-of-control patterns and some of their possible assignable causes[5,6].

The unnatural patterns studied in this research are defined as follows. The reader is referred to the Western Electric Handbook[6] for a detailed description of these unnatural patterns.

1. Trends. The major characteristic of trends is a continuous movement in one direction. They could be either upward trends or downward trends. The possible causes for this condition are operator fatigue, tool wear, equipment deterioration, and many others.
2. Sudden shifts. A shift may be defined as a sudden or abrupt change in the average of the process. Shift patterns include upward shift and downward shift. Shift patterns could be caused by an alteration in the process setting, a difference in raw materials, a minor failure of a machine part, the introduction of new workers, materials, a change in the inspection method or standards, and so forth.
3. Cycles. Cycles can be recognized by a series of high portions or peaks interspersed with low portions or troughs. Cycles may result from the periodic rotation of operators, operator fatigue, regular rotation of operators and/or machines, or heat or stress buildup.
4. Mixtures. In a mixture the points show a tendency to fall near the high and low edge of the pattern with an absence of normal fluctuations near the middle. A mixture is actually a combination of data from separate distributions. Some of the causes for this condition are items from different suppliers, machines, or workers. Sometimes the mixtures could be caused by over-control of the process.
5. Systematic variation. One of the characteristics of a natural pattern is that the point-to-point fluctuations are unsystematic or unpredictable. In systematic variations a low point is always followed by a high one or vice versa. Control chart may reveal systematic variation due to difference between shifts, difference between test sets and difference between production lines where product is sampled in rotation.

Interpreting unnatural patterns is a difficult task for quality control personnel. Some of the major problems associated with the analysis of control chart patterns can be summarized as follows. First, the random noise might contaminate the present pattern, the effect may change with the magnitude of the unnatural pattern. Second, a pattern may sometimes resemble other patterns. For instance, a short trend may be misinterpreted as a shift or vice versa.

Third, a pattern may be a subset of other patterns. Fourth, the problem is much more complicated if there is more than one pattern of interest, or if the signal-to-noise ratio of a pattern is low. Based on the above discussion, it shows pattern recognition a crucial problem in the implementation of control charts.

In the past, the zone rules or run rules have been the major tool for interpreting control charts. Although the zone rules or run rules have been proven to be effective in detecting out-of-control situations the interpretation of process data is still a very difficult task. The major difficulty lies in the fact that there is no one-to-one mapping between a supplementary rule and an unnatural pattern. In practice, the types of unnatural patterns that a process may experience are not known in advance. There might be several patterns associated with a particular rule. For instance, the possible patterns associated with the rule 'eight points in a row on both sides of the centerline with none in zones C' suggested by Nelson[5], could be a mixture or a systematic variation. These rules might indicate that an unnatural pattern is present, but do not explicitly indicate which pattern really occurs. In addition, some of the supplementary rules result in more false alarms without significantly improving the performance of control charts. A typical example is the commonly used trend rule. Davis and Woodall[1] evaluated the trend rule and concluded that the trend rule is not effective in detecting a linear trend. The limitations of supplementary rules have motivated interests in developing algorithmic approach for control chart pattern recognition.

3. A Graphical Method for the Analysis of Control Chart Patterns

The proposed method is a graphical tool for control chart pattern recognition. It provides a graphical depiction of some important features of unnatural patterns. The key feature of the proposed method that set it apart from a time-series graph is the provision of methods to detect unnatural pattern as well as to describe the characteristic parameters of these patterns (e.g., trend slope, cyclic amplitude, etc.). This paper will focus on the unnatural pattern on $\bar{X}$ control charts. The following notation is used to describe the operational details of the proposed method:

FACS(j)	Forward average of cumulative sum computed at sampling time j
BACS(j)	Backward average of cumulative sum computed at sampling time j
m	The number of observations available for analysis

X_i A single observation or the average of several observations taken at sampling time i

In mathematical terms, the FACS(j) and BACS(j) can be defined as follows:

$$\text{FACS}(j) = \frac{\sum_{i=1}^{j} X_i}{j}$$

$$\text{BACS}(j) = \frac{\sum_{i=m}^{j} X_i}{m-j+1}$$

The FACS and BACS can be thought of as a data smoother used to discover the important features contained in the data. The plots of FACS's and BACS's make the shape of a pattern explicit. Hence, they facilitate the recognition process and yields increased accuracy. Some notable features can be easily observed from the FACS and BACS curves of noise-free data. Therefore, in the rest of this section we will use noise-free data to describe the detailed pattern information observable from the FACS and BACS curves.

Figure 1 displays the FACS and BACS curves of an upward trend without noise.

The following comments can be made on this figure. It can be seen that the FACS and BACS curves will be parallel with a significant slope when a trend is present. It is not difficult to show that the trend slope can be computed by $2D/(m-1)$, where D is the distance between FACS and BACS curves. In the presence of noise, the average distance between the FACS and BACS curves can estimate the slope. Note that the upward trend and the downward trend patterns share the same characteristics in terms of the average of cumulative sum. Therefore the same interpretation can be applied to downward trend patterns.

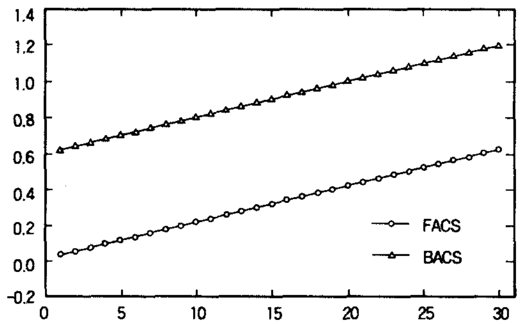

Fig. 1 FACS and BACS curves of a noise-free trend

The FACS and BACS curves of a noise-free upward shift are shown in Figure 2. As can be seen from Figure 2, the distance between FACS and BACS curves becomes small at either end. This feature differentiates shift patterns from trend patterns. We can see that the maximal distance (say D^*) occurs at the change point. The magnitude of shift can be computed as D^*. Note that the same phenomenon applies to downward shift patterns.

Figure 3 shows the plots of FACS and BACS of a noise-free cycle. Note that the average of a complete cycle is equal to 0.

Therefore, the lowest point of the FACS curve will be equal to 0. The period of a cycle can be estimated by the interval between two valleys of the FACS curve. The amplitude of a cycle is the sum of the first and the second peak values on the FACS curve.

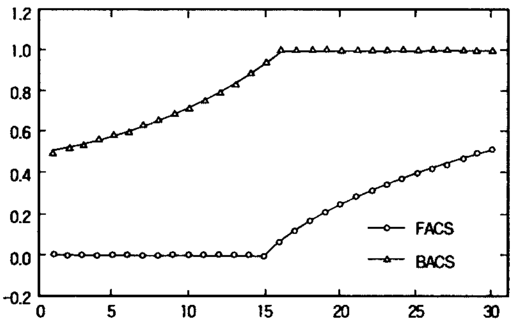

Fig. 2 FACS and BACS curves of a noise-free shift

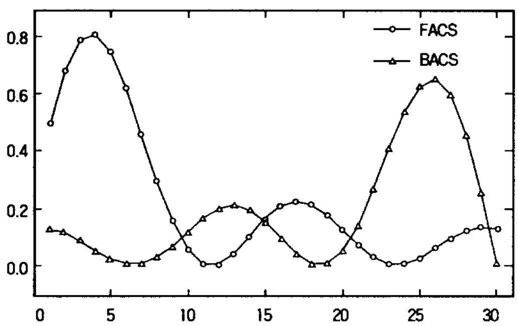

Fig. 3 FACS and BACS curves of a noise-free cycle

Figure 4 displays the FACS and BACS curves of a noise-free mixture. Plateaus, peaks and valleys can be observed on both curves. The presence of plateaus can be used to differentiate cyclic patterns from mixture patterns. The level of plateau indicates the offset from the in-control mean. The positions of peak and valley will indicate the change point of mean level.

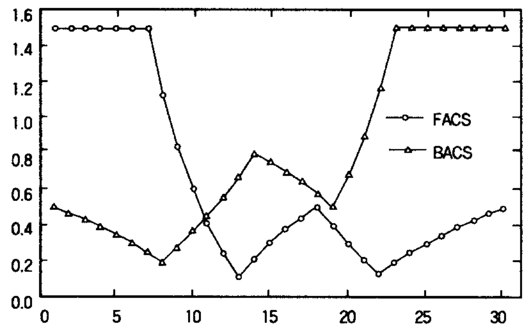

Fig. 4 FACS and BACS curves of a noise-free mixture

In systematic variations a low (high) point is always followed by a high (low) one. The plots of FACS's and BACS's might not reveal any significant feature. Therefore, FACS (odd) and BACS(odd) are computed for samples with odd number of sampling index.

The same computation is applied to samples with even number of sampling index. Figure 5 displays the FACS and BACS curves of a noise-free systematic pattern. This figure has four horizontal lines.

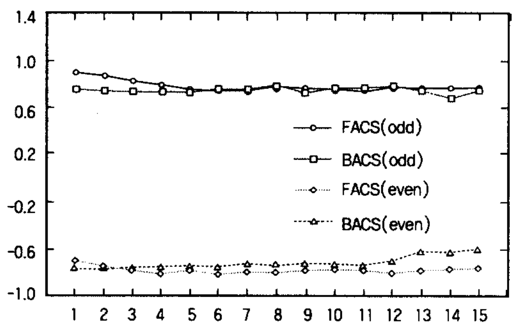

Fig. 5 FACS and BACS curves of a noise-free systematic pattern

Some notable characteristics can be seen from Figure 5. First, FACS and BACS curves for are very close. There is no intersection between FACS(odd) (or BACS(odd) and FACS (even) (or BACS(even)). Second, FACS or BACS curves are parallel with horizontal axis.

4. Examples

In this section some selected examples are used to illustrate the operation of the proposed method. For each example, there are 30 samples available for analysis. Without loss of generality, standardized values are used for purposes of illustration.

Figure 6 displays an example of trend. The slope of the trend is about $0.04\sigma_{\bar{x}}$. Figure 7 depicts the FACS and BACS curves. From Figure 6 we can see that FACS and BACS curves are almost parallel. It can be concluded that the control chart exhibits a trend pattern. The slope is estimated as $2D/(m-1)$, which is 0.0456.

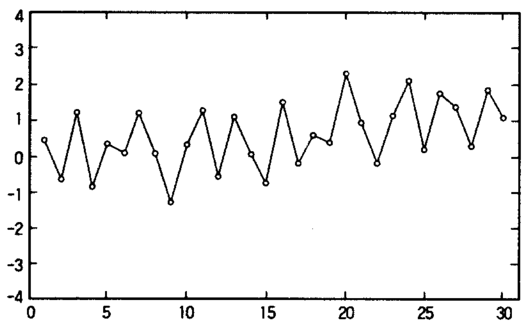

Fig. 6 Example of a trend

Figure 8 illustrates a chart exhibiting shift pattern. The shift in the mean (about $1.0\sigma_{\bar{x}}$) occurs from the sample 16 onwards. Figure 9 depicts the FACS and BACS curves. It is seen from Figure 8 that FACS and BACS curves are getting closer at one end. The magnitude of shift is estimated as the maximal D, which is 0.9877. The maximal D occurs at sample 15, we might conclude that the shift occurs at sample 16 (a correct conclusion).

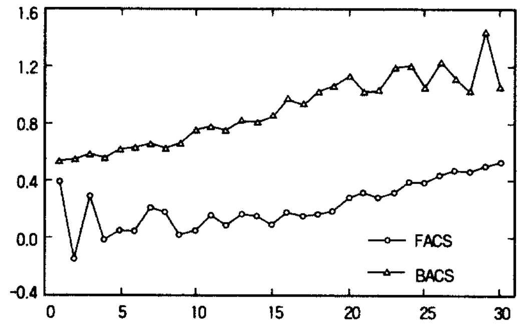

Fig. 7 FACS and BACS curves of a trend

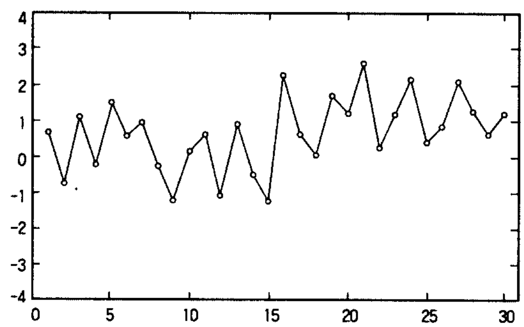

Fig. 8 Example of a shift

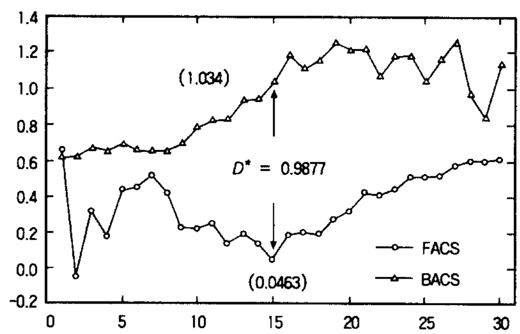

Fig. 9 FACS and BACS curves of a shift

Figure 10 displays an example of cycle with period=12. The amplitude of the cycle is about $1.0\sigma_{\bar{x}}$. Figure 11 depicts the plots of FACS's and BACS's. It is seen in Figure11 that there are two local minima occurring at samples 12 and 24. Therefore, the period is computed as 12. The amplitude is estimated as the sum of the first (sample 3) and the second (sample 16) peak values on the FACS curve, which is about 1.22.

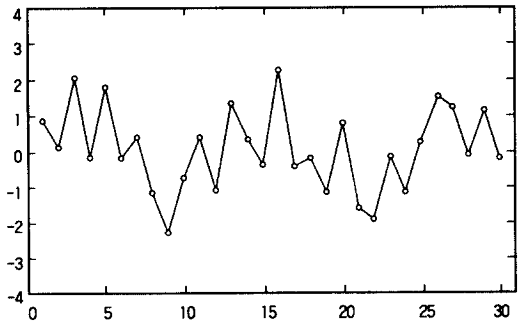

Fig. 10 Example of a cycle

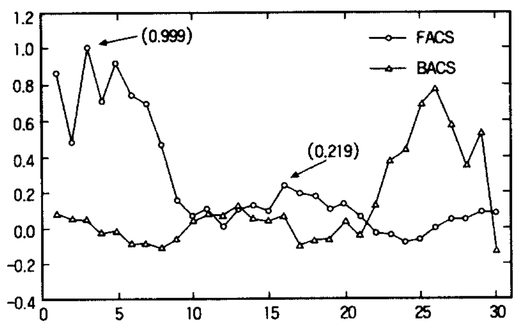

Fig. 11 FACS and BACS curves of a cycle

An example of mixture is presented in Figure 12. The shift in the mean is about $1.5\sigma_{\bar{x}}$.

Figure 13 depicts the FACS and BACS curves. The offset is estimated as $1.5\sigma_{\bar{x}}$.

There are some obvious plateaus, peaks and valleys on both curves. We conclude that samples 1-7, 14-18 and 23-30 are from the same distribution.

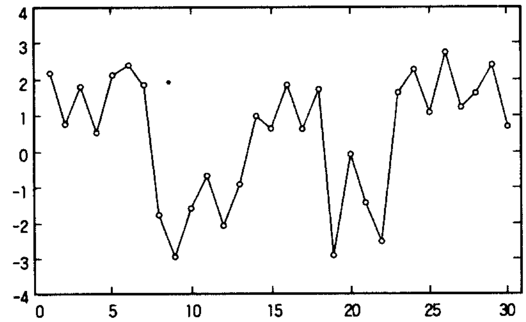

Fig. 12 Example of a mixture

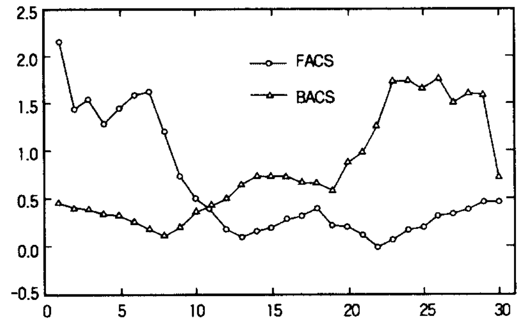

Fig. 13 FACS and BACS curves of a mixture

Figure 14 shows an example of systematic pattern. The FACS and BACS curves are displayed in Figure 15. It is clear that FACS and BACS curves are nearly parallel without any significant slope. It is therefore concluded from Figure 15 that the control chart exhibits a systematic variation.

The above examples have demonstrated that the proposed method is able to accurately identify several typical unnatural patterns and also to identify key parameters of the specific pattern involved. It should be pointed out that data stream consisting of natural and unnatural patterns will result in a weak feature, especially when the number of samples from natural pattern is larger. Hence, a moving window analysis method is suggested to implement the proposed approach. The analysis will be based on the most recent m samples. The pattern feature will gradually strengthen as the analysis window move forward through the data stream.

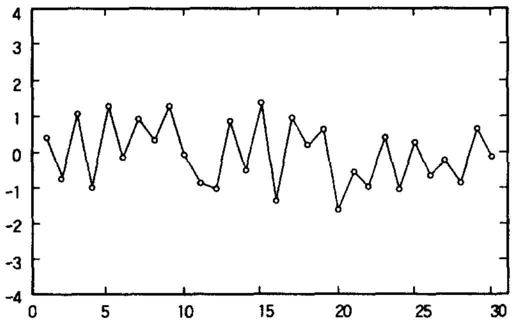

Fig. 14 Example of a systematic pattern

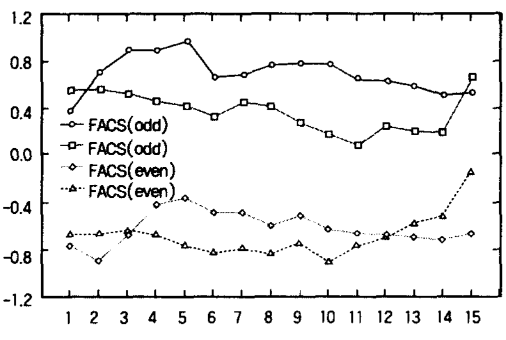

Fig. 15 FACS and BACS curves of a systematic pattern

Conclusion

A new approach using a graphical method for control chart pattern recognition is presented. The proposed approach is capable of detecting unnatural patterns as well as providing detailed pattern information, such as shift magnitude, trend slope, etc., which is important for diagnostic search. Several examples are used to illustrate the operation of the proposed approach. It can be concluded that the proposed approach offers a great potential as an aid in the analysis of control chart patterns. The proposed approach can be further improved by developing more accurate method in estimating the key parameters of the unnatural pattern.

References

1. Davis, R.B. and Woodall, W.H., Performance of the Control Chart Trend Rule under Linear Drift, *Journal of Quality Technology*, 20, 260-262.(1988).
2. Duncan, A.J., *Quality Control and Industrial Statistics*(4th), Illinois: Richard D. Irwin, Inc, 1974.
3. Grant, E. L. and Leavenworth, R. S., *Statistical Quality Control*, New York: McGraw-Hill Book Co.1988.
4. Nelson. L.S., The Shewhart Control Chart-Tests for Special Cause, *Journal of Quality Technology*, 16(4), 237-239 (1984).
5. Nelson. L.S., Interpreting Shewhart X-bar Control Charts, *Journal of Quality Technology*, 17(2), 114-116 (1985).
6. Western Electric Company, *Statistical Quality Control Handbook*, Indianapolis, Indiana: Western Electric Co. Inc (1958).