International Journal of Reliability and Application
Vol.1, No.1, pp.15–26, 2000.

Aperiodic Preventive Maintenance Model and Parameter Estimation †

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Abstract. This paper considers an aperiodic preventive maintenance (PM) model for repairable systems, in which the time intervals between two consecutive preventive maintenances are unequal. To propose such an aperiodic PM model, we assume that each PM reduces the current hazard rate by a certain amount which depends on the number of PMs performed previously. If the system fails between PMs, the minimal repair is performed and the hazard rate remains unchanged after the repair. We give the exact expressions for the hazard rate function for the aperiodic PM model. Based on the proposed aperiodic PM model, we suggest the maximum likelihood method to estimate the parameters characterizing the model and apply the method to the case of Weibull distribution. Numerical examples for estimating the parameters are presented for the purpose of illustration.

Key Words : *aperiodic preventive maintenance, hazard rate, minimal repair, restoration interval, HPP, NHPP, maximum likelihood method.*

1. INTRODUCTION

Preventive maintenance (PM) is used to slow the degradation process of the repairable system and keep the system operating without failure during its mission period. Since most of the repairable systems are subject to deteriorate with time in practice, efficient maintenance of the system is critical to reduce the failure of the system and to improve the productivity of the system. Thus, it is desirable to

†This paper was conducted by the research fund provided by Korean Council for University Education, support for 2000 Domestic Faculty Exchange Program.

develop a PM model, under which the system keeps its operation at the necessary level of reliability and the operating cost is minimized over a finite time span.

Because of the wide range of applications and great importance of the PM policy in reliability theory, many authors have studied the problem of modeling the PM policies and of choosing an optimal PM policy by minimizing the average cost per unit time of operating the system. Malik(1979) proposes a general approach to model the improvement effect of maintenance and introduces improvement factor in the proportional age reduction model with a reliable preventive maintenance scheduling. The measure of PM effect is explained by the proportion of reduced age, which is referred to as improvement factor. Shin, Lim and Lie(1996) employ the maximum likelihood method to estimate the parameters of the failure process and the maintenance effect in Malik's proportional age reduction model. Nakagawa(1979) proposes several optimal policies when the PM is imperfect. Murthy and Nguyen(1981) study the optimal age replacement policy with imperfect preventive maintenance. The preventive is imperfect in the sense that it can cause failure of a non-failed system. Nakagawa(1986) studies both periodic and sequential PM policies to determine the optimal PM intervals and the optimal number of PMs to be performed before replacing the system by a new one.

This paper extends the periodic PM model proposed by Canfield(1986) to the case when the system undergoes the PMs at different intervals. Canfield(1986) discusses a periodic PM model of a system for which the PM slows the rate of degradation, while the hazard rate keeps monotone increase. Park, Jung and Yum(2000) determine the optimal period and the optimal number of PMs for Canfield's(1986) periodic PM model so that the expected cost rate per unit time for an infinite time span is minimized. In this paper, we propose an aperiodic PM model (Nakagawa(1986) called it sequential PM model), for which the system is maintained preventively at constant intervals $x_k(k = 1, 2, \cdots, N)$ and is replaced by a new system at the Nth PM. Each PM is assumed to relieve stress temporarily and to slow the degradation process of the system. If the system fails between PMs, it undergoes only minimal repair and hence, the hazard rate remains unchanged after any of these minimal repairs is completed. Since the effect of PM may depend not only on the age of the system, but also on the number of PMs performed previously, it is more reasonable to perform the PMs at different intervals to make the PM policy more effective.

In Section 2, we present the assumptions and the exact expressions for hazard rate function to propose a new aperiodic PM model. Section 3 estimates the parameters characterizing the aperiodic PM model by applying the maximum likelihood method. We give more detailed discussions of the method when the failure time of the system follows Weibull distribution. Section 4 presents two numerical examples, simulation results and real data analysis, to explain the proposed method.

2. ASUMPTION AND APERIODIC PM MODEL

Assumptions

1) The system is maintained preventively at constant intervals x_k $(k = 1, 2, \cdots, N)$ and is replaced by a new system at the Nth PM. Thus, the PM is done at successive times $0 < x_1 < x_1 + x_2 < \cdots < x_1 + x_2 + \cdots + x_N$ and the system is replaced at time $x_1 + x_2 + \cdots + x_N$.

2) If the system fails between PMs, it undergoes only minimal repair and hence, the failure rate remains undisturbed by any of these minimal repairs.

3) Each PM reduces operational stress to that existing τ_k time units previous to PM intervention, where τ_k is the restoration interval which is less than or equal to x_k, the PM intervention interval.

4) The level of system hazard function depends on the extent of system degradation.

Let F denote the life distribution function with its corresponding density function f and let $\bar{F}(t) = 1 - F(t)$ be its survival function. Then, we have

Definition 1. The hazard rate of a life distribution F is defined as

$$h(t) = f(t)/\bar{F}(t)$$

for t such that $\bar{F}(t) > 0$ if $f(t)$ exists.

Definition 2. A life distribution F is IFR(DFR) if $h(t)$ is nondecreasing (nonincreasing) in $t \geq 0$.

In this paper, we consider only the case when $h(t)$ is strictly increasing and thus, the system degrades as it ages. If the system wears out with time, the rationale for the proposed aperiodic PM model is that it may need more frequent maintenances to the extent that budget and man power permit. Let x_k and y_k be the time interval between the $(k-1)$st and the kth PM and the kth PM time, respectively. That is, $y_k = \sum_{i=1}^{k} x_i$. In addition, we let τ_k and ρ_k be the restoration interval and the improvement factor at the kth PM. Thus, we assume that $\tau_k = \rho_k x_k$,where $0 \leq \rho_k \leq 1$. We use $h(t)$ and $h_{pm}^k(t)$ to denote the hazard rate without PM and the hazard rate between the kth and the $(k+1)$st PM, respectively. When the kth PM is performed effectively at time y_k, the hazard rate function changes from $h_{pm}^{k-1}(t)$ to $h_{pm}^k(t)$, where $h_{pm}^{k-1}(t) \geq h_{pm}^k(t)$ for all $t > y_k$ for $k = 1, 2, \cdots, N$.

For Canfield's(1986) periodic PM model, the PM reduces operational stress to that existing τ time units previous to the PM intervention, where τ is a restoration interval. Thus, the hazard rate under Canfield's model is given as

$$h_p(t) = h_p(k\Delta t) + h(t - k\tau) - h(k(\Delta t - \tau)) \tag{2.1}$$

for $k = 0, 1, 2, \cdots$, where $k\Delta t < t \leq (k+1)\Delta t$ and $h_p(0) = h(0)$. By assumptions (1) and (3), we generalize Canfield's model to the case when the PMs are performed at different intervals. Applying (2.1), the hazard rate under the aperiodic PM model is defined as

$$\begin{aligned} h_{pm}^k(t) =& h_{pm}^{k-1}\Big(\sum_{i=1}^{k} x_i\Big) + h\Big(t - \sum_{i=1}^{k} \tau_i\Big) - h\Big\{\sum_{i=1}^{k}(x_i - \tau_i)\Big\} \\ & \text{for } \sum_{i=1}^{k} x_i < t \leq \sum_{i=1}^{k+1} x_i,\ k = 0, 1, 2, \cdots . \end{aligned} \tag{2.2}$$

By substituting recursively, (2.2) can be rewritten as

$$h_{pm}^k(t) = \begin{cases} h(t), & \text{for } 0 \leq t \leq x_1 \\ \sum_{i=1}^{k}\Big\{h\Big(\sum_{j=1}^{i-1}(x_j - \tau_j) + x_i\Big) & \\ -h\Big(\sum_{j=1}^{i}(x_j - \tau_j)\Big)\Big\} + h\Big(t - \sum_{i=1}^{k}\tau_i\Big), & \text{for } \sum_{i=1}^{k} x_i < t \leq \sum_{i=1}^{k+1} x_i \end{cases} \tag{2.3}$$

If we set $\tau_k = \tau$ and $x_k = x$ for all k, then (2.3) is reduced to the following failure rate, which is equivalent to (2.1) by replacing x by Δt.

$$h_{pm}^k(t) = \begin{cases} h(t), & \text{for } 0 \leq t \leq x \\ \sum_{i=1}^{k}\Big\{h((i-1)(x-\tau) + x) & \\ -h(i(x-\tau))\Big\} + h(t - k\tau), & \text{for } kx < t \leq (k+1)x \end{cases} \tag{2.4}$$

Furthermore, if we let $\tau_k = x_k = \tau = x$ for all k, then the PM is performed periodically and the restoration interval at each PM is equal to the periodic PM interval. In this case, (2.3) is reduced to

$$h_{pm}^k(t) = \begin{cases} h(t), & \text{for } 0 \leq t \leq x \\ k\big\{h(x) - h(0)\big\} + h(t - kx), & \text{for } kx < t \leq (k+1)x \end{cases} \tag{2.5}$$

The hazard rates (2.4) and (2.5) have been proposed and studied by Canfield(1986). In addition to the different PM intervals, we generalize the hazard rate model, given in (2.2), further by incorporating the improvement factor in the model. By replacing τ_k of (2.2) by $\rho_k x_k$ for $k = 0, 1, 2, \cdots$, we obtain

$$\begin{aligned} h_{pm}^k(t) =& h_{pm}^{k-1}\Big(\sum_{i=1}^{k} x_i\Big) + h\Big(t - \sum_{i=1}^{k} \rho_i x_i\Big) - h\Big\{\sum_{i=1}^{k}(x_i - \rho_i x_i)\Big\}, \\ & \text{for } \sum_{i=1}^{k} x_i < t \leq \sum_{i=1}^{k+1} x_i \end{aligned} \tag{2.6}$$

By substituting recursively, (2.6) can be rewritten as

$$h_{pm}^{k}(t) = \begin{cases} h(t), & \text{for } 0 \le t \le x_1 . \\ \sum_{i=1}^{k}\Big\{h\Big(\sum_{j=1}^{i-1}(x_j-\rho_j x_j)+x_i\Big)-h\Big(\sum_{j=1}^{i}(x_j-\rho_j x_j)\Big)\Big\}+h\Big(t-\sum_{i=1}^{k}\rho_i x_i\Big), & \\ \qquad\qquad \text{for } \sum_{i=1}^{k} x_i < t \le \sum_{i=1}^{k+1} x_i . & \end{cases} \tag{2.7}$$

If $\rho_k = 0$ in (2.7), then the PM does not improve the system at all at the kth PM and thus if $\rho_k = 0$ for all k, then the state of the system is as bad as old at each PM. This model is referred to as BAO model for later use. In this case, $h_{pm}^{k}(t) = h(t)$ for $\sum_{i=1}^{k} x_i < t \le \sum_{i=1}^{k+1} x_i$ and $k = 0, 1, 2, \cdots$. A typical plot of the hazard rate under the aperiodic PM model is shown in Figure 2.1. Here, we assume that $\rho_k = \rho$ for all k in (2.7).

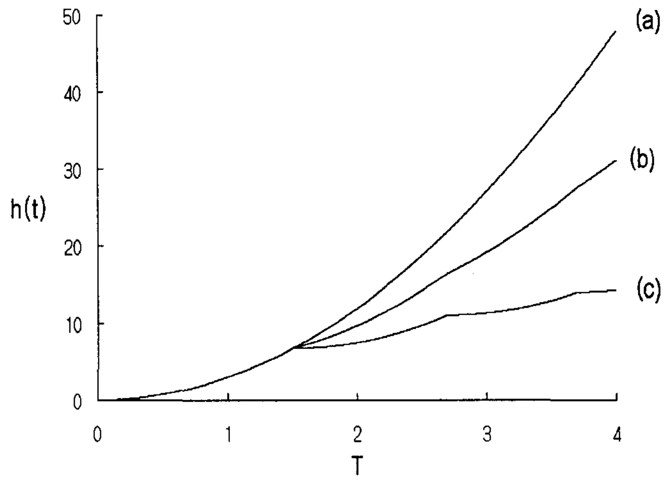

Figure 2.1. Hazard rate under the aperiodic PM model with (a) $\rho = 0$, (b) $\rho = 0.5$ and (c) $\rho = 1$.

3. PARAMETRIC ESTIMATION

The inter-failure times in nonhomogeneous Poisson process(NHPP) are dependent on the total time elapsed from the origin so that NHPP is usually adopted to model the failures of aging system. When the failures are modeled by NHPP, the aging system is assumed to have an increasing intensity function which reflects the decreasing trend in inter-failure time.

Methods of data analysis under various parametric settings of the intensity functions in NHPP are summarized in Ascher and Feingold(1984). Cox and Lewis(1966), Brown(1972), and others discuss statistical procedures for time-dependent Poisson processes.

Let $\{T_i : i = 1, 2, \cdots\}$ be a stochastic process of Poisson type with the intensity function, $\lambda(t)$, which is defined as the rate of occurrence of the events and we may interpret T_i as the time of occurrence of the ith event. Suppose that the failures are censored at y^* and the number of failures before the censoring time is equal to n. It is known that the conditional joint probability density function(pdf) of the first n failure times, given there are n occurrences in time $(0, y^*]$ is given by

$$f(t_1, t_2 \cdots, t_n \mid N(y^*) = n) = \frac{n! \prod_{k=1}^{n} \lambda(t_k)}{\left[\int_0^{y^*} \lambda(s)ds\right]^n},$$

and thus, the unconditional joint pdf of $T_1, \cdots, T_n$ is given by

$$f(t_1, t_2 \cdots, t_n) = \Big\{\prod_{k=1}^{n} \lambda(t_k)\Big\} \exp\Big\{-\int_0^{y^*} \lambda(s)ds\Big\}.$$

Thus, the likelihood function can be written as

$$L = \Big\{\prod_{k=1}^{n} \lambda(t_k)\Big\} \exp\Big\{-\Lambda(y^*)\Big\}, \tag{3.1}$$

where $\Lambda(t) = \int_0^t \lambda(u)du$. If the process is observed until the nth failure instead of fixing the censoring time, then the likelihood function is obtained by replacing y^* of (3.1) by t_n, the nth failure time.

In this section, we estimate the parameters of the intensity function and the measure of the PM effect for the aperiodic PM model by applying the maximum likelihood method on the basis of NHPP assumption for the failures within each PM interval. For our aperiodic PM model, the intensity function is piecewise continuous and is expressed by

$$h(t) = h_{pm}^k(t) \quad \text{for } y_k < t \le y_{k+1} \quad \text{and} \quad k = 0, 1, 2, \cdots,$$

and the PM times provide mutually disjoint subintervals such as $(y_0 = 0, y_1]$,$(y_1, y_2]$, $\cdots$,$(y_k, y_{k+1}]$, $\cdots$.

For the purpose of parameter estimation, we consider the case that all the improvement factors are equal. That is, $\rho_k = \rho$ for all k. Thus, the hazard rate under our consideration has the following expression.

$$h_{pm}^{k}(t) = h_{pm}^{k-1}\Big(\sum_{i=1}^{k} x_i\Big) + h\Big(t - \sum_{i=1}^{k} \rho x_i\Big) - h\Big\{\sum_{i=1}^{k}(1-\rho)x_i)\Big\},$$
$$\text{for } \sum_{i=1}^{k} x_i < t \leq \sum_{i=1}^{k+1} x_i, k = 0, 1, 2, \cdots . \tag{3.2}$$

Replacing $y_k = \sum_{i=1}^{k} x_i$ and substituting recursively, it is easy to show that (3.2) is reduced to

$$h_{pm}^{k}(t) = \sum_{i=1}^{k}\Big\{h(y_i - \rho y_{i-1}) - h(y_i - \rho y_i)\Big\} + h(t - \rho y_k), \text{ for } y_k < t \leq y_{k+1} \tag{3.3}$$

Next, we let N_k be the number of failures during the interval $(y_k, y_{k+1}], k = 0, 1, 2, \cdots$ and $\mathbf{T_k} = (T_{k1}, T_{k2}, \cdots, T_{kN_k})$ denote the failure times in this interval. From (3.1) we can write the joint pdf of $(\mathbf{T_k}, N_k)$ as

$$f(t_{k1}, t_{k2}, \cdots, t_{kn_k}) = \Big\{\prod_{j=1}^{n_k} h_{pm}^{k}(t_{kj})\Big\} \exp\Big\{-H_{pm}^{k}(y_{k+1})\Big\}, \tag{3.4}$$

where $H_{pm}^{k}(t) = \int_{y_k}^{t} h_{pm}^{k}(s)ds$. When the PM interventions are pre-scheduled, they are not random and thus, the failure time vectors $\mathbf{T_1}, \mathbf{T_2}, \cdots$ are independent. For notational convenience, we take the censoring time y^* to be equal to y_{n+1}. Then the joint pdf of the failure times is simply the product of the joint pdfs given in (3.4) by property of NHPP. Thus, the likelihood function is represented as

$$L = f(\mathbf{t_0}, \mathbf{t_1} \cdots, \mathbf{t_n}) = \prod_{k=0}^{n}\prod_{j=1}^{n_k} h_{pm}^{k}(t_{kj}) \exp\Big\{-\sum_{k=0}^{n} H_{pm}^{k}(y_{k+1})\Big\} \tag{3.5}$$

To examine and illustrate the parametric approach more explicitly, we consider a specific model for which the failure time of the system has Weibull distribution. Let $\{T_i : i = 1, 2, \cdots\}$ be a stochastic process of Poisson type with the Weibull intensity function and the PM times provide disjoint subintervals $(y_0 = 0, y_1]$,$(y_1, y_2]$, $\cdots$,$(y_n, y_{n+1}]$, $\cdots$. Suppose the failures are censored at y_{n+1} and $m = \sum_{k=0}^{n} n_k$, the total number of failures during the whole observation period.

The Weibull distribution has the following form of hazard function.

$$h(t) = \gamma\theta^{-\gamma}t^{\gamma-1}$$

for $t > 0$, where $\theta > 0$ and $\gamma > 0$ are scale and shape parameters, respectively. Under the aperiodic PM model, given in (3.3), the hazard rate is piecewise continuous and it is expressed as

$$h_{pm}^k(t) = \gamma\theta^{-\gamma}\left[\sum_{i=1}^{k}\left\{(y_i - \rho y_{i-1})^{\gamma-1} - (y_i - \rho y_i)^{\gamma-1}\right\} + (t - \rho y_k)^{\gamma-1}\right],$$

$$y_k < t \le y_{k+1} \qquad (3.6)$$

Combining (3.5) and (3.6), the log-likelihood function can be written as

$$\begin{aligned}\ln L(\theta,\gamma,\rho) =& m\ln\gamma - m\gamma\ln\theta \\ &+ \sum_{k=0}^{n}\sum_{j=1}^{n_k}\ln\left[\sum_{i=1}^{k}\left\{(y_i - \rho y_{i-1})^{\gamma-1} - (y_i - \rho y_i)^{\gamma-1}\right\} + (t_{kj} - \rho y_k)^{\gamma-1}\right] \\ &- \theta^{-\gamma}\sum_{k=0}^{n}\left[\sum_{i=1}^{k}\gamma\left\{(y_i - \rho y_{i-1})^{\gamma-1} - (y_i - \rho y_i)^{\gamma-1}\right\}(y_{k+1} - y_k)\right. \\ &\left. + (y_{k+1} - \rho y_k)^{\gamma} - (y_k - \rho y_k)^{\gamma}\right]. \end{aligned} \qquad (3.7)$$

To obtain the maximum likelihood estimators(MLE) of θ, γ and ρ , we first find the estimator of θ by solving the equation $\partial \ln L/\partial\theta = 0$. It is easy to obtain

$$\begin{aligned}\hat{\theta} = &\left[m^{-1}\sum_{k=0}^{n}\left\{\sum_{i=1}^{k}\gamma\Big((y_i - \rho y_{i-1})^{\gamma-1} - (y_i - \rho y_i)^{\gamma-1}\Big)(y_{k+1} - y_k)\right.\right. \\ &\left.\left. + (y_{k+1} - \rho y_k)^{\gamma} - (y_k - \rho y_k)^{\gamma}\right\}\right]^{1/\gamma} \end{aligned} \qquad (3.8)$$

By substituting $\hat{\theta}$ of (3.8) for θ in (3.7), we eliminate θ from the log likelihood function. Next, we take the partial derivatives of the log likelihood function of (3.7) with respect to γ and ρ to obtain

$$\partial \ln L(\gamma,\rho)/\partial\gamma = m/\gamma - mg_1/g_2 + \sum_{k=0}^{n}\sum_{j=1}^{n_k}(g_3/g_4)$$

and

$$\partial \ln L(\gamma,\rho)/\partial\rho = -mg_5/g_2 + (\gamma - 1)\sum_{k=0}^{n}\sum_{j=1}^{n_k}(g_6/g_4),$$

where

$$\begin{aligned} g_1 =& \sum_{k=0}^{n}\sum_{i=1}^{k}\Big[\left\{(y_i - \rho y_{i-1})^{\gamma-1} - (y_i - \rho y_i)^{\gamma-1}\right\}(y_{k+1} - y_k) \\ &+ \gamma\left\{(y_i - \rho y_{i-1})^{\gamma-1}\ln(y_i - \rho y_{i-1}) - (y_i - \rho y_i)^{\gamma-1}\ln(y_i - \rho y_i)\right\}(y_{k+1} - y_k)\Big] \\ &+ \sum_{k=0}^{n}\left\{(y_{k+1} - \rho y_k)^{\gamma}\ln(y_{k+1} - \rho y_k) - (y_k - \rho y_k)^{\gamma}\ln(y_k - \rho y_k)\right\} \end{aligned}$$

$$g_2 = \sum_{k=0}^{n}\sum_{i=1}^{k}\gamma\Big((y_i - \rho y_{i-1})^{\gamma-1} - (y_i - \rho y_i)^{\gamma-1}\Big)(y_{k+1} - y_k) + \sum_{k=0}^{n}\Big\{(y_{k+1} - \rho y_k)^{\gamma} - (y_k - \rho y_k)^{\gamma}\Big\},$$

$$g_3 = \sum_{i=1}^{k}\Big\{(y_i - \rho y_{i-1})^{\gamma-1}\ln(y_i - \rho y_{i-1}) - (y_i - \rho y_i)^{\gamma-1}\ln(y_i - \rho y_i)\Big\} + (t_{kj} - \rho y_k)^{\gamma-1}\ln(t_{kj} - \rho y_k),$$

$$g_4 = \sum_{i=1}^{k}\Big\{(y_i - \rho y_{i-1})^{\gamma-1} - (y_i - \rho y_i)^{\gamma-1}\Big\} + (t_{kj} - \rho y_k)^{\gamma-1},$$

$$g_5 = \gamma(\gamma-1)\sum_{k=0}^{n}\sum_{i=1}^{k}\Big\{(y_i)(y_i - \rho y_i)^{\gamma-2} - (y_{i-1})(y_i - \rho y_{i-1})^{\gamma-2}\Big\}(y_{k+1} - y_k) - \gamma\sum_{k=0}^{n}(y_k)\Big\{(y_{k+1} - \rho y_k)^{\gamma-1} - (y_k - \rho y_k)^{\gamma-1}\Big\},$$

and

$$g_6 = \sum_{i=1}^{k}\Big\{(y_i)(y_i - \rho y_i)^{\gamma-2} - (y_{i-1})(y_i - \rho y_{i-1})^{\gamma-2}\Big\} - (y_k)(t_{kj} - \rho y_k)^{\gamma-2}.$$

The maximum likelihood estimators, $\hat{\gamma}$ and $\hat{\rho}$, can then be obtained by solving the equations $\partial \ln L(\gamma,\rho)/\partial\gamma = 0$ and $\partial \ln L(\gamma,\rho)/\partial\rho = 0$ simultaneously.

4. NUMERICAL EXAMPLES

Applying the aperiodic PM model proposed in Section 2 to the actual data, we present numerical calculations for the estimation of parameters characterizing the model.

4.1 SIMULATION RESULTS

Simulations are carried out to investigate the accuracy of the estimation of parameters in the model. The failure times are generated for a specific parameter set and the parameter estimations are performed using the generated failure data. To generate the failure times, we utilize the thinning method suggested by Lewis and Shedler(1979). They suggest thinning a given NHPP with a majorizing intensity function to generate a NHPP with a general intensity function $\lambda(t)$ in a fixed interval. The thinning method assumes $\lambda(t) \le \pi(t)$, where $\pi(t)$ is a known majorizing intensity function on $(0,\infty)$. The generation process is as follows. Generate event times for a NHPP with intensity $\pi(t)$ within a fixed interval $(0, x_0)$. The time x_0 should be finite, no matter how large, so that the expected number of events in the interval

$(0, x_0)$, denoted as $E[N(0, x_0)]$, is finite. Denote them as $0 < T_1 < T_2 < \cdots$. Also, generate $U_1, U_2, \cdots$, which are independent uniform $U(0, 1)$ variates. We choose the T_i's satisfying $U_i \pi(T_i) \leq \lambda(T_i)$, which in turn form a subsequence $T_{i_1}, T_{i_2}, \cdots$. The resulting subsequence forms a NHPP with intensity function $\lambda(t)$.

As mentioned in Section 3, the MLEs $\hat{\theta}, \hat{\gamma}$ and $\hat{\rho}$ can then be obtained by solving the equations $\partial \ln L / \partial \gamma = 0$, $\partial \ln L / \partial \rho = 0$ and $\partial \ln L / \partial \theta = 0$ iteratively. To find the maximum likelihood estimators of the parameters for the model, the log-likelihood function given in (3.7) should be maximized numerically. For the Weibull hazard function, the input values of the parameter set is fixed at $\theta = 1, \gamma = 3, \rho = 0.8$, $y_0 = 0$, $y_1 = 0.8$, $y_2 = 1.6$, $y_3 = 2.4$ $y_4 = 3.1$, $y_5 = 3.8$, $y_6 = 4.4$, $y_7 = 5$, $y_8 = 5.5$, $y_9 = 5.75$ and $y_{10} = 6$. Failure times are generated for this parameter set and estimations are performed on the generated failure data. Simulation is carried out 100 times and the results of MLEs, its standard derivations and CV's are shown in Table 4.1. Here, CV denotes the coefficient variation and is calculated as the ratio of standard deviation to mean in percentage.

Table 4.1. Simulation results

Varible	Mean	Std Dev	CV
γ	2.899923	0.676695	23.33491
θ	0.974864	0.164105	16.83358
ρ	0.759594	0.167102	21.99891

4.2 REAL DATA ANALYSIS

This subsection presents real data analysis to explain our proposed method. The failure data set of Table 4.2 consists of 15 failures and 4 maintenance data cited by Shin, Lim and Lie(1996). They used this data to estimate the parameters of the failure process and the maintenance effect in Malik's proportional age reduction model. Applying this data, we also estimate the parameters in our aperiodic PM model. Table 4.2 summarizes the failure and maintenance data collected from the trouble reports and the operator's daily record from 1989 to 1994. The system is observed to be operated for 612 days and 15 failures and 3 major overhauls are identified. The MLEs of θ and γ for both the proposed model and BAO model are compared in Table 4.3. The estimated hazard functions, $\hat{h}(t)$, in BAO model and our proposed aperiodic PM model are shown in Figure 4.2.

Table 4.2. Failure maintenance data of a central cooler system

1.16	3.86	4.92	5.64
1.51	3.87	4.94	5.90
1.54*	3.95	5.01	6.09
2.13	4.07	5.12*	6.12†
2.63*	4.63	5.37	

Time scale: cumulative running days/100
* Major overhaul
† Censoring time

Table 4.3. Estimates of parameters for central cooler system in two models

Model	$\hat{\gamma}$	$\hat{\theta}$
BAO ($\rho = 0$)	2.12810	1.71437
Proposed ($\rho = 1$)	2.89336	1.76757

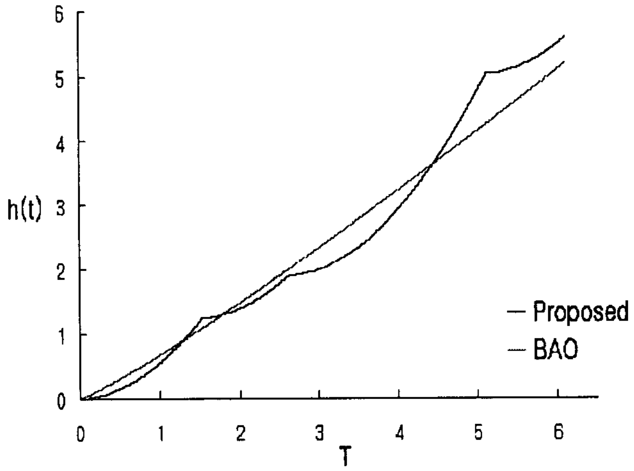

Figure 4.2. Estimated hazard functions in two models (central cooler system).

REFERENCES

Ascher, H. and Feingold, H.(1984), *Repairable System Reliability: Modeling, Inference, misconceptions and their causes*, Marcel Dekker Inc., New york.

Brown, M.(1972), *Statistical Analysis of Non-homogeneous Poisson Processes, In Stocastic Point Processes*, P. A. W. Lewis, Ed. New york: Wiley.

Canfield, R. V.(1986), Cost Optimization of Periodic Preventive Maintenance, *IEEE Transactions on Reliability*, **35**, 78-81.

Cox, D. R. and Lewis, P. A. W.(1966), *The Statistical Analysis of Series of Events*, London: Methuen.

Park, D. H., Jung, G. M., Yum, J. K.(2000), Cost Minimization for Periodic Maintenance Policy of a System Subject to Slow Degradation, *Reliability Enginnering and System Safety*, **68**, 105-112.

Lewis, P. A. W. and Shedler, G. S.(1979), Simulation of Nonhomogenuous Poisson Processes by Thinning, *Naval Research logistic Quarterly*, **68**, 403-413.

Malik, M. A. K.(1979), Reliable Preventive Maintenance Scheduling, *AIIE Trans*, **11**, 221-228.

Murthy, D. N. P. and Nguyen, D. G.(1981), Optimum Age-Policy with Imperfect Preventive Maintenance, *IEEE Transactions on Reliability*, **30**, 80-81.

Nakagawa, T.(1979), Imperfect Preventive Maintenance, *IEEE Transactions on Reliability*, **28**, 402-407.

Nakagawa, T.(1986), Periodic and Sequential Preventive Maintenance Policies, *Journal of Applied Probability*, **23**, 536-542.

Shin, Lim and Lie(1996), Estimating Parameters of Intensity Function and Maintenance Effect for Repairable Unit, *Reliability Engineering and System Safety*. **54**. 1-10.