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OPTIMAL DESIGN MODEL FOR A DISTRIBUTED HIERARCHICAL NETWORK WITH FIXED-CHARGED FACILITIES

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ABSTRACT

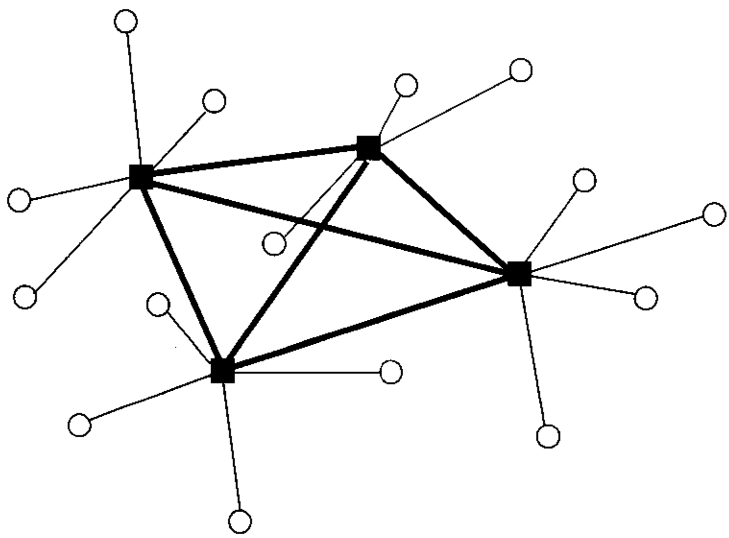
We consider the design of a two-level telecommunication network having logical full-mesh/star topology, with the implementation of conduit systems taken together. The design problem is then viewed as consisting of three subproblems: locating hub facilities, placing a conduit network, and installing cables therein to configure the logical full-mesh/star network. Without partitioning into subproblems as done in the conventional approach, the whole problem is directly dealt with in a single integrated framework, inspired by some recent successes with the approach. We successfully formulate the problem as a variant of the classical multicommodity flow model for the fixed charge network design problem, aided by network augmentation, judicious commodity definition, and some flow restrictions. With our optimal model, we solve some randomly generated sample problems by using CPLEX MIP program. From the computational experiments, it seems that our model can be applied to the practical problem effectively

1. INTRODUCTION

The widespread deployment of optical fibre systems, characterized by the high speed and the large equipment cost, makes the architecture that uses facility-hubbing the de facto standard for present-day communication networks [9, 10]. Such networks consist of hub-level and access-level components. The hub-level component connects the hub facilities, and the access-level component connects demand points to a node in the hub-level network. In this paper, we are concerned with the most general structure among its kind: the hub-level network is logically full-meshed, and each access-level (local) network attached to an established hub is of logical star type. In order to implement the logical network, we need a physical infrastructure such as conduit systems to install cables. An example of the logical full-mesh star network and its physical implementation are illustrated in Figure 1.

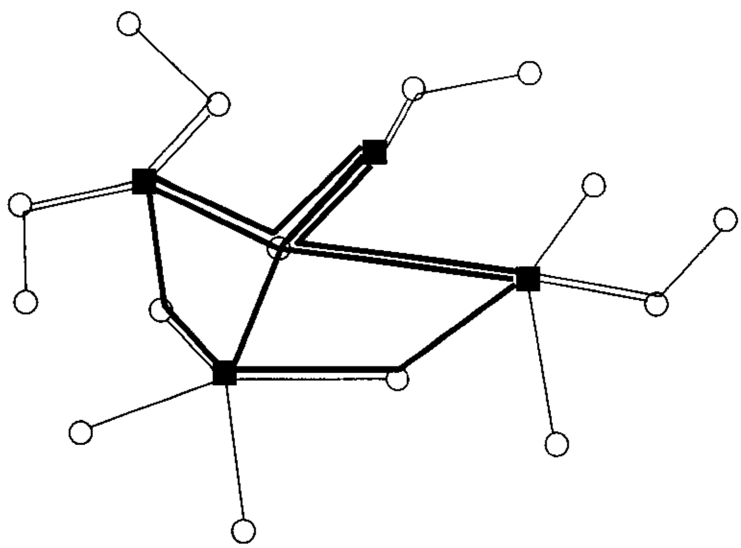
The design of a logical full-mesh star with two level hierarchical structure was studied by Chung *et.al.* [2]. However, the physical implementation plan of the logical network has not been studied yet. In this paper, we consider the design of a physical network with two-level hierarchy having logical full-mesh/star topology. When constructing such a network, the designer must make the following major decisions: the number and location of hub facilities in the hub-level network, the configuration of the infrastructure for the hub-level and the access-level networks, and the cable installation plan within that network. From the complex trade-off relations between the cost incurring network elements, hub facility, conduit and cable, the conventional solution approach for network design is to decompose the whole problem into subproblems easier to solve. But the critical shortcomings of the approach is that the quality of the generated solution is not guaranteed. This has motivated a new line of network design studies of directly dealing with the whole problem without partitioning. This kind of network design problem including hub location decision is a relatively recent but active area of research [2, 3, 4, 6, 7, 11] . Table 1 gives a brief summary of these papers.

This paper can be viewed as an extended version of Chung *et.al.* [2] where the physical implemetation constraints and the variable cost to install a cable on the infrastructure are added. Tcha and Yoon [7] proposed an integrated design model of the physical network design problem for a centralized network incorporating the three major network costs. Recently, Yoon *et.al.* [11] addressed the design problem of distributed fiber transport network, where they integrated the above three decisions into a single framework of formulation. However, they assumed that the entire region be partitioned into a few predetermined regions and that in each of the regions one and only one hub be opened, which makes the design problem much easier to tackle. This paper generalizes the above researches, that is, relaxes the regional restrictions for Yoon *et.al.* [11], and adds the physical implementaion to Chung *et.al.* [2]. The resulting configuration of the physical



■ : Hub Node ○ : User Node ▬ : Hub Link —— : Access Link

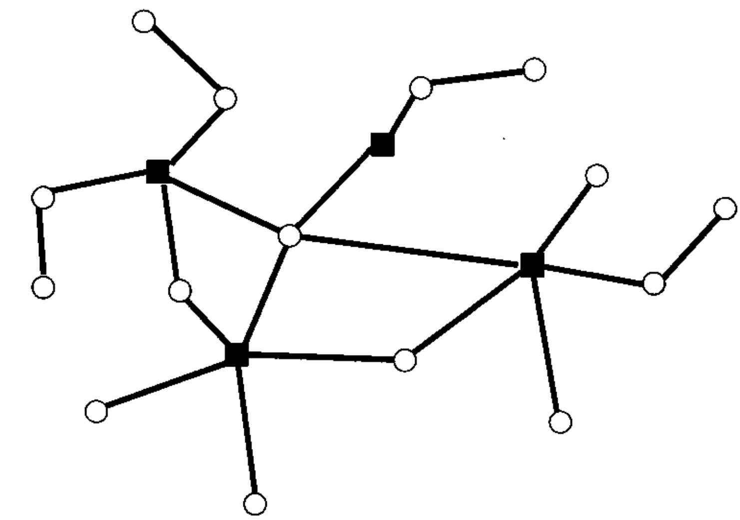
a) Logical full-mesh star network configuration



■ : Hub Node ○ : User Node

▬ : Cable for Hub Link —— : Cable for Access Link

b) Physical cable installation



c) Physical network configuration

Figure 1. Example of a logical full mesh star and its physical implementation

Table 1. Summary of Hierarchical Network Design Studies with Fixed-charged Facilities

Researchers	Problems	Network Topology	Costs	Decisions	Algorithms Employed
W. Tcha and M.G. Yoon['95]	Conduit and cable installation problem	Logically star-star Physically no restrictions	Fixed Cost on Hub Fixed Cost on Arc Variable cost on Arc	Hub Location Arc Selection User Assignment Path Selection	Dual Ascent Method (Centralized Network)
Yoon et.al.['98]	Distributed fiber transport network design	No topological restrictions	Fixed Cost on Hub Fixed Cost on Arc Variable cost on Arc	Hub Location Arc Selection User Assignment Path Selection	Dual Ascent Method (Distributed Network)
J.G. Kim and D.W. Tcha['92]	Backbone network design with tree-star configuration	Tree-star	Fixed Cost on Hub Fixed Cost on Arc	Hub Location Arc Selection User Assignment	Dual Ascent Method
S.H. Chung et.al.['91]	Hierarchical network with full meshed structure	Full mesh-star	Fixed Cost on Hub Fixed Cost on Arc	Hub Location Arc Selection User Assignment	Branch and Bound using Dual Ascent Method
J.R. Current['88]	Hierarchical network with transhipment facilities	Tree-tree	Fixed Cost on Hub Fixed Cost on Arc	Hub Location Arc Selection	Lagrangeran Relaxation
Pirkul and V. Nagarajan['92]	Centralized communication network design with tree-star topology	Tree-star	Fixed Cost on Hub Fixed Cost on Arc	Hub Location Arc Selection	Lagrangeran Relaxation

network is not restricted to a certain type.

Our network design problem may be viewed as consisting of three problems: Given a network with a set of users (terminals) and a set of potential hub sites, the problem is to locate hub facilities, to place a conduit network (physical network), and to find a cable installation plan within the conduit network to configure the logical full-mesh/star network. Three kinds of costs are considered: the fixed cost of establishing a hub, the fixed cost of placing a conduit system over a link, and the cost of installing cables over a link. Each user has to be connected to an established hub, and thus each established hub has its own local network of logical star type attached to it. All the established hubs have to be interconnected each other. Note that candidate hub sites not chosen to be opened may still be used as a junction point for the conduit system without incurring any fixed cost, and that any conduit systems may be shared by both the hub network and the local networks.

This paper is organized as follows. In Section 2, the given network is first augmented by introducing dummy nodes and dummy arcs. On the augmented network, two types of commodities are defined: one for the logical full mesh connection of established hubs, and the other for the logical star connections of users. Nodal fixed costs are translated to arcs to give rise to an arc-cost formulation. On top of this manipulation, some flow restrictions are additionally made, so that the resulting integrated model becomes a variant of the classical multicommodity flow formulation of the network design problem. Section 3 shows a computaional experiments for randomly generated problems. Using CPLEX MIP program, we solve a total of 63 test problems. Some solution strategies for a large scale problem and concluding remarks are finally discussed in Section 4.

2. Design Model

2.1 Network Augmentation

Consider an undirected network $G=(N_0,E_0)$ where N_0 and E_0 represent a set of nodes and edges respectively. N_0 consists of the set of user nodes I and the set of hub nodes (candidate hub sites) J. To establish a hub at candidate site $j \in J$, fixed cost g_j is incurred. When a candidate hub site is not chosen to be established, it can still be used as a transit (or junction) node without incurring the associated fixed cost. Fixed cost f_{ij} is incurred when placing conduit over edge $\{i, j\} \in E_0$. The capacities of established facilities, hubs and conduit systems, are all assumed to be unlimited.

We now augment the original network by introducing dummy nodes and edges as follows:

- Associate each hub node $j \in J$ with a dummy hub node j'. Let J' be the set of those dummy hub nodes. Also, add a super dummy node 0 and define $N = N_0 \cup J' \cup \{0\}$.
- Add dummy edges connecting each hub node and the corresponding dummy hub node. And add dummy edges connecting all the dummy nodes. Then define $E = E_0 \cup \{\{j, j'\}, j \in J\} \cup \{\{j', 0\}, j' \in J'\} \cup \{\{l', m'\}, l', m' \in J'\}$.

The augmented network is now visualized as consisting of a super dummy and two planes: the dummy plane at the upper level, and the real plane at the lower level. Then edges on the real plane are all real, while those on and above the dummy plane and between the two planes are all dummy. An illustration of the augmented network is given in Figure 2.

For our complex network design problem, we shall use the multicommodity flow formulation for which the powerful dual-based solution approach is known well suited. For user connectivity, we define the set of user commodities as follows:

$$K_u = \{k : o(k) \in I, d(k) = 0\},$$

where $o(k)$ and $d(k)$ represent the origin and destination nodes of the commodity k, respectively. Note that each user commodity flow, the nonbifurcated one, has to pass through the dummy plane, particularly through a candidate hub site and the corresponding dummy hub, to get to its destination super dummy node, implying that the hub site is chosen to be established and the user is attached to it.

Since established hubs should be interconnected each other, the set of hub commodities are defined as

$$K_h = \{k : o(k) \in J', d(k) \in J'\}.$$

Note that both the origin and the destination of every hub commodity are located on the dummy plane, i.e., on J', thereby making a hub commodity defined for any pair of hub nodes, established or not. It would be instructive to identify the flow path of a hub commodity, both the origin and the destination hub of which are established: It starts at its dummy hub origin, gets down along the dummy edge to the corresponding real hub node one level below, and moves on the real plane to the real hub destination, and jumps up again along the dummy edge to the final dummy destination one level above. The whole commodity set K is then defined as $K_u \cup K_h$.

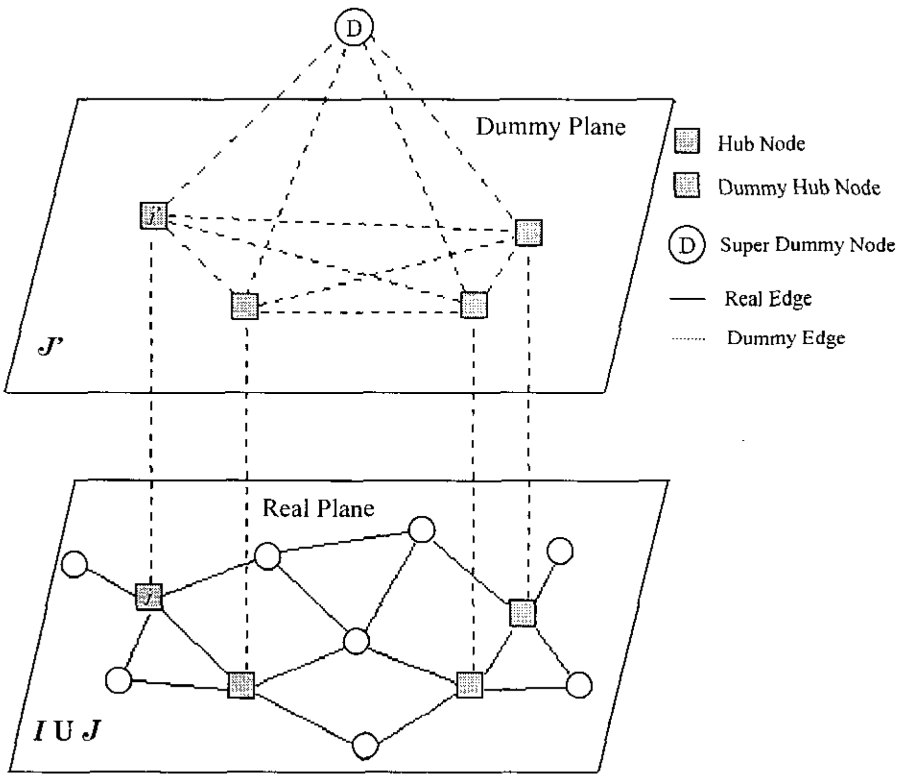


Figure 2 . Augmentation for the network

2.2 Cost Transformation and Flow Restrictions

From the nonbifurcation property of every commodity flow at optimum, we can make costs defined only on edges and arcs. The fixed cost of establishing a hub can be transferred to that of establishing its only dummy edge connecting to the corresponding dummy hub node. We then have the following fixed cost structure on dummy edges:

$$f_{jj'} = g_i, \quad j \in J, \tag{1}$$

$$f_{l'm'} = 0, \quad l', m' \in J' \cup \{0\}. \tag{2}$$

Let A be a set of (directed) arcs induced by associating each edge in E with two arcs of opposite directions. To avoid confusion, an edge and an arc between two nodes i and j will be denoted by $\{i,j\}$ and (i,j) respectively.

The installation cost of cables required for the circuit demand of commodity k on an arc (i,j) is given by c_{ij}^k, which is set equal to c_{ji}^k. On dummy arcs, no cable installation costs should be associated with. But for the convenience of the formulation, an infinitely large cost is assigned to each dummy arc on the dummy plane for all commodities except the one having the two end nodes as its origin and destination nodes. This definition forces each commodity to get to its destination, being the super dummy or any dummy hub, without unnecessarily hovering around on the dummy plane. So we have the following cable installation costs on dummy arcs:

$$c_{jj'}^k = c_{j'j}^k = 0, \qquad j \in J, \qquad j' \in J', k \in K \tag{3}$$

$$c_{l'0}^k = c_{0l'}^k = 0, \qquad l' \in J', \qquad k \in K \tag{4}$$

$$c_{l'm'}^k = \begin{cases} 0, & l' = 0(k), m' = d(k), k \in K_h, \\ \infty, & \text{otherwise}. \end{cases} \tag{5}$$

Under this cost structure, three kinds of flow paths of a hub commodity are possible: 1) Going up to the super dummy first, 2) Taking the direct dummy arc, and 3) Going along the bridging dummy arc one level down to the real plane first. Obviously the third kind corresponds to the case where both hubs of the hub commodity are established and interconnected.

Now focusing on a dummy arc $(l',0)$, suppose that a user commodity flow is found on the arc. This implies that the corresponding hub l is established to connect that user node. Hence, every hub commodities originating from l' is either the second or the third kind above, and thus not allowed to take that dummy arc, giving rise to the relation (6). These two kinds of hub commodities are further distinguished from each other: the second type have a dummy destination not established while only the third corresponds to the real one having both origin and destination established. From the viewpoint of a dummy hub node corresponding to an established (open) one, it should not be the destination of the second type hub commodities, resulting in the following constraint (7).

$$x_{l'0}^{k} + x_{l'0}^{k} \le y_{l'0}, \qquad k' \in K_u,\ k \in K_h, \quad o(k) = l', \quad l' \in J', \tag{6}$$

$$x_{m'l'}^{k} + x_{l'0}^{k} \le y_{m'l'}, \qquad k' \in K_u,\ k \in K_h, \quad d(k) = l', \quad \{m', l'\} \in E' \tag{7}$$

where, y_{ij} denotes the 0-1 variable concerning the establishment of edge $\{i, j\}$, x_{ij}^{k} denotes the variable denoting the flow of commodity k transported on arc (i, j), and E' is the set of edges on the dummy plane.

Under this constraints, it would be instructive to distinguish again hub commodities according to their path types: those with both origin and destination (o-d) open take the third type path, those with the o-d closed and open respectively the first type path, and those with their o-d open and closed respectively as well as those with both o-d closed the second type path.

2.3 Multicommodity Flow Model

Incorporating all the maneuvers, we now present the multicommodity flow model for our design problem of networks with logical full-mesh/star configuration, which looks quite similar to those for a host of existing network design studies in the literature [1, 4, 5, 8].

$$(P)\ \text{Min} \quad \sum_{\{i,j\} \in E} f_{ij} y_{ij} + \sum_{k \in K} \sum_{(i,j) \in A} c_{ij}^{k} x_{ij}^{k} \tag{8}$$

$$\text{s.t} \quad \sum_{j \in N} x_{ij}^{k} - \sum_{j \in N} x_{ji}^{k} = \begin{cases} 1, & i = o(k), \\ -1, & i = d(k), \\ 0, & \text{other } i \in N, \end{cases} \qquad \forall k \in K, \tag{9}$$

$$x_{i0}^{k} \le \begin{cases} y_{i0} - x_{i0}^{k'}, & k' \in K_u,\ k \in K_h,\ i = o(k),\ i \in J', \\ y_{i0}, & \text{otherwise}, \end{cases} \tag{10}$$

$$x_{ij}^{k} \le \begin{cases} y_{ij} - x_{j0}^{k'}, & k' \in K_u,\ k \in K_h,\ j = d(k),\ \{i, j\} \in E', \\ y_{ij}, & \text{otherwise}, \end{cases} \tag{11}$$

$$x_{ji}^{k} \le \begin{cases} y_{ij} - x_{i0}^{k'}, & k' \in K_u,\ k \in K_h,\ i = d(k),\ \{i, j\} \in E', \\ y_{ij}, & \text{otherwise}, \end{cases} \tag{12}$$

$$y_{ij} \in \{0,1\}, \qquad \{i, j\} \in E, \tag{13}$$

$$x_{ij}^{k} \ge 0, \qquad (i, j) \in A, \quad k \in K. \tag{14}$$

The objective function of (P) has only two arc-cost terms: the first corresponds to the fixed costs of establishing both hubs and conduit systems, the second is for cable installation. The flow conservation constraints (9) enforce the network connectivity for both hub and user commodities. The flow restrictions of (6) and (7) are included in the forcing constraints (10), (11) and (12) respectively.

3. Computational Experiments

The formulation (P) for our comprehensive problem was solved by CPLEX MIP program on a PC (Pentium II 233MHz). The test problems were generated randomly to obtain problems with differing levels of cost tradeoffs. We first randomly located the prespecified number of candidate hub and user sites from (100 x 100) grid in the plane. On top of an arbitrarily obtained spanning tree covering all those sites, additional edges were placed until the total number of edges reaches the specific level.

To define various cost data, we first set a base cost c_{ij} on each arc $\{i,j\}$, which was the Euclidean distance between nodes i and j on the above plane. The fixed cost of arc i,j was obtained by multiplying the base cost by the scaling factor f which was the same for all ones in a particular network. The variable cost of each commodity, k, on arc (i,j) was then obtained by multiplying the base cost c_{ij} by the demand U_k. Each demand $U_k, k \in K$ was randomly selected from an interval (10.0, 50.0). The hub establishment costs, g_j, were chosen randomly from interval (a,b). In real-world networks, the base costs are not necessarily incurred in proportion to Euclidean distance. To consider the non-Euclidean distance for an arc (i,j), we randomly select a scaling factor V_{ij} from an interval (0.5, 2.5). Hence, the base cost on each arc (i,j) for non-Euclidean distance can be defined by multiplying the base cost c_{ij} for Euclidean distance by the scaling factor V_{ij}.

In the randomly generated problems, we solved a total of 126 test problems for both Euclidean and non-Euclidean distance cases, and each problem is grouped into seven different subsets by the problem size, i.e., the number of user and hub nodes, arcs and commodities. Each subset is divided by the range of hub establishment costs, and further splitted into three smaller subsets having different scaling factors f. Table 2 lists the summary of the test problems ranging

Table 2. Computational Results

a) Euclidean Distance Case

|I|x|J|x|E| x|K|[a]	Hub Cost	*f* Ratio	IP Optimum Value	LP Gap[b]	% Gap (%)[c]	Computation Time (Sec.)	# of Established Hubs	Hub Cost Ratio(%) [d]
5x10x20x20	1,000 - 5,000	3.0	20,287	861.0	4.2	2.86	3	32.2
		5.0	21,295	1,142.5	5.3	1.65	3	30.6
		10.0	23,815	1,905.0	8.0	3.45	3	27.4
	5,000 - 10,000	3.0	30,148	142.5	0.5	1.19	2	45.9
		5.0	31,196	1,341.5	4.3	1.26	2	44.4
		10.0	33,815	1,838.0	5.4	1.38	2	40.9
	20,000 - 50,000	3.0	48,460	0.0	0.0	1.02	1	44.1
		5.0	49,504	0.0	0.0	1.17	1	43.2
		10.0	52,114	0.0	0.0	1.30	1	41.0
5x10x34x20	1,000 - 5,000	3.0	19,623	368.5	1.8	2.01	2	20.4
		5.0	20,731	688.0	3.3	2.79	2	19.3
		10.0	23,176	1,266.0	5.4	5.74	2	17.3
	5,000 - 10,000	3.0	27,709	179.0	0.6	1.66	1	20.7
		5.0	28,939	357.0	1.2	1.62	1	19.8
		10.0	31,581	691.7	2.2	2.15	1	18.1
	20,000 - 50,000	3.0	44,274	0.0	0.0	1.82	1	48.3
		5.0	45,413	0.0	0.0	1.46	1	47.1
		10.0	47,853	0.0	0.0	1.45	1	44.7
5x20x35x30	1,000 - 5,000	3.0	34,111	2.5	0.0	3.34	3	15.2
		5.0	35,469	44.5	0.1	3.02	3	14.6
		10.0	38,853	291.0	0.7	3.86	3	13.4
	5,000 - 10,000	3.0	51,768	44.5	0.1	3.03	2	24.9
		5.0	53,161	83.5	0.1	3.55	2	24.3
		10.0	56,536	178.5	0.3	4.64	2	22.8
	20,000 - 50,000	3.0	82,929	145.0	0.2	4.11	1	25.8
		5.0	84,497	497.0	0.6	4.63	1	25.3
		10.0	88,293	1,260.0	1.4	4.46	1	28.5
5x20x60x30	1,000 - 5,000	3.0	33,358	25.0	0.1	8.19	4	21.9
		5.0	34,742	90.0	0.2	6.34	4	21.0
		10.0	38,188	319.5	0.8	20.02	4	19.1
	5,000 - 10,000	3.0	46,694	5.5	0.0	6.70	2	23.8
		5.0	48,175	10.0	0.0	4.28	2	23.1
		10.0	51,870	103.6	0.2	11.09	2	21.4

|I|x|J|x|E| x|K|[a]	Hub Cost	*f* Ratio	IP Optimum Value	LP Gap[b]	% Gap (%)[c]	Computation Time (Sec.)	# of Established Hubs	Hub Cost Ratio(%)[d]
	20,000 - 50,000	3.0	77,324	0.0	0.0	9.39	1	27.7
		5.0	78,944	0.0	0.0	12.08	1	27.1
		10.0	82,974	0.0	0.0	10.13	1	25.8
10x25x45x70	1,000 - 5,000	3.0	46,842	0.0	0.0	35.09	6	27.6
		5.0	48,630	16.5	0.0	58.18	6	26.5
		10.0	53,100	159.3	0.3	102.99	6	24.3
	5,000 - 10,000	3.0	69,625	1,036.8	1.5	71.63	3	29.3
		5.0	71,462	1,438.8	2.0	48.74	3	28.6
		10.0	76,052	3,432.3	4.5	95.76	3	26.8
	20,000 - 50,000	3.0	117,369	0.0	0.0	65.31	2	46.0
		5.0	119,309	0.0	0.0	70.71	2	45.3
		10.0	124,159	26.5	0.0	84.28	2	43.5
10x30x62x75	1,000 - 5,000	3.0	48,206	239.0	0.5	210.71	5	21.5
		5.0	50,161	534.6	1.1	277.25	6	25.5
		10.0	54,792	1,221.1	2.2	567.16	6	23.3
	5,000 - 10,000	3.0	67,187	22.7	0.0	110.57	4	36.8
		5.0	69,161	188.3	0.3	89.01	4	35.7
		10.0	74,030	701.3	0.9	147.26	4	33.4
	20,000 - 50,000	3.0	118,660	0.0	0.0	77.22	1	33.1
		5.0	120,762	0.0	0.0	93.04	1	32.5
		10.0	126,017	106.5	0.1	97.35	1	31.2
10x30x90x75	1,000 - 5,000	3.0	46,620	371.9	0.8	180.39	5	22.2
		5.0	48,602	678.7	1.4	197.47	5	21.3
		10.0	53,557	1,491.7	2.8	538.35	5	19.3
	5,000 - 10,000	3.0	63,238	24.5	0.0	177.95	3	27.2
		5.0	65,296	34.0	0.0	120.69	3	26.3
		10.0	70,407	204.0	0.2	132.43	3	24.4
	20,000 - 50,000	3.0	114,823	0.0	0.0	119.23	1	34.2
		5.0	116,995	0.0	0.0	133.25	1	33.6
		10.0	122,425	0.0	0.0	151.56	1	32.1

a : |I|, |J|, |E|, |K| denote the number of user nodes, candidate hub nodes, arcs and commodities respectively.

B : LP Gap = IP Optimal Value – LP Optimal Value.

c : % Gap = LP Gap / IP Optimal Value x 100 (%).

d : Hub Cost Ratio = Hub Establishment Cost / Total Cost x 100 (%).

b) Non-Euclidean Distance Case

|I|x|J|x|E| x|K|	Hub Cost	f Ratio	IP Optimum Value	LP Gap	% Gap (%)	Computation Time (Sec.)	# of Established Hubs	Hub Cost Ratio(%)
5x10x20x20	1,000 - 5,000	3.0	12,560	338.5	2.7	1.50	3	51.9
		5.0	13,084	501.0	3.8	1.70	3	49.9
		10.0	14,403	940.5	6.5	1.85	3	45.3
	5,000 - 10,000	3.0	18,293	0.0	0.0	0.90	2	75.7
		5.0	18,817	0.0	0.0	0.97	2	73.6
		10.0	20,136	32.5	0.2	1.03	2	68.8
	20,000 - 50,000	3.0	34,927	0.0	0.0	0.92	1	61.2
		5.0	35,447	0.0	0.0	0.89	1	60.3
		10.0	36,753	0.0	0.0	1.03	1	58.2
5x10x34x20	1,000 - 5,000	3.0	11,815	0.0	0.0	1.29	2	33.9
		5.0	12,367	44.4	0.4	1.53	2	32.4
		10.0	13,589	196.2	0.5	2.99	2	29.5
	5,000 - 10,000	3.0	16,720	0.0	0.0	1.73	1	45.0
		5.0	17,334	37.7	0.2	1.16	1	43.4
		10.0	18,622	59.0	0.3	1.62	1	28.9
	20,000 - 50,000	3.0	32,833	0.0	0.0	1.24	1	65.1
		5.0	33,402	0.0	0.0	0.94	1	64.0
		10.0	34,622	0.0	0.0	1.19	1	61.8
5x20x35x30	1,000 - 5,000	3.0	20,706	1.0	0.0	3.21	4	35.3
		5.0	21,380	24.0	0.1	2.56	4	34.1
		10.0	23,075	200.9	0.9	3.07	4	31.7
	5,000 - 10,000	3.0	32,141	0.0	0.0	2.96	2	34.6
		5.0	32,887	104.0	0.3	3.06	2	33.8
		10.0	34,724	333.0	1.0	3.63	2	37.2
	20,000 - 50,000	3.0	52,167	0.0	0.0	3.15	1	41.0
		5.0	52,945	0.0	0.0	3.16	1	40.4
		10.0	54,908	0.0	0.0	2.98	1	39.0
5x20x60x30	1,000 - 5,000	3.0	20,058	39.0	0.2	11.84	3	25.9
		5.0	20,754	58.5	0.3	10.68	3	25.0
		10.0	22,506	190.5	0.8	21.88	3	23.0
	5,000 - 10,000	3.0	28,912	3.0	0.0	12.27	2	38.5
		5.0	29,645	4.3	0.0	12.87	2	37.5
		10.0	31,496	48.5	0.2	11.28	2	35.3

|I|x|J|x|E| x|K|	Hub Cost	f Ratio	IP Optimum Value	LP Gap	% Gap (%)	Computa-tion Time (Sec.)	# of Estab-lished Hubs	Hub Cost Ratio(%)
	20,000 - 50,000	3.0	49,364	0.0	0.0	9.20	1	43.3
		5.0	50,168	0.0	0.0	10.52	1	42.6
		10.0	52,186	0.0	0.0	15.41	1	41.0
10x25x45x70	1,000 - 5,000	3.0	29,484	404.0	1.4	32.01	4	27.6
		5.0	30,401	653.5	2.1	33.25	5	35.0
		10.0	32,697	1178.7	3.6	61.61	5	32.6
	5,000 - 10,000	3.0	44,278	161.2	0.4	25.85	3	41.0
		5.0	45,195	295.4	0.7	31.30	3	40.2
		10.0	47,491	683.7	1.4	39.11	3	38.2
	20,000 - 50,000	3.0	73,428	0.0	0.0	26.26	1	36.1
		5.0	74,476	0.0	0.0	31.02	1	35.6
		10.0	77,064	0.0	0.0	29.70	1	34.4
10x30x62x75	1,000 - 5,000	3.0	29,281	6.4	0.0	102.28	5	35.4
		5.0	30,287	206.7	0.7	141.76	5	34.2
		10.0	32,745	702.0	2.1	246.14	5	31.7
	5,000 - 10,000	3.0	42,650	39.5	0.1	129.98	3	40.3
		5.0	43,704	157.3	0.4	84.57	3	39.4
		10.0	46,288	440.3	0.9	121.89	4	53.4
	20,000 - 50,000	3.0	74,358	0.0	0.0	120.72	1	38.5
		5.0	75,414	0.0	0.0	92.34	1	38.0
		10.0	78,032	0.0	0.0	137.83	1	36.7
10x30x90x75	1,000 - 5,000	3.0	28,437	252.7	0.9	164.86	4	30.7
		5.0	29,413	374.0	1.3	126.06	4	29.7
		10.0	31,827	654.7	2.1	238.52	4	27.4
	5,000 - 10,000	3.0	40,218	11.7	0.0	162.70	2	29.4
		5.0	41,246	14.7	0.0	228.15	3	41.8
		10.0	43,802	101.0	0.2	200.35	3	39.3
	20,000 - 50,000	3.0	72,012	0.0	0.0	147.69	1	35.3
		5.0	73,058	0.0	0.0	270.44	1	34.8
		10.0	75,674	0.0	0.0	178.61	1	33.6

a : $|I|$, $|J|$, $|E|$, $|K|$ denote the number of user nodes, candidate hub nodes, arcs and commodities respectively.

b : LP Gap = IP Optimal Value – LP Optimal Value.

c : % Gap = LP Gap / IP Optimal Value x 100 (%).

d : Hub Cost Ratio = Hub Establishment Cost / Total Cost x 100 (%).

from 5 hub nodes, 10 user nodes 45 arcs and 15 commodities to 10 hub nodes, 30 user nodes, 90 arcs and 40 commodities. The details of the associated computational results for randomly generated problems are summarized in Table 2 for both Euclidean and non-Euclidean distance cases.

The computation times are vary from a few seconds for the smaller problems to over 100 seconds for some of the larger problem sets. They increase with the problem size, i.e., the number of noces, arcs and commodities, and decrease as the hub establishment costs become larger. Noting the reality that the cost of hub facilities in telecommunication systems are much larger than the cable and conduit placement costs, our model may solve some large scale real-world problems within a few minuites on PC. Most of LP gaps reported are extremely small. That is the optimal value of the LP relaxed problem of $[P]$ is almost close that of $[P]$. It means there are some possibility to develop an efficient heuristic algorithm by applying a dual ascent method well-known as a powerful tool for very large scale network design problems[1, 7, 8, 11]. Table 2 also reports the number of established hubs and the ratio of hub establishment cost to the total cost. As expected, the number of established hubs decreases with the hub cost. We can not find any significant difference between Euclidean and non-Euclidean distance cases from Table 2.

We are unfortunately unable to find any other published studies on the same subject as ours for comparison of computational results. Despite the absence of comparision with other works, the computational results show that our model can be effectively applied to the real-world network design problems of this kind.

4. Discussions

We considered the network design problem, probably more practical and comprehensive than any design study in the literature, in that it, besides having the most general architecture of the logical full-mesh/star type, covers in one framework the following three issues: locating hub facilities, placing a conduit network, and installing cables within the established conduit network to configure the logical full-mesh/star network. Despite the inherent complexity, we successfully formulated the problem as a variant of the classical uncapacitated network design model, enabled by the network augmentation, judicious commodity definition therein, and some flow restrictions.

The reader's attention is again called upon to the fact that our design problem is so complex to include as subproblems in an integrated framework the two NP-complete problems: one on hub facility location and the other on design of an uncapacitated network. Despite the excessive computational burden expected for such complex network design problems, the optimal solution can be obtained within a few minuits for a large-sized network. Even though the computation time increases with the problem size, most of all problems can be solved by CPLEX program within an appropriate computation time. This means the practical problem can be tractable effectively with our model.

Even the CPLEX MIP program efficiently solves the large scale problem up to 10 hub nodes and 30 user nodes with our model, however, it needs to develop an efficient heuristic algorithm for more large scale problems. From the LP gaps in Table 2 and due to the resemblance of (P) to the classical network design models, the dual ascent method which is known very powerful for that class of problems [1, 2, 4, 8, 11] can be suggested as an efficient one. Further study for an efficient algorithm still remains to be done.

REFERENCES

[1] Balakrishnan, A., T.L. Magnanti and R.T. Wong, "A Dual-ascent Procedure for Large-scale Uncapacitated Network Design," *Operations Research*, Vol. 37, 716-740, 1989.

[2] Chung, S.H., Y.S. Myung and D. Tcha, "Optimal Design of a Distributed Network with a Two-level Hierarchical Structure," *European J. Operational Research*, Vol. 62, 105-115, 1992.

[3] Current, J.R., "The Design of a Hierarchical Transportation Network with Transshipment Facilities," *Transportation Science*, Vol. 22, 270-277, 1988.

[4] Lee, C.H., H.B. Ro and D. Tcha, "Topological Design of a Two-level Network with Ring-Star Configuration," *Computers and O.R.*, Vol. 20, No. 6, 625-637, 1993.

[5] Minoux, M., "Network Synthesis and Optimum Network Design Pro-blem: Models, Solution Methods and Applications," *Networks*, Vol. 19, 313-360, 1989.

[6] Pirkul, H. and V. Nagarajan, "Locating Concentrators in Centralized Computer Networks," *Annals of O.R.*, Vol. 36, 247-262, 1992.

[7] Tcha, D. and M.G. Yoon, "Conduit and Cable Installation for a Centralized Network with Logical Star-Star Topology," *IEEE Tr. on Communications*,

Vol. 45, 958-967, 1995

[8] Wong, R.T., "A Dual Ascent Approach for Steiner Tree Problems on a Directed Graph," *Mathematical Programming*, Vol. 28, 271-287, 1984.

[9] Wu, T.H., D.J. Kolar and R.H. Cardwell, "Survivable Network Architectures for Broadband Fiber Optic Networks: Model and Performance Comparison," *J. of Lightwave Technology*, Vol. 6, 1698-1709, 1988.

[10] Wu, T.H., *Fiber Network Service Survivability*, Artech House, 1992.

[11] M.G. Yoon, Y.H. Baek and D.W. Tcha, "Optimal Design of a Distributed Fiber Transport Network with Hubbing Topology," *European Journal of Operational Research*, Vol. 104, 510-520, 1998.