

# Final Settlement Prediction Methods of Embankments on Soft Clay

Lee, Dal Won\* · Lim, Seong Hun\*

*\*Dept. of Agric. Eng., Chungnam National University, Taejeon, Korea*

**Abstract** □ Analyses, in which load was regarded as instant load and gradual step load, respectively, were performed with data measured on a gradually loaded field, and the results were inspected to find the effect of load conditions, and the final settlements which were predicted by Hyperbolic, Tan's, Asaoka's, and Monden's methods were compared with each other.

Settlement curves in which load was regarded as instant load and gradual step load begin to coincide at twice the time of duration of embankment.

On the ground installed vertical drain, from the results of Hyperbolic, Tan's, Asaoka's, Monden's, Curve fitting I, and Curve fitting II (simple, carrillo) methods it was concluded that Asaoka, Curve fitting I, and Curve fitting II methods are reliable for prediction final settlement with back analysis.

---

**Keywords** □ Asaoka, consolidation, Curve fitting, embankment, final settlement, Hyperbolic, Monden, prefabricated vertical drain

## I. Introduction

Recently, there is an expansion of required industrial and residential areas as well as a plan of social overhead capital in Korea. And coastal land is needed much more than the past because well conditioned lands are being exhausted.

Soils in these area are characteristic of low shear strength and subside so that it may cause future problems which have to be studied.

Since Terzaghi's consolidation theory(1943), many theses for verification of the theory were presented and applied to field. But uncertainty is raised from the poor representation of properties in the target soil in analysis or distribution and the stress change of the specimens during the sampling, carrying and experimental processes, and thus the analysis results are not in agreement with the real behavior of the ground. Especially the prediction method of final settlement which caused by gradual step load does not be estab-

lished yet. Miyakawa(1961) presented a settlement curve shape as a hyperbolic curve. Hoshino(1962) modified the hyperbolic method with the proposal that settlement including shear strain is proportional to the square root of time. Monden(1963) used the fact that the relation curve  $T_v$  to  $\ln(1-U)$  is a straight line. Asaoka(1978) proved that the relationship of  $S_j$  and  $S_{j+1}$ , based on Mikasa's consolidation theory, is a straight line. Tan(1993) established the theoretical system on which final settlement is determined with hyperbolic function expanded from Sridharan's proposal.

The purpose of this paper is a proposal of the best believable method to predict the final settlement from the measured settlement data. To do so the results predicted with Hyperbolic, Tan's, Asaoka's, Monden's, and Curve fitting methods were compared and analyzed.

## II. Analysis methods and ground properties

### 1. Final settlement prediction methods

1) Hyperbolic method is developed based on the consideration that the settlement curve shape is hyperbolic, and the equation is the following.

$$S_t = S_0 + t / (\alpha + \beta \cdot t)$$

2) Tan adopted Sridharan's  $C_v$  determination method for the prediction of the final settlement. This method is based on the assumption that relation curves  $T_v$  to  $T_v/U$  and time to (measured settlement)/time have the

same shape.

$$S_f = S_0 + \frac{a'}{S_i}$$

3) Asaoka divided measured settlement into the same time interval  $\Delta t$  and plotted  $S_j$  on a horizontal axis versus  $S_{j+1}$  on a vertical axis. The final settlement is predicted with this line.

$$S_f = \frac{\beta_0}{1 - \beta_1}$$

4) Monden's method : In this method, the degree of consolidation  $U$  is changed until the relative curve  $T_v$  to  $\ln(1-U)$  becomes a straight line.

$$\ln(1-U) = \frac{-8 \cdot T_h}{F(n)} = \frac{-8 \cdot C_h}{F(n)} \frac{t}{de^2}$$

where

$a, \beta$ : intercept and slope of the regression line on  $y$  axis when time  $t$  is put on  $x$  axis and the time over the settlement  $t/S$  is put on  $y$  axis, respectively.

$a'$  : slope of the straight line segment on  $x$ - $y$  plane when the time factor  $T$  is put on  $x$  axis and the time factor over the degree of consolidation  $T/U$  is put on  $y$  axis.

$\beta_0, \beta_1$  : intercept and slope of the regression line on  $y$  axis when  $S_j$  is put on  $x$  axis and  $S_{j+1}$  is put on  $y$  axis, respectively.

$C_h$  : coefficient of radial consolidation.

$de$  : diameter of effective circle.

$F(n)$  : function to drain spacing ratio.

$S_0$  : the initial settlement at time started.

$S_f$  : final settlement.

$S_i$  : slope of the straight line segment on x-y plane when the time  $t$  is put on x axis and the time over the settlement  $t/S$  is put on y axis.

$S_t$  : the settlement at any time  $t$ .

$T_h$  : time factor on radial consolidation.

$U$  : degree of consolidation.

5) Curve fitting method : This method searches  $C_h$  and  $C_c$  directly with a trial and error method in settlement equation at each time. This equation is classified into two types. One regards the embankment load as instant and the other regards as a gradual step load.

$$S_t = \sum_{i=1}^n \left\{ \frac{C_c}{1+e_0} \times H \times \log \frac{P_0 + \Delta P_i}{P_0} \times [1 - \exp(-8 \times \lambda \times t)] \right\} \quad (1)$$

The settlement at any time is calculated by Terzaghi's settlement equation. And the degree of consolidation on which Barron's, Hansbo's, Yoshikuni's, and Onoue's equations are put together with  $\lambda$  for the purpose of ignoring smear effect and well resistance and then each settlement is added to the total settlement at any time as shown in Fig. 1, where  $n$  is the embankment step. The value of  $C_c$  and  $\lambda$  may be changed until that the sum of the differences between the  $S_t$  calculated by equation (1) and the measured settlement at time  $t$  is the smallest. But these simple difference calculations cause errors because the measured time interval during embankment

construction is smaller than that in the latter part of consolidation. Therefore the value of  $C_c$  and  $\lambda$  were changed until that the sum of the difference areas formed by the measured and the estimated curves and the time interval lines is the smallest. This concept is shown on Fig. 2.

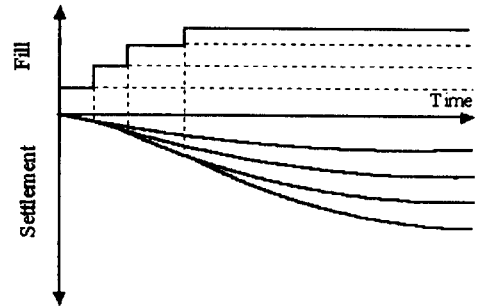


Fig. 1. Settlement caused by gradual step load

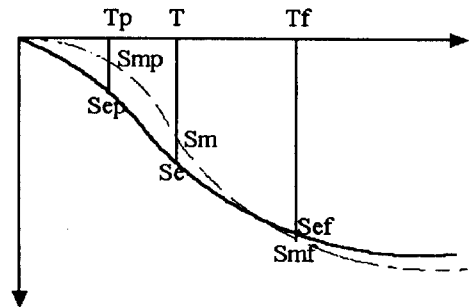


Fig. 2. Difference area between measured and estimated settlement

This procedure is implemented by a computer program developed by authors.

## 2. Ground Properties

The analyzed data was measured on housing land preparation project in YangSan. Spacing of vertical drains, depths of clay and heights of embankment on each method are shown on

following Table 1.

A method(Menard drain) is that vertical plastic board drain was reformed to the perforated circular cross sectional ductile gophered thin plastic pipe encircled by nonwoven geotextile.

B method(Pack drain) is sand pile on which sand was packed in synthetic fiber bag to protect the sand pile from sheared necking.

C method(Plastic board drain) is that perforated plastic board was used instead of sand pile.

The mechanical properties on the ground, as tested in laboratory are shown on Table 2.~ Table 4. The properties inputted in analysis are

Table 1. Spacing of drain and height of embankment

Method	Spacing of drain	Remark
A Method (Menard drain)	A1,B1 : 1.0×1.0m	1. Depth of drain : 25.5m
	A2,B2 : 1.2×1.2m	
B Method (Pack drain)	A3,B3 : 1.4×1.4m	
	A4,B4 : 1.6×1.6m	2. Height of embankment : 5.0m
C Method (Plastic board drain)	C1 : 1.0×1.0m	
	C2 : 1.0×1.0m	
	C3 : 1.0×1.0m	
	C4 : 1.0×1.0m	
	C5 : 1.5×1.5m	3. Different companies' products were used in C1,C5, C2,C6, C3,C7 and C4,C8, respectively.
	C6 : 1.5×1.5m	
C7 : 1.5×1.5m		
C8 : 1.5×1.5m		

Table 2. Mechanical properties of ground at A method site

Depth(m)	$e_0$	$C_c$	$V_{cc}$	$C_v(\text{cm}^2/\text{s})$	$P_0(\text{tf}/\text{m}^2)$
0 ~ 3	1.137	0.42	0.460	$1.50 \times 10^{-3}$	1.185
3 ~ 11	1.796	0.80	0.891	$3.47 \times 10^{-4}$	4.77
11 ~ 20	1.520	0.62	0.778	$3.41 \times 10^{-4}$	10.185
20 ~ 30	1.265	0.46	0.695	$1.43 \times 10^{-3}$	16.90
Weighted mean value	1.470	0.595	0.749	$5.68 \times 10^{-4}$	10.305

the mean values weighted to each layer thickness. The unit weights, as used on a Curve fitting method are shown on Table 5. where  $e_0$  is the initial void ratio,  $C_c$  is the compression index,  $V_{cc}$  is the virgin compression index,  $C_v$  is the consolidation coefficient, and  $P_0$  is the in situ effective overburden pressure.

Table 3. Mechanical properties of ground at B method site

Depth(m)	$e_0$	$C_c$	$V_{cc}$	$C_v(\text{cm}^2/\text{s})$	$P_0(\text{tf}/\text{m}^2)$
0 ~ 3	1.137	0.42	0.500	$2.17 \times 10^{-3}$	1.187
3 ~ 8	1.739	0.53	0.630	$3.53 \times 10^{-4}$	3.948
8 ~ 20	1.893	0.79	0.852	$2.22 \times 10^{-4}$	9.153
20 ~ 31	1.430	0.32	0.646	$1.19 \times 10^{-3}$	18.027
Weighted mean value	1.631	0.545	0.709	$4.25 \times 10^{-4}$	15.268

Table 4. Mechanical properties of ground at C method site

Depth(m)	$e_0$	$C_c$	$V_{cc}$	$C_v(\text{cm}^2/\text{s})$	$P_0(\text{tf}/\text{m}^2)$
0 ~ 3	1.196	0.43	0.515	$1.05 \times 10^{-3}$	1.155
3 ~ 13	1.892	0.83	0.967	$3.12 \times 10^{-4}$	5.210
13 ~ 21	1.549	0.63	0.776	$3.43 \times 10^{-4}$	10.71
21 ~ 30	1.353	0.46	0.592	$1.58 \times 10^{-3}$	16.415
Weighted mean value	1.569	0.626	0.758	$5.19 \times 10^{-4}$	9.760

Table 5. Unit weight of sand mat and embankment

Method	Classification	Unit weight $\gamma_s(\text{tf}/\text{m}^3)$	
		Sand mat	Embankment
A		1.997	1.914
B		1.876	1.938
C		1.957	1.961

The height of embankment and settlements used on analysis on each method are shown

on Fig. 3 ~ Fig. 6. In Fig. 3, A method was

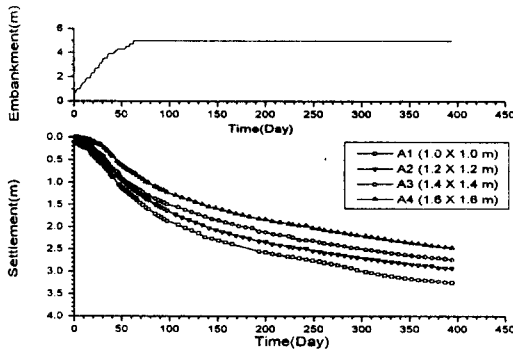


Fig. 3. Distribution of time vs settlement at A method site

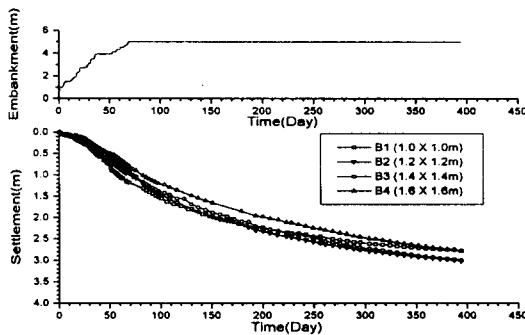


Fig. 4. Distribution of time vs settlement at B method site

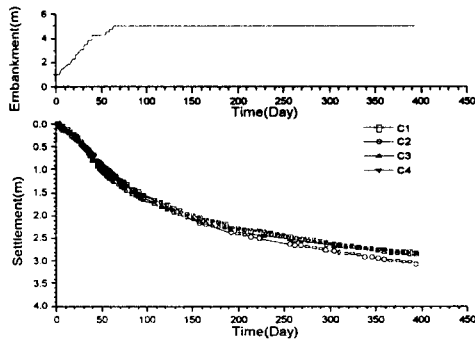


Fig. 5. Distribution of time vs settlement at C method site (spacing : 1.0 x 1.0m)

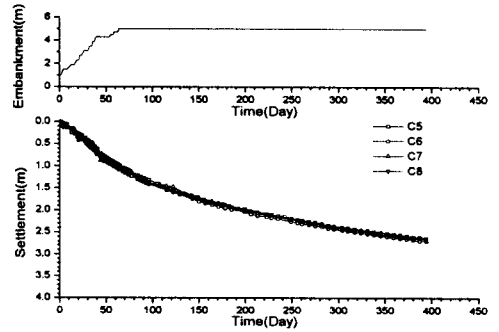


Fig. 6. Distribution of time vs settlement at C method site (spacing : 1.5 x 1.5m).

used. The period of embankment construction is 62 days and then the site was left alone for 332 days. On A1, A2, A3 and A4, the spacing of drains is 1.0 x 1.0m, 1.2 x 1.2m, 1.4 x 1.4m and 1.6 x 1.6m, respectively. As shown in Fig. 3, the measured settlements at A method during 394 days range from 3.246 to 2.467m and the settlement enlarged with the tighter spacing of drains. The settlement rate at any time is high during embankment construction and then becomes low as time passes. Settlements using the B method during 394 days range from 2.776 to 3.021m and it is shown that spacing of drains has no relation on settlement. Fig. 5 and 6 presents the settlements using C method with the spacing of drains as 1.0 x 1.0m and 1.5 x 1.5m, respectively.

The settlements during 394 days range from 2.630 to 3.041m as shown in Fig. 5. and the settlements are different with the kinds of drains. In Fig. 6, the settlements during 394 days are almost same values as from 2.630 to 2.695m irrespective of the kinds of drains.

### III. Results and discussion

#### 1. Settlement curve variation according to load condition

Fig. 7 shows a part of the settlement curves which one is estimated on the condition of the instant load and the other gradual step load as derived by a computer program with the whole data from the beginning of embankment construction to the end of measured data.

As shown in the figure, the line a indicates an imaginary instant embankment, and point b is the beginning time of construction and point c is the time of embankment construction end, and point d is two times the embankment

construction period and also it is the beginning point at which two settlement curves coincide, and lastly point e is the end of measurement.

The kink line in Fig. 7(a), (b) are the measured settlement curves. In front part of the settlement curves, the difference between the measured and the estimated curves is larger on A method than that on B method.

The estimated curves, one as calculated as an instant load and the other as a gradual step load, are the almost same value after the point d even though the front parts are very different from each other. Fig. 8 and Fig. 9 show the processes that hyperbolic and Asaoka's methods are done, respectively. In

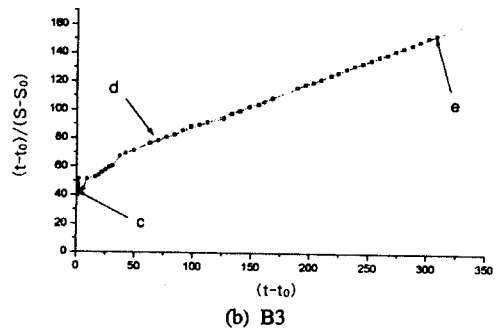
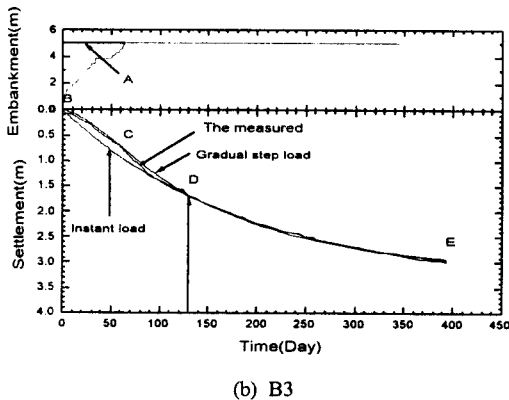
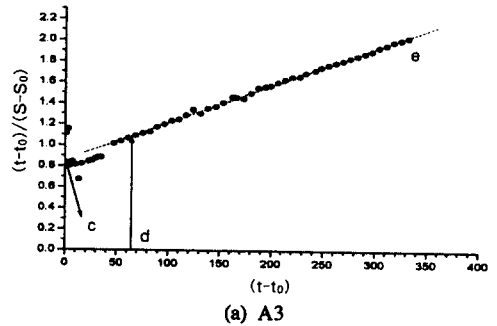
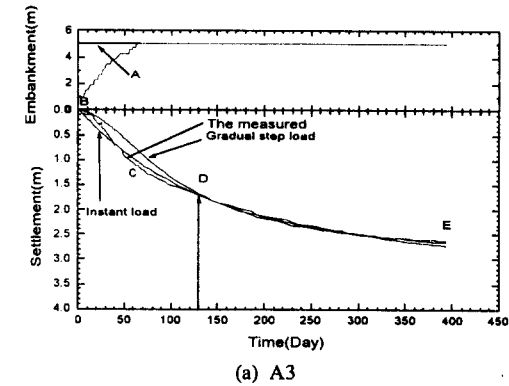


Fig. 7. Relationship instant and gradual step loads

Fig. 8. Hyperbolic method

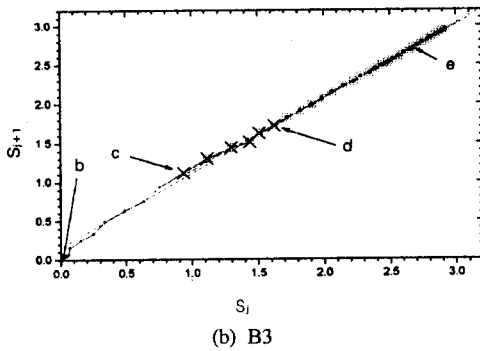
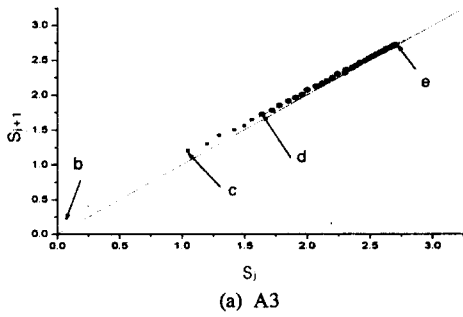


Fig. 9. Asaoka's method

these figures, the analyzed data is the same as

in Fig. 7. The points b, c, d and e in Fig. 8 and 9 are the same meanings in Fig. 7. As expected, the lines are straight after the point d. In Fig. 8, it makes no sense searching straight tendency before the point d and in Fig. 9, the errors before the point d are small comparatively, but despite these small variations, the estimated final settlement changes may be relatively large because of the characteristics of Asaoka's method itself.

Therefore, it is concluded that the effect of gradual step load is eliminated when the time from the starting time(the point b) of embankment construction becomes twice(the point d) the period(the point c) of embankment construction. This is shown by the settlement curve agreement between a gradual step load and an instant load.

For that reason if the whole measured data is used on the analysis with a Curve fitting

Table 6. Predicted final settlements of each method (unit : m)

	Hyperbolic	Tan's method	Asaoka's method	Monden's method	Curve fitting I	Curve fitting II	
						Simple	Carrillo
A1	4.187	3.481	3.369	3.70	3.502	3.296	3.345
A2	3.883	3.223	3.115	3.70	3.038	3.024	3.148
A3	3.920	3.228	2.956	3.10	2.930	2.829	2.878
A4	3.980	3.206	2.637	2.90	2.752	2.607	2.656
B1	3.606	3.000	2.920	3.00	2.916	2.854	2.870
B2	4.115	3.341	3.221	3.30	3.333	3.155	3.206
B3	4.339	3.485	3.655	3.40	3.382	3.232	3.237
B4	4.505	3.588	3.206	3.30	3.260	3.051	3.101
C1	3.776	3.125	2.972	3.10	2.923	2.907	2.933
C2	4.354	3.560	3.181	3.40	3.170	3.053	3.079
C3	3.771	3.168	2.979	3.10	2.923	2.884	2.884
C4	3.617	3.011	3.372	3.10	2.994	2.884	2.893
C5	3.928	3.236	2.942	3.20	2.853	2.762	2.762
C6	4.008	3.291	3.050	3.20	2.958	2.835	2.884
C7	4.268	3.484	2.954	3.20	2.923	2.835	2.835
C8	3.881	3.175	2.926	3.10	2.923	2.835	2.834
Remark					Instant load	Gradual step load	

method, it is reasonable to use the data which is measured after two times the period of embankment construction.

## 2. Reliability estimation of final settlement prediction methods

The methods used in final settlement predictions from measured settlements are Hyperbolic, Tan's, Asaoka's, Monden's, Curve fitting in which load is considered as instant and vertical direction consolidation is ignored, Curve fitting II(simple) in which load is considered as a gradual step load and vertical direction consolidation is ignored, and lastly Curve fitting II(Carrillo) in which load is considered as a gradual step load and vertical consolidation is combined with horizontal consolidation in Carrillo's method. The results are shown in Table 6.

Fig. 10 shows the predicted final settlements according to each method. In the results of A method, the predicted final settlements were larger  $A1 > A2 > A3 > A4$  in order as spacing of drains are small. In the results of B method, the predicted final settlement was the largest in B3 and was the smallest in B1 in which spacing of drains are the smallest irrespective of spacing of drains. In the results of C method, the variation of predicted settlements was large in C1~C4 but small in C5~C6. The predicted final settlements according to analysis methods were larger as follows, Hyperbolic > Tan's > Monden's > Asaoka's > Curve fitting I > Curve fitting II (Carrillo) > Curve fitting II(Simple) in order.

Considering the real field condition in which embankment is constructed with gradual step, it

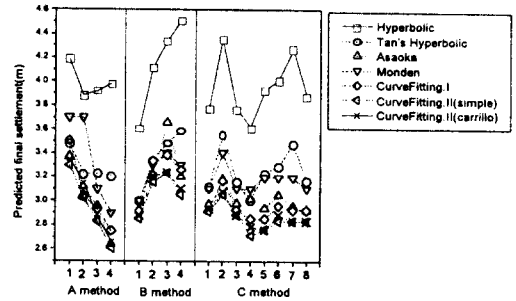


Fig. 10. Predicted final settlements

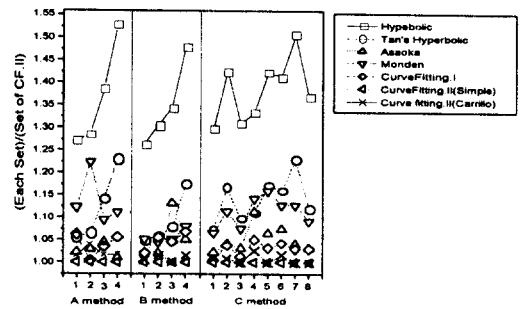


Fig. 11. Difference compared with curve fitting II method

is reasonable that one of Curve fitting II methods have to be used because predicted final settlements differed from the values analyzed in each method. Fig. 11 shows the ratio of each predicted settlement to the predicted, which was the smallest, with Curve fitting II(simple). The ratio is 26~55% in Hyperbolic method, 6~20% in Monden's and Tan's methods and nearly the same in range of 10% in Curve fitting I, II and Asaoka's methods. But the shape of the curves in the figure are irregular in Monden's, Tan's and Hyperbolic methods. Consequently Curve fitting I, II and Asaoka's methods are reliable in predicting final settlement with measured data.



## IV. Conclusions

From the results that were predicted with Curve fitting methods on the conditions of instant and gradual step loads and that used Hyperbolic, Tan's, Asaoka's, and Monden's methods, the following conclusions were obtained.

1. Settlement curves, that were predicted on the conditions of instant and gradual step loads, begin to coincide with each other at twice the time of duration of embankment.

2. Settlement was predicted 26~55% larger in Hyperbolic(simple), 6~20% larger in Tan's and Monden's methods, and nearly the same in range of 10% in Curve fitting I,II and Asaoka's methods.

3. Asaoka's, Curve fitting I, and Curve fitting II methods are reliable for predicting final settlement with back analysis.

## References

1. Asaoka, A., 1978, Observational procedure of settlement prediction, *Soils and Foundations*, 18(4), pp. 87~101.
2. Asaoka, A. & M. Matsuo, 1980, An inverse problem approach to settlement prediction, *Soils and Foundations*, 20(4), pp. 53~66.
3. Barron, R. A., 1948, Consolidation of fine-grained soils by drain wells, *Transactions*, 113(paper No. 2346), pp. 718~742.
4. Carrillo, N., 1942, Simple Two and three dimensional cases in theory of consolidation of soils, *Journal of Mathematics and*

- Physics*, 21(1), pp.1~5.
5. Casagrande, A. & R. E. Fadum, 1940, Notes on soil testing for engineering purposes, Harvard Univ. Graduate School of Engineering, Publication No. 8, quoted by Das, B. M., 1983, *Advanced soil mechanics*, McGraw-Hill Book Company, pp. 292~293.
  6. Hansbo, S., 1979, Consolidation of clay by bandshaped prefabricated drains, *Ground Engineering*, July, 12(5), reprinted in *Geotech. 92 Workshop : Applied ground improvement technique*, SEAGS, AIT. Bangkok, Thailand, December, 1992.
  7. Hansbo, S., 1981, Consolidation of fine grained soil by prefabricated drains, *Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering*, Stockholm, Sweden, Vol. 3, pp. 677~682.
  8. Magnan, J. P., 1983, Back analysis of soil consolidation around vertical drains, *Improvement of ground*, VIII ECSMFE Helsinki '83, Vol. 2, pp. 653~658.
  9. Matyas, E. L. & L. Rothenburg, 1996, Estimation of total settlement of embankments by field measurements, *Canadian Geotechnical Journal*, 33, pp. 834~841.
  10. Mesri, G. & A. Rokhsar, 1974, Theory of consolidation for clays, *Journal of the Geotechnical Engineering Division*, 100 (GT8), pp. 889~904.
  11. Mesri, G. & P. M. Godlewski, 1977, Time and stress compressibility interrelationship, *Journal of the Geotechnical Engineering Division* 103(GT5), pp. 417~430.
  12. Monden, H., 1963, A new time-fitting

- method for the settlement analysis of foundation on soft clays, *Memoir Faculty of Eng. Hiroshima Univ.*, 2-1, 21, pp. 21~29.
13. Onoue, A., 1988, Consolidation by vertical drains taking well resistance and smear into consolidation, *Soil and Foundation*, 28(4), pp. 165~174.
  14. Sridharan, A., N. S. Murthy & K. Prakash, 1987, Rectangular hyperbola method of consolidation analysis, *Geotechnique*, 37(3), pp. 355~368.
  15. Sridharan, A. & K. Prakash, 1985, Improved rectangular hyperbola method for the determination of coefficient of consolidation, *Geotechnical Testing Journal*, 8(1), pp. 37~40.
  16. Sridharan, A. & R. A. Sreepada, 1981, Rectangular hyperbola fitting method for one dimensional consolidation, *Geotechnical Testing Journal*, 4, pp. 161~168.
  17. Tan, S. A. & S. H. Chew, 1996, Comparison of the hyperbolic and Asaoka observational method of monitoring consolidation with vertical drains, *Soils and Foundations*, 36(3), pp. 31~42.
  18. Tan, S. A., 1993, Ultimate settlement by hyperbolic plot for clays with vertical drains, *Journal of the Geotechnical Engineering*, 119(3), pp. 950~1066.
  19. Tan, S. A., 1994, Hyperbolic method for settlement in clays with vertical drains, *Canadian Geotechnical Journal*, 31, pp. 125~131.
  20. Tan, S. A., 1995, Validation of hyperbolic method for settlement in clays with vertical drains, *Soils and Foundations*, 35(1), pp. 101~113.
  21. Taylor, D. W., 1942, Research on consolidation of clays, Department of Civil and Sanitary Engineering, Massachusetts Institute of Technology, Publication No. 82, quoted by Das, B. M., 1983, *Advanced soil mechanics*, pp. 293~294.
  22. Terzaghi, K., 1943, *Theoretical soil mechanics*, John Wiley & Sons, pp. 286~289.
  23. Tewatia, S. K., 1998, Comparison of the hyperbolic and Asaoka observational methods of monitoring consolidation with vertical drains, Discussion, *Soils and Foundations*, 38(2), pp. 224~227.
  24. Yoshikuni, H., 1992, Basic consolidation theory of vertical drain method, Faculty of Engineering Hiroshima University Higashi-Hiroshima, 724, Japan, pp. 53~59, reprinted in *Geotech 92 Workshop : Applied ground improvement technique*, SEAGS, AIT. Bangkok, Thailand, December, 1992.
  25. Yoshikuni, H. & H. Nakanodo, 1974, Consolidation of soils by vertical drain wells with finite permeability, *Soils and Foundations*, 14(2), pp. 35~46.