A Fracture Mechanics Analysis on Fatigue Life Estimation of DEN Plate

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DEN판재의 피로수명 평가에 관한 파괴역학적 연구

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Abstract : This paper mainly deals with fatigue life estimation and prediction in notched structures. The fatigue crack initiation life and the fatigue crack growth behavior in the DEN specimens were predicted using S.I.F. K solution derived in this study and the Paris' crack growth equation. Predicted results showed good agreement with experimental crack growth behaviors under constant-load-amplitude.

요 약 : 본 연구는 노치가 있는 구조물의 피로수명 예측 평가에 대하여 연구하였다. DEN시험편에 대한 피로균열 발생수명과 피로균열 성장수명은 SIF K해석에서 유도한 식과 Paris의 균열성장 식을 사용하였다. 예측 평가 결과는 일정하중 진폭 하에서의 실험적 균열성장거동과 잘 일치하는 결과를 보였다.
Key Words : fatigue life estimation, fracture mechanics, DEN plate

Nomenclature

b	:	W-d(mm)
C	:	coefficient of Paris' power law
d	:	notch depth(mm)
da/dN	:	crack growth rate
$\Delta\sigma$	:	alternate stress
K_t	:	stress concentration factor
n	:	exponent of Paris' power law
N	:	number of cycles
N_f	:	number of specimens fracture cycles(or total fatigue fracture life)
N_i	:	number of crack initiation cycles
N_p	:	number of crack propagation cycles(or fatigue crack growth life)
R	:	load ratio(or stress ratio)
r	:	radius of notch root(mm)
S.I.F. K	:	stress intensity factor
σ_{max}	:	peak stress at notch root
A_a	:	actual cross section area of specimens
LEFM	:	linear elastic fracture mechanics
W	:	half width of specimens(mm)

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1. Introduction

Fatigue crack propagation in engineering materials has been the subject of considerable research. By the use of fracture mechanics principles it is possible to predict the number of cycles spent in growing a crack to some specified length or to final failure. The aircraft industry has been instrumental in the effort to understand and predict fatigue crack growth. They have developed the safe-life or fail-safe design approach. In this method, a component is designed such that if a crack forms, it will not grow to a critical size between specified inspection intervals. Thus, by knowing the material growth rate characteristics and with regular inspections, a cracked component may be kept in service for an extended useful life.

Total fatigue life[N_f] of almost all machine components and structures becomes the sum of fatigue crack initiation[N_i] and fatigue crack growth life[N_p].[1] It has been shown that fatigue cracks emanate from notch and other flaws in components, particularly in aircraft structures, nuclear pressure vessels and motor cars.[2~5] From an engineering point of view, it is convenient to separate the total fatigue life(N_f) into two parts: the crack initiation life(N_i) and the crack propagation life (N_p), which is the time it takes for growing cracks to fail.[6~10] We adopted a working stress of structures lower than the yielding stress of the materials. If the fatigue crack initiation and growth are predicted, they can effectively be used for safety design, repair and control. Therefore, the development of a fatigue life estimation system for notched materials is required. In the present study, the main points are the construction of a fatigue life estimation system for notched materials and the adequate verification of this system.

2. Procedure for Fatigue Life Estimation

The fatigue life of notched materials can be estimated by the following procedure.

[1] Estimation of a fatigue crack initiation(N_i):
After fatigue test, the fatigue crack initiation life can be estimated when the notch root radius and depth are variable.

[2] The Stress Intensity Factor[S.I.F. K] analysis of the notch tip:
S.I.F. K can be analyzed by the FEM method according to the varying notch root radius and depth.[11]

[3] Relation of da/dN-ΔK analysis for fatigue crack at notch tip:
The da/dN-ΔK curve can be obtained with the fatigue crack growth rate da/dN by Nisitani's[12] formula.

[4] Fatigue crack propagation life[N_p] estimate from da/dN-ΔK curve:
The fatigue crack propagation life can be estimated by numerical integral analysis from the da/dN-ΔK curve.[13]

[5] Total fatigue life[N_f] with notched plate:
The total fatigue life[N_f] becomes the sum of the fatigue crack initiation life and the fatigue crack growth life i.e., $N_f = N_i + N_p$.

3. Estimation of Fatigue Crack Initiation Life

Almost all machine components and structural members contain some form of geometrical or microstructural discontinuities. These discontinuities, or stress concentrations, often result in maximum local stresses σ_{max} at the discontinuities which are many times greater than the nominal stress of the number, σ_n. In ideally elastic members, the ratio of these stresses is designated as K_t, the theoretical stress concentration factor.

$$K_t = \sigma_{max} / \sigma_n \tag{1}$$

where

σ_{max} : maximum local stress

σ_n : nominal stress($= P_{max} / A_a$)

This theoretical stress concentration factor is solely dependent on geometry and mode of loading. Heywood[14] proposed the stress concentration factor formula for the DEN plate. This is obtained by photoelastic test results. From these tests Heywood proposed the following empirical relationship :

$$K_t = 1 + \left[\frac{W/b - 1}{(1.55\,W/b - 1.3)} \frac{b}{r} \right]^n \tag{2}$$

$$n = \frac{(W/b - 1) + 0.5\sqrt{d/r}}{(W/b - 1) + \sqrt{d/r}}$$

where

d : notch depth(mm)

r : radius of notch root(mm)

W : half width of specimens

b : W-d

The maximum local stress can be obtained from Eq.(1). Table 1 is a summary of the stress concentration factor of the DEN plates. The values of K_t are obtained from Eq.(2) and are dependent on the geometry

Table 1. Summary of stress concentration factor(Kt) from Eq.(2)

d(mm)	7.00	10.50	12.25	14.00	15.75	17.50	19.25
d/W	0.20	0.30	0.35	0.40	0.45	0.50	0.55
b(mm)	28.00	24.50	22.75	21.00	19.25	17.50	15.75
W/b	1.250	1.429	1.538	1.667	1.818	2.000	2.222
A_s(mm^2) / r(mm)	375.20	328.30	304.85	281.4	257.95	234.5	211.05
0.5	6.157	6.485	6.537	6.551	6.531	6.460	6.369
1.0	4.673	4.905	4.953	4.961	4.929	4.879	4.806
1.5	4.001	4.189	4.218	4.224	4.197	4.145	4.070
2.0	3.591	3.752	3.772	3.772	3.743	3.691	3.618
2.5	3.307	3.448	3.465	3.457	3.427	3.375	3.304

of the notch shape. In the present study, the load ratio, $R(=P_{min}/P_{max})$ is 0.1 in the fatigue test.[13]

The alternating stress($\nabla\sigma$) can be obtained from the maximum(σ_{max}) and the minimum(σ_{min}) local stresses. Table 2 is a summary of the alternating stress($\nabla\sigma$) and the crack initiation cycles(N_i) depending on variable notch shape. Fig.1 shows the contents in table 2 which is the relation by the log-log coordinates between actual alternating stress($\nabla\sigma$) and fatigue crack initiation life(N_i) depending on the stress concentration at the tip of the notch according to the depth and the radius of notch root.

To obtain the formula for the relationship N_i and $\nabla\sigma$ according to the notch root radius from Fig.1, the relation $\nabla\sigma$ and N_i is nondimensionalized. That is, $\nabla\sigma$

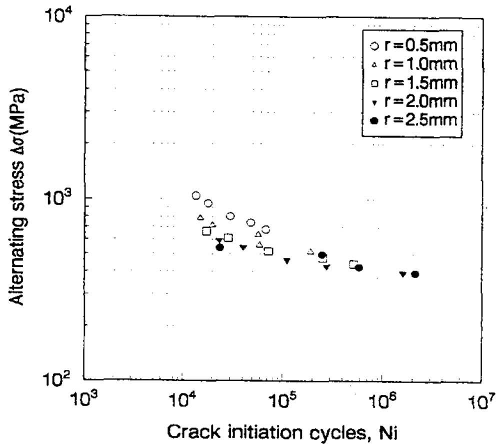

Fig. 1. Alternating stress($\Delta\sigma$) vs. number of crack initiation cycles(N_i)

Table 2. Summary of alternating stress($\Delta\sigma$) and crack initiation cycles(N_i) in the fatigue test[13]

r(mm)	d/w	0.3	0.35	0.40	0.50	0.55
0.5	$\Delta\sigma$ (MPa)	681	739	801	949	1,039
	N_i	67,500	48,000	30,000	18,000	13,500
1.0	$\Delta\sigma$ (MPa)	515	559	637	717	785
	N_i	189,000	58,500	56,700	20,000	15,000
1.5	$\Delta\sigma$ (MPa)	439	477	516	609	664
	N_i	516,000	252,000	72,000	28,700	17,500
2.0	$\Delta\sigma$ (MPa)	393	427	461	542	591
	N_i	1,596,700	276,000	110,000	40,500	23,400
2.5	$\Delta\sigma$ (MPa)		391	423	496	538
	N_i	∞	2,115,220	580,000	247,500	23,700

comes to be $\nabla\sigma/420$ and N_i comes to be $N_i/6\times10^5$. The results shown in Table 3. Fig. 2 shows the contents in table 3 by log-log coordinates. The data points are concentrated linearly to the 6×10^5 cycles(N_i) and 420 MPa($\nabla\sigma$). The data can be expressed as an exponential function form.

$$\log(\triangle\sigma/420) = \alpha \cdot \log(\Rightarrow/6\times10^5) + \log\beta \qquad (3)$$

where

α : slope of data points

$\log\beta = 0$

Table 3. Results of non-dimensional $\Delta\sigma$ and N_i

r(mm)	d/W	0.3	0.35	0.40	0.50	0.55
0.5	$\Delta\sigma$/420	1.621	1.760	1.907	2.260	2.474
	Ni/6×10^5	0.113	0.080	0.050	0.030	0.023
1.0	$\Delta\sigma$/420	1.226	1.330	1.517	1.707	1.869
	Ni/6×10^5	0.315	0.098	0.095	0.033	0.025
1.5	$\Delta\sigma$/420	1.045	1.136	1.229	1.450	1.581
	Ni/6×10^5	0.860	0.420	0.120	0.048	0.029
2.0	$\Delta\sigma$/420	0.936	1.017	1.098	1.290	1.407
	Ni/6×10^5	2.661	0.460	0.183	0.068	0.039
2.5	$\Delta\sigma$/420	0.751	0.931	1.007	1.181	1.281
	Ni/6×10^5	∞	3.525	0.967	0.413	0.040

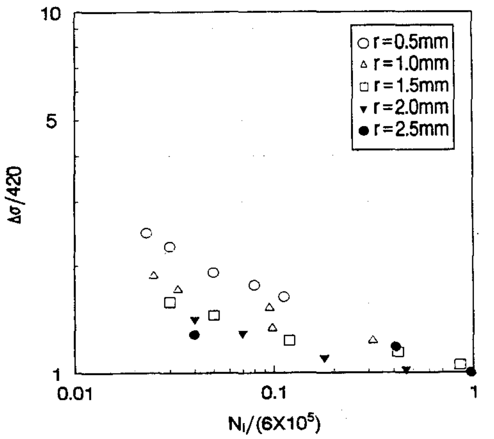

Fig. 2. Non-dimensional alternating stress vs. crack initiation cycles($\Delta\sigma$)/420 - $N_i/6\times10^5$ relation)

N_i is shown as follows:

$$N_i = (\frac{\Delta\sigma}{420})^{1/\alpha} \cdot 6\times10^5 \qquad (4)$$

where slope α is a variable according to the notch root radius(r).

If $\nabla\sigma$, α values are given, the fatigue crack initiation life can be evaluated. To obtain the fatigue crack initiation life, the slope α has to be a function of r. Table 4 shows the results of slope α and variable notch root radius. Fig. 3 shows the relationship α and r of table 4. The equation shown as a fractional equation in Fig. 3 is as follows:

Table 4. Results of slope(α) and notch root radius(r)

r(mm)	0.5	1.0	1.5	2.0	2.5
α	-0.2297	-0.1681	-0.1232	-0.0952	-0.0812

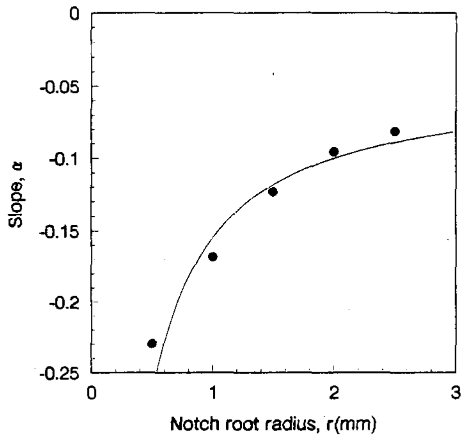

Fig. 3. Slope(α) vs. notch root radius(r)

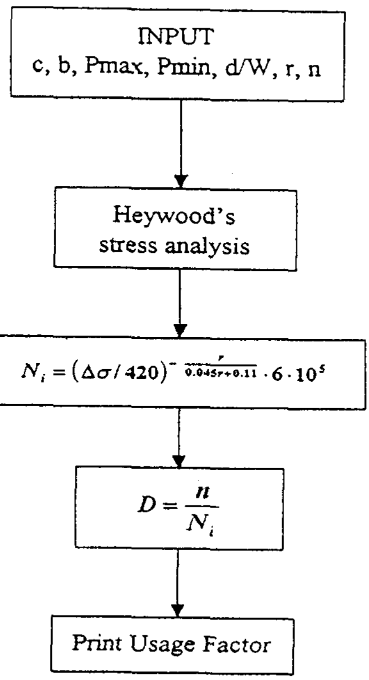

Fig. 4. simplified evaluation procedure of the crack initiation cycles.

$$\alpha = -0.11 \cdot r^{-1} - 0.045 \qquad (5)$$

where, if $r \to \infty$ then $\alpha \to -0.045$. Putting Eq.(5) into Eq.(4) results in an equation that can be solved for N_i as follows:

$$N_i = (\frac{\Delta\sigma}{420})^{-(\frac{r}{0.045r+0.11})} \cdot 6\times10^5 \qquad (6)$$

4. The Estimation of Fatigue Crack Growth Life

Behavior of the fatigue crack growth(da/dN-ΔK) is divided into the 3 Regions as in Fig.5. Most of the current applications of LEFM concepts to describe crack growth behavior are associated with Region II in Fig. 5. In this region the slope of the log da/dN versus log ΔK curve is approximately linear. The Paris[15]

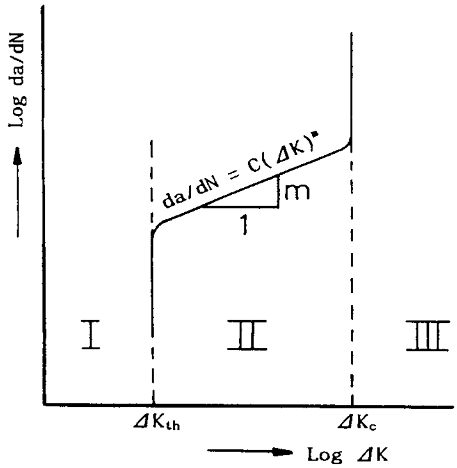

Fig. 5. da/dN - ΔK relation

equation, which was proposed in the early 1960s, is the most widely accepted. The values of the exponent, m, of metals are usually between 3~4. In the present study, da/dN is calculated by the secant method proposed in ASTM E647 and m is estimated as 4. The value of ΔK analysis is estimated by Eq.(7).[12)]

$$\Delta K = \Delta\sigma\sqrt{\pi a}\,(1.122-0.154B+0.807B^2-1.894B^3+2.494B^4) \qquad (7)$$

where

a : half length of crack

B : a/W

The generalized formula for da/dN is shown as follows:

$$da/dN=f(\Delta K) \qquad (8)$$

In the present study, fatigue crack growth life was estimated by Eq.(9):

$$da/dN=C(\Delta K)m \qquad (9)$$

Fatigue crack growth life N_p is shown as follows:

$$N_p=\int_{a_i}^{a_p}\frac{da}{f(\Delta K)} \qquad (10)$$

where

a_i : initiated crack length

a_p : fracture crack length

Using Paris formula da/dN is shown as follows:

$$da/dN = C(\Delta K)^m$$

$$N_p=\int_{a_i}^{a_p}\frac{da}{C(\Delta K)^m} \qquad (11)$$

where ΔK is obtained from Eq.(7), *m*=4 is put into Eq.(11) and then N_p is estimated from Eq.(11). When estimating fatigue crack growth life, generally the final crack length doesn't have a great effect on the fatigue crack growth life. In case of the fatigue crack growth where the final crack length greatly changes for high strength steel, the fracture life is short. Generally an alternative approximate method is useful for estimating the fatigue crack life in a constant-amplitude loading condition. Fig. 6 shows the simplified evaluation procedure of the crack growth cycles.

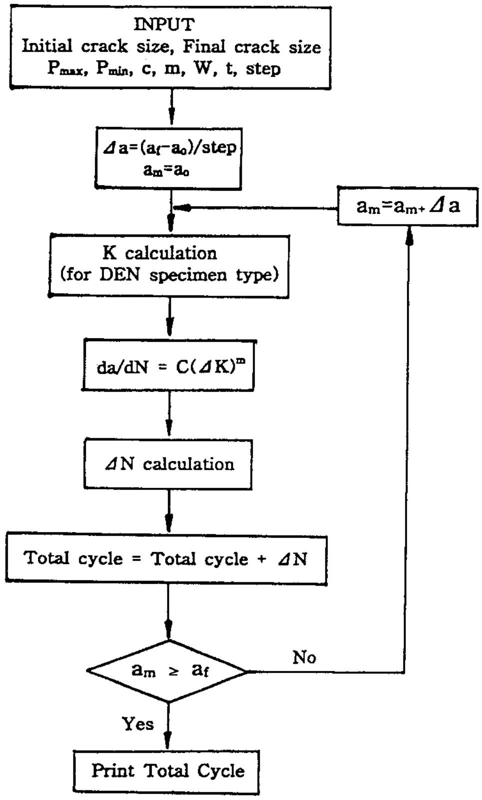

Fig. 6. Simplified evaluation procedure of the crack growth cycles

5. Fatigue Life Evaluation Method for the DEN Plate

The number of fracture cycles(N_f) involves crack initiation cycles(N_i) and crack propagation cycles(N_p) to obtain the crack initiation cycles. Eq.(6) was used for the regression analysis of experimental results of DEN specimens to determine the crack initiation life in the notched materials. And then the system for estimating crack initiation life is constructed as in Fig. 4.

The evaluation method of crack growth life is shown in part 4 and Fig. 6. Fatigue life evaluation system for the notched material is structured by combining Fig.4 with Fig.6. Fig. 7 shows the simplified life evaluation procedure of the notched plate. The case studies were done for r=1.50mm, d/W=0.30, 0.35, 0.40, 0.50 and da/dW=0.40, r=1.00, 1.50, 2.00, 2.50mm to verify adequacy. In these cases, Table 5 and Table 6 show the results of the case study at r=1.50,

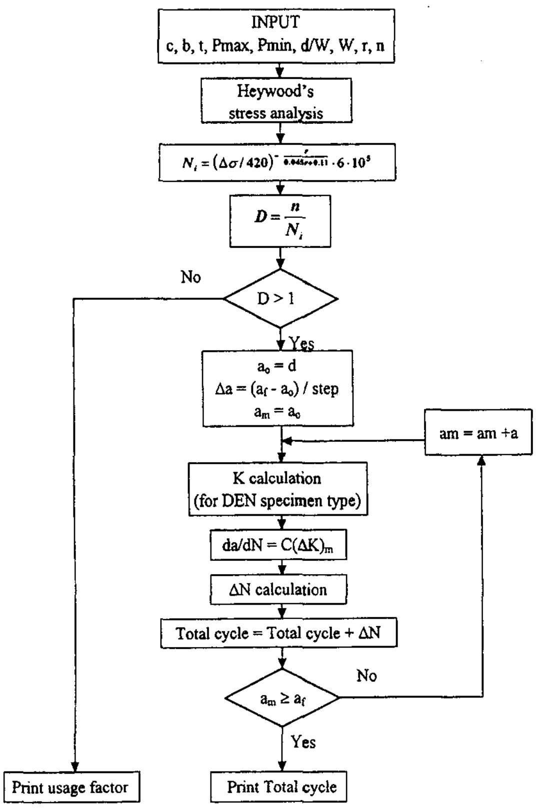

Fig. 7. Simplified life evaluation of the notched plate

Table 5. Results of case study at r=1.5 mm

d/W	Results of test $N_{F,TEST}$	Results of Estimation $N_{F,EST}$	error %
0.30	639,500	456,882	29
0.35	502,000	256,536	49
0.40	128,030	164,933	-29
0.50	68,000	100,147	-47

※ $N_{F,EST} = N_{i,EST} + N_{P,EST}$

Table 6. Results of case study at d/W=0.4

r(mm)	Results of test $N_{F,TEST}$	Results of Estimation $N_{F,EST}$	Error %
1.0	110,000	63,042	43
1.5	128,030	164,933	-29
2.0	210,660	330,583	-57
2.5	713,570	691,691	31

※ $N_{F,EST} = N_{i,EST} + N_{P,EST}$

and d/W=0.40. Also, the results of test and results of estimation are shown in tables 5 and 6. The results of estimation match well with that of test and the accuracy is within 57%. However, the error may vary with the selected materials and the size of notch root radius.

6. Results

The results of this study can be summarized as follows:

[1] Considering the stress concentration factor on the notch tip, when the maximum stress is estimated and the material constant is evaluated from the relation between maximum stress and crack initiation life, then the crack initiation life estimation was performed using these results. By this method, the fatigue life can be estimated considering these results, for in the fatigue life estimation of notched material, in order to predict the life with accuracy, the increasing stress of the notched material is considered.

[2] The fatigue crack initiation life can be predicted using the following formula.

$$N_i = \left(\frac{\Delta\sigma}{420}\right)^{-\left(\frac{r}{0.045r+0.11}\right)} \cdot 6\times 10^5$$

[3] The total fatigue life(N_f) can be estimated by

adding the fatigue crack initiation life(N_i) and the fatigue crack propagation life(N_p).

[4] A fatigue life estimation system was established. Based on the predicted results for the case study at r=1.50mm and d/W=0.40, the ratio of the predicted results to experimental results was found to be within 57%.

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