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커멘슈레이트 시간지연을 갖는 선형시스템의 출력궤환 H_∞ 제어기 설계

(An Output Feedback H_∞ Controller Design for Linear Systems with Commensurate Time Delay)

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요 약

본 논문은 상태변수와 입력변수에 커멘슈레이트 시간지연을 갖는 선형시스템의 출력궤환 H_∞ 제어문제를 취급한다. 본 연구에서 제시한 출력궤환 제어기도 역시 제어기의 상태변수에 커멘슈레이트 시간지연항을 갖는다. 제어기는 컨벡스 최적화 기법을 사용하여 쉽게 풀 수 있는 선형행렬 부등식의 해를 이용하여 합성할 수 있다. 제안하는 방법의 효용성을 예시하기 위하여 수치예를 제시한다.

Abstract

This paper deals with an H_∞ output feedback control problem for linear systems with commensurate time delay in both state and input variables. The proposed output feedback controller also has commensurate time delay terms in the controller state. The controller can be synthesized based on the solution of the linear matrix inequalities(LMI) which can be easily solved using the convex optimization method. In order to demonstrate the efficacy of the proposed method, numerical examples are presented.

I. Introduction

The stability analysis and control of linear systems with delayed states are problems of practical and theoretical interest since time delays are frequently encountered in physical processes and very often is the cause for instability and poor performance of control systems. In the last decade, the H_∞ controller design method for the linear time delay systems has been developed remarkably[1-7]. In [1-7], a memoryless state feedback controller was used in order to stabilize linear state delayed systems. L. Xie et al[9] has designed a robust memoryless state fedback controller based on the Lyapunov-Razumikhin function approach. Their methods are dependent on the size of the delay and are given in terms of LMIs. Jeung et al[8] considered an H_∞ output feedback control problem for linear systems with time varying delayed states. They synthesized the output feedback controller by solving coupled LMIs. Unfortunately, there is no general method to solve coupled LMIs. By fixing coupled terms a priori, coupled LMIs can be solved by using the

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convex optimization method[10]. Attention has been also paid on the robust stabilization of linear systems with multiple time delay[15]. In [15], a robust full state feedback controller is designed using the convex optimization method. A system with commensurate time delay is a class of a system with multiple time delay. When all the time delays $d_1, d_2, \cdots, d_n$ can be written as $d_i = k_i d$, $i=1,\cdots,n$, where k_i is an integer and $d>0$ is a real number, we say that this multiple time delayed system is a system with commensurate time delay. Hence any multiple time delayed system can be approximated to a system with commensurate time delay within arbitrary accuracy. However, there are only a few publications on the study of a system with commensurate time delay[16]. In this paper, an H_∞ output feedback controller is proposed for linear systems with commensurate time delay. The proposed controller stabilizes the linear time delay systems and attenuates the disturbance attenuation level below the prescribed level. The proposed controller is also a linear system with commensurate time delay and is synthesized by solving an LMI problem. Since the controller has delayed controller states, it is expensive to implement the proposed controller in the real control system. But we will show that better performance can be expected when the proposed irrational controller is used.

We use fairly standard notation. The symbol $R(C)$ denotes the field of real(complex) numbers. $R^n(C^n)$ denotes the n dimensional real(complex) vector space and $R^{n\times m}(C^{n\times m})$ denotes the set of all $n\times m$ real(complex) matrices. We will use A^T and A^* to denote the transpose and the conjugate transpose of matrix A respectively. $A^\perp$ denotes an orthogonal complements of A. In a block symmetric matrix, $*$ in the (i,j) block denotes the transpose of the submatrix in the (j,i) block. I and 0 denote the identity matrix and the zero matrix respectively. $\bar{\sigma}(A)$ denotes the maximum singular value of matrix A. When $A\in C^{n\times m}$, A_R and A_I denote the real part and the imaginary part of the matrix A respectively.

II. Problem Statement

We consider the following linear system with commensurate time delay.

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \sum_{i=1}^{N} A_i\, x(t-id) + B_1 w(t) \\ &\quad + B_{20}u(t) + \sum_{i=1}^{N} B_{2i}u(t-id) \\ z(t) &= C_1x(t) + D_{11}w(t) + D_{12}u(t) \\ y(t) &= C_2x(t) + D_{21}w(t) \end{aligned} \tag{1}$$

where $x(t)\in R^n$ is the state, $w(t)\in R^{n_w}$ is the disturbance input, $u(t)\in R^{n_u}$ is the control input, $z(t)\in R^{n_z}$ is the error signal, $y(t)\in R^{n_y}$ is the measured variable and $id>0$, $(i=1,\cdots,N)$ are delays in the system. In addition, $A, A_i, \cdots, D_{21}$ are constant matrices with appropriate dimensions.

We want to design a strictly proper output feedback controller in the form of

$$\begin{aligned} \dot{x}_k(t) &= A_kx_k(t) + \sum_{i=1}^{N} A_{ki}x_k(t-id) + B_ky(t) \\ u(t) &= C_kx_k(t) \end{aligned} \tag{2}$$

where $x_k\in R^n$ is the controller state whose dimension is same as that of the plant state. Note that the output feedback controller (2) is also a linear system with commensurate time delay.

When the controller (2) is applied to the delayed system (1), the resulting closed loop system can be written as follows.

$$\begin{aligned} \dot{x}_c(t) &= A_cx_c(t) + \sum_{i=1}^{N} A_{ci}\, x_c(t-id) + B_cw(t) \\ z &= C_cx_c(t) + D_cw(t) \end{aligned} \tag{3}$$

where

$$x_c = \begin{bmatrix} x \\ x_k \end{bmatrix},\quad A_c = \begin{bmatrix} A & B_{20}C_k \\ B_kC_2 & A_k \end{bmatrix},$$

$$A_{ci} = \begin{bmatrix} A_i & B_{2i}C_k \\ 0 & A_{ki} \end{bmatrix},\quad B_c = \begin{bmatrix} B_1 \\ B_kD_{21} \end{bmatrix},$$

$C_c = [\,C_1 \;\; D_{12}C_k\,]$, $D_c = D_{11}$

Note that our method can not be applicable to a controller design for systems with time varying delay.

Our goal is to design the output feedback controller (2) such that the closed loop system (3) is internally asymptotically stable and $\| T_{zw} \|_\infty < \gamma$ where T_{zw} is the transfer function from the disturbance input $w(t)$ to the error signal $z(t)$ and γ is the prescribed disturbance attenuation level.

Ⅲ. H_∞ Norm Bound

In this section, we will develope a sufficient condition guaranteeing $\| T_{zw} \|_\infty < \gamma$ in the closed loop system (3). First, we define a linear system with complex uncertain parameters associated with the linear time delay system (3).

$$\begin{aligned} \dot{x}_c(t) &= A_{cz}x_c(t) + B_c w(t) \\ z(t) &= C_c x_c(t) + D_c w(t) \end{aligned} \tag{4}$$

where $A_{cz} = A_c + \sum_{i=1}^{N} A_{ci}\, z^{-i}$ and z is a complex uncertain parameter with $|z| = 1$. One can easily observe that the linear parameter dependent system (4) can be uniquely obtained from the linear time delay system (3) and vice versa. We begin with the discrete system version of strictly positive real condition[11] which will be used later.

Lemma 1 : The two statements are equivalent.

① An $m \times m$ stable transfer function $H(z) = D_G + C_G(zI - A_G)^{-1}B_G$ is strictly positive real. Thus $H(z) + H^*(z) > 0$ for all $|z| = 1$.

② There exists an $R = R^T > 0$ such that LMI (5) holds.

$$\begin{bmatrix} R - A_G^T R A_G & C_G^T - A_G^T R B_G \\ C_G - B_G^T R A_G & D_G + D_G^T - B_G^T R B_G \end{bmatrix} > 0 \tag{5}$$

(proof) It was already shown that $H(z)$ is strictly positive real if and only if there exist $R_1 = R_1^T > 0$, $M = M^T > 0$, K and L such that (6)-(8) hold[12].

$$R_1 - A_G^T R_1 A_G = Q^T Q + M \tag{6}$$

$$C_G - B_G^T R_1 A_G = W^T Q \tag{7}$$

$$D_G + D_G^T - B_G^T R_1 B_G = W^T W \tag{8}$$

(①=>②) Let $H_\varepsilon(z) = H(z) - \frac{\varepsilon}{2} I$. Obviously there exists a sufficiently small $\varepsilon > 0$ such that $H_\varepsilon(z) + H_\varepsilon^*(z) > 0$ for all $|z| = 1$. Accordingly there exist $R > 0$, $M_\varepsilon > 0$, Q_ε and W_ε satisfying

$$\begin{aligned} R - A_G^T R A_G &= Q_\varepsilon^T Q_\varepsilon + M_\varepsilon, \\ C_G - B_G^T R A_G &= W_\varepsilon^T Q_\varepsilon, \\ D_G + D_G^T - \varepsilon I - B_G^T R B_G &= W_\varepsilon^T W_\varepsilon \end{aligned} \tag{9}$$

From (9), we have

$$\begin{aligned} &\begin{bmatrix} R - A_G^T R A_G & C_G^T - A_G^T R B_G \\ C_G - B_G^T R A_G & D_G + D_G^T - B_G^T R B_G \end{bmatrix} \\ &= \begin{bmatrix} M_\varepsilon & 0 \\ 0 & \varepsilon I \end{bmatrix} + \begin{bmatrix} Q_\varepsilon^T \\ W_\varepsilon^T \end{bmatrix} [\,Q_\varepsilon \;\; W_\varepsilon\,] > 0 \end{aligned} \tag{10}$$

(②=>①) Since LMI (5) holds, there exists an invertible $U = [\,U_1 \;\; U_2\,]$ such that

$$\begin{aligned} &\begin{bmatrix} R - A_G^T R A_G & C_G^T - A_G^T R B_G \\ C_G - B_G^T R A_G & D_G + D_G^T - B_G^T R B_G \end{bmatrix} \\ &= U^T U = \begin{bmatrix} U_1^T U_1 & U_1^T U_2 \\ U_2^T U_1 & U_2^T U_2 \end{bmatrix} \end{aligned} \tag{11}$$

After a little straightforward manipulation using (11) and the identity (12)

$$\begin{aligned} R - A_G^T R A_G = &\; (z^{-1}I - A_G^T) R A_G + A_G^T R (zI - A_G) \\ &+ (z^{-1}I - A_G^T) R (zI - A_G) \end{aligned} \tag{12}$$

we have

$$\begin{aligned} H(z) + H^*(z) &= D_G + D_G^T + B_G^T (z^{-1}I - A_G^T)^{-1} C_G^T \\ &\quad + C_G (zI - A_G)^{-1} B_G \\ &= U_2^T U_2 + B_G^T R B_G + B_G^T (z^{-1}I - A_G^T)^{-1} \\ &\quad \cdot (U_1^T U_2 + A_G^T R B_G) \\ &\quad + (U_2^T U_1 + B_G^T R A_G)(zI - A_G)^{-1} B_G \\ &= T^*(z)\, T(z) \end{aligned} \tag{13}$$

where $T(z) = U_2 + U_1(zI - A_G)^{-1}B_G$.

Let $x \in C^m$. Since $[\,U_1 \;\; U_2\,]$ is invertible and $T(z)x$ can be written as

$$T(z)x = [U_1 \;\; U_2]\begin{bmatrix} (zI - A_G)^{-1}B_G x \\ x \end{bmatrix} \qquad (14)$$

$T(z)x = 0$ implies $x = 0$ for all $|z| = 1$. Hence, $x^*(H(z) + H^*(z))x = x^* T^*(z)T(z)x > 0$ for all nonzero $x \in C^m$ and $|z| = 1$. This completes the proof.

We are ready to state a sufficient condition guaranteeing $\| T_{zw} \|_\infty < \gamma$ in the closed loop system (3).

Lemma 2 : The following two statements are equivalent.

① There exists a $P = P^* > 0$ such that LMI (15) holds for all the complex number z satisfying $|z| = 1$.

$$L_z = \begin{bmatrix} A_{cz}^* P + PA_{cz} & PB_c & C_c^* \\ B_c^* P & -\gamma I & D_c^* \\ C_c & D_c & -\gamma I \end{bmatrix} < 0 \qquad (15)$$

② There exists $P = P^* > 0$ and $R = R^* > 0$ such that LMI (16) holds.

$$L = \begin{bmatrix} A_G^* R A_G - R & A_G^* R B_G + C_G^* \\ B_G^* R A_G + C_G & B_G^* R B_G + D_G + D_G^* \end{bmatrix} < 0 \qquad (16)$$

where

$$A_G = \begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ & & \ddots & & \\ 0 & 0 & 0 & \cdots & I \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad B_G = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ W \end{bmatrix}, \quad W = [I \;\; 0 \;\; 0],$$

$$C_G = [C_{GN} \; C_{GN-1} \; \cdots \; C_{G1}], \quad C_{Gi} = \begin{bmatrix} PA_{ci} \\ 0 \\ 0 \end{bmatrix},$$

$$D_G = \frac{1}{2}\begin{bmatrix} A_c^* P + PA_c & PB_c & C_c^* \\ B_c^* P & -\gamma I & D_c^* \\ C_c & D_c & -\gamma I \end{bmatrix}$$

Moreover, if ① or equivalently ② holds, the closed loop system (3) is internally asymptotically stable and $\| T_{zw} \|_\infty < \gamma$.

(proof) The proof of asymptotic stability part will be omitted. Using the definitions of A_{cz}, LMI (15) can be written as

$$L_z = D_G + D_G^* + \sum_{i=1}^{N} C_{Gi} W z^{-i} + \sum_{i=1}^{N} W^* C_{Gi}^* z^i \qquad (17)$$

Note that (A_G, B_G, C_G, D_G) is a state space realization of an FIR filter transfer function $D_G + \sum_{i=1}^{N} C_{Gi} W z^{-i}$. Hence (17) becomes

$$\begin{aligned} L_z = \; & D_G + C_G(zI - A_G)^{-1}B_G \\ & + D_G^* + B_G^*(z^{-1}I - A_G^*)^{-1}C_G^* \\ & < 0 \end{aligned} \qquad (18)$$

Since LMI (18) holds for all the complex number z with $|z| = 1$, the discrete system $(A_G, B_G, -C_G, -D_G)$ is strictly positive real. From the strictly positive real condition, we conclude that ① is equivalent to ②.

Define $G(s, z) = D_c + C_c(sI - A_{cz})^{-1}B_c$. LMI (16) implies $sup_{|z|=1}\, sup_w\, \bar{\sigma}(G(jw, z)) < \gamma$. Hence we obtain

$$\| T_{zw} \|_\infty \le \; sup_{|z|=1}\, sup_w\, \bar{\sigma}(G(jw, z)) < \gamma.$$

This completes the proof.

From the proof of Lemma 2, one can observe that any P satisfying LMI (15) can be a solution of LMI (16). Note that P and R in LMI (16) is not necessarily restricted to being Hermitian. In fact, Lemma 3 states that they can be replaced by symmetric positive definite matrices.

Lemma 3 : Suppose that there exist the Hermitian positive definite matrices P and R such that LMI (16) holds. Then the real part of the matrices P and R also satisfy LMI (16).

(proof) LMI (16) can be written as $L = L_R + jL_I < 0$ where

$$L_R = \begin{bmatrix} A_G^T R_R A_G - R_R & A_G^T R_R B_G + C_{GR}^T \\ B_G^T R_R A_G + C_{GR} & B_G^T R_R B_G + D_{GR} + D_{GR}^T \end{bmatrix},$$

$$L_I = \begin{bmatrix} A_G^T R_I A_G - R_I & A_G^T R_I B_G - C_{GI}^T \\ B_G^T R_I A_G + C_{GI} & B_G^T R_I B_G + D_{GI} - D_{GI}^T \end{bmatrix}$$

Let $L \in C^{k \times k}$. Since $L < 0$ and $L_I^T = -L_I$, for any nonzero $x = x_R + jx_I$ ($x \in C^k$, $x_R, x_I \in R^k$),

$$\begin{aligned} x^* L x = \; & (x_R^T - jx_I^T)(L_R + jL_I)(x_R + jx_I) \\ = \; & [x_R^T \;\; x_I^T]\begin{bmatrix} L_R & L_I \\ -L_I & L_R \end{bmatrix}\begin{bmatrix} x_R \\ x_I \end{bmatrix} < 0 \end{aligned} \qquad (19)$$

From (19) we conclude that $L_R+jL_I<0$ implies $\begin{bmatrix} L_R & L_I \\ -L_I & L_R \end{bmatrix}<0$. Accordingly $L_R+jL_I<0$ implies $L_R<0$. When P_R and R_R are used in LMI (16), we have $L=L_R$. This completes the proof.

Remark 1 : As a special case of Lemma 2, we consider $N=1$ in the system (3). In this case, $A_G=0$, $B_G=W$, $C_G=C_{G1}$. Accordingly LMI (16) is equivalent to

$$\begin{aligned} & B_G^T R B_G + D_G + D_G^T + C_G R^{-1} C_G^T < 0 \\ & \Leftrightarrow \\ & \begin{bmatrix} A_c^T P + PA_c + R & PA_{c1} & PB_c & C_c^T \\ A_{c1}^T P & -R & 0 & 0 \\ B_c^T P & 0 & -\gamma I & D_c^T \\ C_c & 0 & D_c & -\gamma I \end{bmatrix} < 0 \end{aligned} \tag{20}$$

LMI (20) is a well known H_∞ norm bounding condition for linear systems with delayed states.

Ⅳ. Output Feedback Controller

In this section, we derive an output feedback controller (2) based on LMI (15).

Partition P and P^{-1} as

$$P=\begin{bmatrix} Y & N \\ N^T & \bigstar \end{bmatrix},\ P^{-1}=\begin{bmatrix} X & M \\ M^T & \bigstar \end{bmatrix} \tag{21}$$

where M, N are nonsingular and ★ means irrelevant. Then one can observe that $X\in R^{n\times n}$, $Y\in R^{n\times n}$ are symmetric positive matrices and $MN^T=I-XY$.

Define Π_1 and Π_2 as follows.

$$\Pi_1=\begin{bmatrix} X & I \\ M^T & 0 \end{bmatrix},\ \Pi_2=\begin{bmatrix} I & Y \\ 0 & N^T \end{bmatrix} \tag{22}$$

LMI (15) can be written as

$$L_z=\sum_{i=-N}^{N} L_i z^{-i}<0 \quad \text{for all} \quad |z|=1. \tag{23}$$

where

$$L_0=\begin{bmatrix} A_c^T P + PA_c & PB_c & C_c^T \\ B_c^T P & -\gamma I & D_c^T \\ C_c & D_c & -\gamma I \end{bmatrix},$$

$$L_i=L_{-i}^T=\begin{bmatrix} PA_{ci} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},\ (i=1,\cdots,N)$$

If we perform a congruence transformation with $diag(\Pi_1, I, I)$ on both inequalities (23), we obtain

$$\sum_{i=-N}^{N} T_i z^{-i}<0 \quad \text{for all} \quad |z|=1. \tag{24}$$

where

$$\begin{aligned} T_0 &= diag(\Pi_1^T, I, I)\ L_0\ diag(\Pi_1, I, I) \\ &= \begin{bmatrix} T_{011} & * & * & * \\ \hat{A}+A^T & T_{022} & * & * \\ B_1^T & B_1^T Y + D_{21}^T \hat{B}^T & -\gamma I & * \\ C_1 X + D_{12}\hat{C} & C_1 & D_{11} & -\gamma I \end{bmatrix} \end{aligned} \tag{25}$$

$$T_{011}= AX+XA^T+B_{20}\hat{C}+\hat{C}^T B_{20}^T$$
$$T_{022}= A^T Y+YA+\hat{B}C_2+C_2^T\hat{B}^T$$

$$\begin{aligned} T_i &= diag(\Pi_1^T, I, I)\ L_i\ diag(\Pi_1, I, I)=T_{-i}^T \\ &= \begin{bmatrix} A_i X + B_{2i}\hat{C} & A_i & 0 & 0 \\ \hat{A}_i & YA_i & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} = C_{Gi}W \end{aligned} \tag{26}$$

$$C_{Gi}=\begin{bmatrix} A_i X + B_{2i}\hat{C} & A_i \\ A_i & YA_i \\ 0 & 0 \\ 0 & 0 \end{bmatrix},\quad W=\begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \end{bmatrix}$$

$$\hat{A}=YAX+YB_{20}C_k M^T+NB_k C_2 X+NA_k M^T \tag{27}$$

$$\hat{B}=NB_k \tag{28}$$

$$\hat{C}=C_k M^T \tag{29}$$

$$\hat{A}_i=YA_i X+YB_{2i}\hat{C}+NA_{ki}M^T \tag{30}$$

Define

$$A_G=\begin{bmatrix} 0 & I & 0 & \cdots & 0 \\ 0 & 0 & I & \cdots & 0 \\ & & & \ddots & \\ 0 & 0 & 0 & \cdots & I \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix},\ B_G=\begin{bmatrix} 0 \\ \vdots \\ 0 \\ W \end{bmatrix},$$

$$C_G=[\,C_{GN}\ C_{GN-1}\ \cdots\ C_{G1}\,],\ D_G=T_0/2.$$

Then LMI (24) holds if and only if the discrete system realization $(A_G, B_G, -C_G, -D_G)$ is strictly positive real. From the strictly positive realness condition of $(A_G, B_G, -C_G, -D_G)$, we can derive a solvability condition of the H_∞ output feedback control problem. We state our main result in Theorem 4.

Thorem 4 : Suppose that there exist $X=X^T>0$, $Y=Y^T>0$, $R=R^T>0$, $\widehat{A}$, $\widehat{B}$, $\widehat{C}$ and $\widehat{A}_i$ $(i=1,\cdots,N)$ such that following LMIs hold.

$$① \begin{bmatrix} A_G^T R A_G - R & A_G^T R B_G + C_G^T \\ B_G^T R A_G + C_G & B_G^T R B_G + D_G + D_G^T \end{bmatrix} < 0 \qquad (31)$$

$$② \begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0 \qquad (32)$$

Then there exists an output feedback controller such that the closed loop system is internally asymptotically stable and $\| T_{zw} \|_\infty < \gamma$.

(proof) There exists $P=P^T>0$ satisfying (21) if and only if LMI (32) holds[13],[14]. Since LMI (31) holds, LMI (24) holds for all the complex number $|z|=1$. If A_k, B_k, C_k and A_{ki} $(i=1,\cdots,N)$ are chosen such that (27)-(30) are satisfied, the closed loop system achieves $\| T_{zw} \|_\infty < \gamma$. This completes the proof.

Consider the special case of $N=1$ and $B_{21}=0$ in the linear delay system (1). In [8], it was proved that solvability of the matrix inequalities (32)-(35) guarantees the existence of a rational output feedback controller satisfying $\| T_{zw} \|_\infty < \gamma$.

$$\begin{bmatrix} W_1 & 0 & 0 \\ W_2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T \begin{bmatrix} XA^T + AX + A_1 S A_1^T & * & * & * \\ C_1 X & -\gamma I & * & * \\ B_1^T & D_{11}^T & -\gamma I & * \\ X & 0 & 0 & -S \end{bmatrix} \cdot \begin{bmatrix} W_1 & 0 & 0 \\ W_2 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} < 0 \qquad (33)$$

$$\begin{bmatrix} W_3 & 0 & 0 \\ W_4 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}^T \begin{bmatrix} A^T Y + YA + Q & * & * & * \\ B_1^T Y & -\gamma I & * & * \\ C_1 & D_{11} & -\gamma I & * \\ A_1^T Y & 0 & 0 & -Q \end{bmatrix} \cdot \begin{bmatrix} W_3 & 0 & 0 \\ W_4 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} < 0 \qquad (34)$$

$$SQ = I \qquad (35)$$

where $[\, W_1^T \; W_2^T]^T$ and $[\, W_3^T \; W_4^T]^T$ are orthogonal complements of $[\, B_{20}^T \; D_{12}^T]^T$ and $[\, C_2 \; D_{21}]^T$ respectively. The matrix inequalities (32)-(35) are not LMIs since (35) is not linear. There is no efficient method to obtain X, Y, S and Q satisfying (32)-(35). When either S or Q is fixed to a symmetric positive matrix, then (32)-(35) become LMIs whose solution can be effectively computed by using the existing LMI solver. If S or Q is determined a priori in order to solve (32)-(35) efficiently, solvability of (32)-(35) becomes more conservative. But, if the irrational controller as expressed in (2) is to be designed, the controller can be synthesized by using solutions of LMIs (31) and (32). We will show that solvability of (31) and (32) is less conservative than solvability of (32)-(35). Consider LMIs (32)-(34) with $SQ \geq I$ which are apparently less conservative than the matrix inequalities (32)-(35).

Lemma 5 : Suppose that LMIs (32)-(34) with $SQ \geq I$ are solvable. Then LMIs (31)-(32) are also solvable.

(proof) When $N=1$ and $B_{21}=0$, LMI (31) becomes

$$L_1 = \begin{bmatrix} -R & C_G^T \\ C_G & B_G^T R B_G + D_G + D_G^T \end{bmatrix} = M_1 + U_1 [\, \widehat{A} \; \widehat{A}_1]\, V_1 + (U_1 [\, \widehat{A} \; \widehat{A}_1]\, V_1)^T \qquad (36)$$

where

$$M_1 = \begin{bmatrix} -R_1 & * & * & * & * & * \\ -R_2^T & -R_3 & * & * & * & * \\ A_1 X & A_1 & T_{011} + R_1 & * & * & * \\ 0 & YA_1 & A^T + R_2^T & T_{022} + R_3 & * & * \\ 0 & 0 & B_1^T & B_1^T Y + D_{21}^T \widehat{B}^T & -\gamma I & * \\ 0 & 0 & C_1 X + D_{12} \widehat{C} & C_1 & D_{11} & -\gamma I \end{bmatrix}$$

$$R = \begin{bmatrix} R_1 & R_2 \\ R_2^T & R_3 \end{bmatrix}, \quad U_1^T = [\,0 \;\; 0 \;\; 0 \;\; I \;\; 0 \;\; 0\,],$$

$$V_1 = \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 \\ I & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Possible $U_1^{\perp}$ and $V_1^{T\perp}$ are given by

$$U_1^{\perp} = \begin{bmatrix} I & 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix}, \quad V_1^{T\perp} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & I \end{bmatrix}.$$

It is well known that solvability of $L_1 < 0$ is equivalent to solvability of $U_1^{\perp T} M_1 U_1^{\perp} < 0$ and $V_1^{T\perp T} M_1 V_1^{T\perp} < 0$. $V_1^{T\perp T} M_1 V_1^{T\perp}$ can be expressed as

$$V_1^{T\perp T} M_1 V_1^{T\perp} = M_2 + U_2 \hat{B} V_2 + (U_2 \hat{B} V_2)^T \tag{37}$$

where

$$M_2 = \begin{bmatrix} -R_3 & * & * & * \\ YA_1 & A^T Y + YA + R_3 & * & * \\ 0 & B_1^T Y & -\gamma I & * \\ 0 & C_1 & D_{11} & -\gamma I \end{bmatrix},$$
$$U_2 = \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix}, \quad V_2^T = \begin{bmatrix} 0 \\ C_2^T \\ D_{21}^T \\ 0 \end{bmatrix}$$

Accordingly, we have

$$U_2^{\perp T} M_2 U_2^{\perp} = \begin{bmatrix} -R_3 & * & * \\ 0 & -\gamma I & * \\ 0 & D_{11} & -\gamma I \end{bmatrix} \tag{38}$$

$$V_2^{T\perp T} M_2 V_2^{T\perp} = \begin{bmatrix} I & 0 & 0 \\ 0 & W_3 & 0 \\ 0 & W_4 & 0 \\ 0 & 0 & I \end{bmatrix}^T \begin{bmatrix} -R_3 & * & * & * \\ YA_1 & A^T Y + YA + R_3 & * & * \\ 0 & B_1^T Y & -\gamma I & * \\ 0 & C_1 & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & W_3 & 0 \\ 0 & W_4 & 0 \\ 0 & 0 & I \end{bmatrix} \tag{39}$$

$U_1^{\perp T} M_1 U_1^{\perp}$ can be also written as

$$U_1^{\perp T} M_1 U_1^{\perp} = M_3 + U_3 \hat{C} V_3 + (U_3 \hat{C} V_3)^T \tag{40}$$

where

$$M_3 = \begin{bmatrix} -R_1 & * & * & * & * \\ -R_2^T & -R_3 & * & * & * \\ A_1 X & A_1 & AX + XA^T + R_1 & * & * \\ 0 & 0 & B_1^T & -\gamma I & * \\ 0 & 0 & C_1 X & D_{11} & -\gamma I \end{bmatrix},$$
$$U_3 = \begin{bmatrix} 0 \\ 0 \\ B_{20} \\ 0 \\ D_{12} \end{bmatrix}, \quad V_3^T = \begin{bmatrix} 0 \\ 0 \\ I \\ 0 \\ 0 \end{bmatrix}$$

Solvability of LMI (40) is equivalent to solvability *of (41) and (42).*

$$V_3^{T\perp T} M_3 V_3^{T\perp} = M_4 + U_4 R_2 V_4 + (U_4 R_2 V_4)^T \tag{41}$$

$$U_3^{\perp T} M_3 U_3^{\perp} = M_5 + U_5 R_2 V_5 + (U_5 R_2 V_5)^T \tag{42}$$

where

$$M_4 = \begin{bmatrix} -R_1 & * & * & * \\ 0 & -R_3 & * & * \\ 0 & 0 & -\gamma I & * \\ 0 & 0 & D_{11} & -\gamma I \end{bmatrix},$$
$$U_4 = -\begin{bmatrix} I \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad V_4^T = \begin{bmatrix} 0 \\ I \\ 0 \\ 0 \end{bmatrix}$$

$U_5^T = -[I\ 0\ 0\ 0\ 0]$, $V_5 = [0\ I\ 0\ 0\ 0]$

$$M_5 = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & W_1 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & W_2 & 0 \end{bmatrix}^T \begin{bmatrix} -R_1 & * & * & * & * \\ 0 & -R_3 & * & * & * \\ A_1 X & A_1 & AX + XA^T + R_1 & * & * \\ 0 & 0 & B_1^T & -\gamma I & * \\ 0 & 0 & C_1 X & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & W_1 & 0 \\ 0 & 0 & 0 & I \\ 0 & 0 & W_2 & 0 \end{bmatrix}$$

Hence solvability of (40) is equivalent to solvability of LMIs (43)-(46).

$$U_4^{\perp T} M_4 U_4^{\perp} = \begin{bmatrix} -R_3 & * & * \\ 0 & -\gamma I & * \\ 0 & D_{11} & -\gamma I \end{bmatrix} \tag{43}$$

$$V_4^{T\perp T} M_4 V_4^{T\perp} = \begin{bmatrix} -R_1 & * & * \\ 0 & -\gamma I & * \\ 0 & D_{11} & -\gamma I \end{bmatrix} \tag{44}$$

$$U_5^{\perp T} M_5 U_5^{\perp} = \begin{bmatrix} I & 0 & 0 \\ 0 & W_1 & 0 \\ 0 & 0 & I \\ 0 & W_2 & 0 \end{bmatrix}^T \begin{bmatrix} -R_3 & * & * & * \\ A_1 & AX + XA^T + R_1 & * & * \\ 0 & B_1^T & -\gamma I & * \\ 0 & C_1 X & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & W_1 & 0 \\ 0 & 0 & I \\ 0 & W_2 & 0 \end{bmatrix} \tag{45}$$

$$V_5^{T\perp T} M_5 V_5^{T\perp} = \begin{bmatrix} I & 0 & 0 \\ 0 & W_1 & 0 \\ 0 & 0 & I \\ 0 & W_2 & 0 \end{bmatrix}^T \begin{bmatrix} -R_1 & * & * & * \\ A_1 X & AX + XA^T + R_1 & * & * \\ 0 & B_1^T & -\gamma I & * \\ 0 & C_1 X & D_{11} & -\gamma I \end{bmatrix} \begin{bmatrix} I & 0 & 0 \\ 0 & W_1 & 0 \\ 0 & 0 & I \\ 0 & W_2 & 0 \end{bmatrix} \tag{46}$$

Accordingly we can conclude that solvability of LMI (31) is equivalent to solvability of LMIs (38), (39), (43)-(46). Let $X>0$, $Y>0$, $S>0$, $Q>0$ be solutions of LMIs (32)-(34) with $SQ \geq I$. Then it is easy to check that X, Y, $R_3 = Q$ and $R_1 = XS^{-1}X$ also satisfy LMIs (38), (39), (43)-(46). This completes the proof.

Example1 : We consider the linear state delayed system (1) with

$$A=\begin{bmatrix}0 & 1\\ -3 & -1\end{bmatrix},\ B_1=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ B_{20}=\begin{bmatrix}0\\ 1\end{bmatrix},$$

$$A_1=\begin{bmatrix}0 & -0.1\\ 0 & 0.1\end{bmatrix},\ A_2=\begin{bmatrix}0 & 0.1\\ 0 & -0.1\end{bmatrix},\ B_{21}=B_{22}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$C_1=\begin{bmatrix}1 & 0\\ 0 & 0\end{bmatrix},\ D_{11}=\begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix},\ D_{12}=\begin{bmatrix}0\\ 1\end{bmatrix},$$

$$C_2=[1\ 0],\ D_{21}=[0\ 1].$$

When the prescribed disturbance attenuation level $\gamma=1.5$,

$$X=\begin{bmatrix}1.0555 & -0.9428\\ -0.9428 & 3.3464\end{bmatrix},\ Y=\begin{bmatrix}1.5067 & 0.3222\\ 0.3222 & 0.6242\end{bmatrix},$$

$$\hat{A}=\begin{bmatrix}-1.7462 & 3.6884\\ -1.1752 & 0.8779\end{bmatrix},\ \hat{B}=\begin{bmatrix}-0.9031\\ 0.1818\end{bmatrix},$$

$$\hat{C}^T=\begin{bmatrix}-0.0878\\ -0.783\end{bmatrix},\ \hat{A}_1=\begin{bmatrix}0.0226 & -0.0443\\ -0.0196 & 0.0584\end{bmatrix},$$

$$\hat{A}_2=\begin{bmatrix}-0.0150 & 0.0552\\ 0.0027 & -0.0104\end{bmatrix}.$$

are possible solutions of LMIs (31) and (32). From (27)-(30), the state space realization of the controller can be obtained as follows :

$$A_k=\begin{bmatrix}-4.5278 & 0.2998\\ -3.5198 & -2.5332\end{bmatrix},\ B_k=\begin{bmatrix}-0.9031\\ 0.1818\end{bmatrix},$$

$$C_k^T=\begin{bmatrix}1.8828\\ 1.8188\end{bmatrix},\ A_{k1}=\begin{bmatrix}-0.1254 & -0.5033\\ 0.0258 & 0.0655\end{bmatrix},$$

$$A_{k2}=\begin{bmatrix}0.0635 & 0.4624\\ -0.0162 & -0.1225\end{bmatrix}.$$

When d =1[sec], Figure 1 shows the singular value of the closed loop system at some frequency band. Since $\| T_{zw} \|_\infty < 1.5$ the controller robustly stabilizes

$$\begin{aligned}\dot{x}(t)=&\ (A+B_1\Delta C1)x(t)+A_1x(t-d)\\ &+A_2x(t-2d)+(B_2+B_1\Delta D_{12})u(t)\\ y(t)=&\ (C_2+D_{21}\Delta C_1)x(t)+D_{21}\Delta D_{12}u(t)\end{aligned} \tag{47}$$

for any Δ with $\|\Delta\| \le 1/1.5$. When the parametric uncertainties $\Delta=\begin{bmatrix}0.6 & 0\\ 0 & -0.6\end{bmatrix}$ and the initial condition $x(t)=\begin{bmatrix}1\\ -1\end{bmatrix}$, $t\in[-2,\ 0]$ are considered, the time domain state response is depicted in Figure 2.

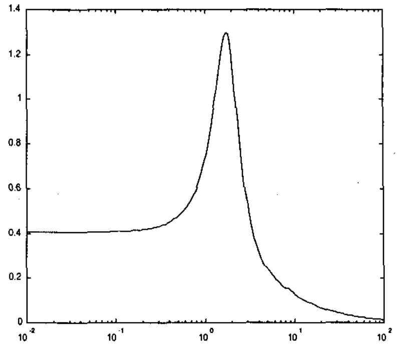

Fig. 1. Singular value of T_{zw} .

Example 2 : Suppose that $A_2=0$ in example 1. We choose Q to be the identity matrix. Then the matrix inequalities (32)-(34) become LMIs. By solving LMIs (32)-(34) we can observe that the minimum achievable disturbance attenua- tuation level $\gamma_{rat}\approx 2.4163$. On the other hand, if the controller is allowed to be irrational we can obtain the minimum achievable disturbance attenuation level $\gamma_{irrat}\approx 1.1338$ by solving LMIs (31) and (32). Since $\gamma_{irrat} < \gamma_{rat}$, we observe that the irrational controller will give better performance or better robustness property than the rational controller. When the delay time d is time varying, it is hard to realize the controller in the form of (3). But we guess that the irrational controller will also give better performance than the rational controller. We leave development of the design method for the time varying delay case to one of the future research topics.

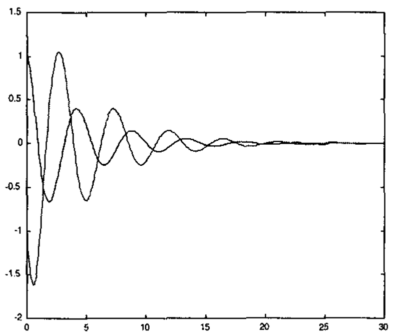

Fig. 2. Time Domain State Response.

V. Concluding Remark

In this paper, we have synthesized an H_∞ output feedback controller for linear systems with commensurate time delay. In order to develope the controller design method, an H_∞ norm bounding condition for the closed loop system has been suggested. The H_∞ norm bounding condition has been derived from the strictly positive real lemma and the bounded real lemma for a complex parameter dependent system. Using the H_∞ norm bounding condition, the existence condition of the output feedback controller has been suggested. The controller parameter can be determined from solutions of two LMIs. The resulting controller is also a linear system with commensurate time delay. We also have shown that the irrational controller will give better performance than the rational controller.

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