International Journal of Management Science
Vol 6, No 2, November 2000

MANUFACTURER'S PROCUREMENT DECISION ANALYSIS IN A SUPPLY CHAIN WITH MULTIPLE SUPPLIERS

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(Received May 2000 ; revision received September 2000)

ABSTRACT

Supply chain management issues faced by a manufacturing company are considered in this paper. The supply chain consists of a manufacturing company and its suppliers. The manufacturer produces multiple products with inputs (e.g., raw materials) from the suppliers, but each product needs a different mix of these inputs. The market demand for the products is uncertain. We develop a mathematical model and algorithm, which can help the manufacturer to solve its procurement decision problem: *how much of raw material to order from which supplier*. The model incorporates such factors as market demand uncertainty, product's input requirement, supplier's as well as manufacturer's capacity, plus other costs comparable with those in a typical newsboy problem. Numerical examples are presented to see the interacting effects among critical parameters and variables.

1. INTRODUCTION

It is an important issue in supply chain management for a manufacturing company to satisfy the market demand as much as possible by balancing the overstocking and understocking costs (Ellarm and Cooper 1990, Rockhold, et al. 1998, Fisher and Raman 1996). For effective supply chain management, however, there is more to consider than just the manufacturer's ability to match its supply with the market demand. In this paper, we consider three levels of a supply chain: the market, the manufacturing company, and suppliers for the manufacturer. The manufacturer's efforts to match the supply with the demand might turn out to be futile unless its suppliers support the manufacturer's strategy in some way.

Our research was motivated by the problem of deciding appropriate stocking and/or production levels of products for a manufacturing company: the manufacturer procures raw materials from multiple suppliers and sells the final products to the market. The market demand is uncertain by nature. Because of this uncertainty, the manufacturing company has to deal with issues of overstocking and understocking simultaneously. In addition, the manufacturer must take into account its suppliers' production/supply capacity. That is, no matter how much eager the manufacturer is to procure from a particular supplier (possibly due to low price and/or high quality of the supply), it can not get all of what it wants unless the supplier's capacity permits.

Consider a computer assembler. Customers in the computer market demand diverse specifications such that the market demand uncertainty is very significant. Since each computer consists of different components, the computer assembler has to procure from multiple suppliers, each supplying all or part of the components at varying prices. When making decisions for appropriate stocking or production levels, the computer assembler also needs to take into account its suppliers' supply capacity. Here the computer assembler must decide *how much of which component to procure from which supplier* for each decision horizon. This is the kind of context we focus on in this research.

This paper is organized as follows. In the next section, we briefly review the literature. Section 3 develops a mathematical model to solve the decision problem for the manufacturing company (such as the computer assembler in our example above). We also propose an algorithm for solving the mathematical problem more efficiently. Numerical examples are provided in Section 4. Two sets of numerical examples are considered: the first aims to demonstrate how to apply the mathematical algorithm to a complex supply chain problem, while the other tries to derive concrete answers to a real-world problem faced by a computer retailer. Finally, we suggest critical managerial implications along with future directions for this line of research.

2. Literature Review

There is an extensive literature on supply chain management: in this section, we focus on the research directly relevant to our model. Nahmias and Schmidt (1984) developed a heuristic algorithm for solving the multi-item newsboy problem by setting up a mathematical model, which dealt with only one constraint, *a single resource (capacity) constraint*. Since it examined only the manufacturer's capacity constraint, it effectively ignored other external factors, e.g., the supplier's capacity constraint. Our mathematical model can be regarded as an extension of Nahmias and Schmidt's. Although like Nahmias and Schmidt's our model takes into account the stochastic nature of the market demand and other relevant cost factors such as understocking and overstocking costs, there exist some significant differences between the two models. That is, we consider multiple constraints including *not only* the manufacturer's capacity constraint, *but also* the product's input requirement and the supplier's capacity constraint.

Researchers have tried to develop mathematical models, exploring the similar extension. In a context similar with ours, Harrison and Van Mieghem (1999) studied dynamic capacity investment decisions, focusing on an operational hedging under demand uncertainty. They considered a firm, which markets multiple products, using several resources and facing product demand uncertainty: see also Li and Tirupati (1994). Concentrating on the contractual aspects of supply chain management, Anupindi and Bassok (1999) suggested several mathematical models, each of which deals with such issues in supply chain management as quantity commitment (incorporating the manufacturer's and supplier's capacity constraints), market demand uncertainty, flexibility, pricing, quality, information sharing, so forth. For a more general literature review, see Kapuscinski and Tayur (1999).

In a more general context, we review references focusing on such issues as stochastic market demand, coordination and information sharing, and strategic success factors for supply chain management. Fisher (1997) suggested that the supply chain strategy should be differentiated according to the characteristics of products demanded in the market: for instance, he emphasized such factors as demand stability, predictability of the uncertain market demand, inventory, and stockout costs. In fact, the market demand uncertainty has been widely studied in the literature. Davis (1993) postulated that there exists uncertainty at each stage of the supply chain. Agrawal and Nahmias (1997) tried to determine the optimal number of suppliers when there exists a supply uncertainty by balancing the tradeoffs between economies of scale and the stockout risk due to the limit on supply capacity.

In relation to the uncertainty, managing information in the supply chain has been a critical issue as well. Lee, et al (1997a, 1997b) extensively surveyed the information distortion phenomena in the supply chain management and suggested a few practical remedies that can be used to cope with such situations. Viewing the supply chain management as an issue concerning multiple supply chain participants rather than the manufacturer only, many researchers have underlined the importance of coordination among the chain members. Based on their case studies, Lee and Billington (1992) put forth that the supply chain is a network coordinating various chain participants in the value creating activities from raw material procurement, transformation, to distribution to the customers. Hines (1998) proposed that the first-tier suppliers should receive the highest attention because of their critical role in helping the manufacturer coordinate more effectively among suppliers in different tiers. By looking into the sensitivity of the value of information, Gavirneni, et al. (1999) explored the effect of information sharing between supply chain partners. Lee and Whang (1999) suggested measures to optimize the supply chain performance even when the decision-making rights are decentralized.

In a broader context, Beamon (1998) reviewed various issues in supply chain management: Mabert and Venkataramanan (1998) were more specific in casting the supply chain management as an issue of managing supply chain linkages. Monczka, et al. (1998) surveyed companies in strategic supplier alliances and provided a list of success factors for such cooperative relationships. In order to determine the variables that affect the supply chain performance, Narasimhan and Jayaram (1998) investigated manufacturing firms in North America, using statistical approaches. In a similar vein, Ross, et al. (1998) employed DEA (Data Envelopment Analysis) to reconfigure a supply network in a real world case. Other researchers used a wide range of mathematical tools: an LP model was adopted by Kalakota, et al. (1998), while Arntzen, et al. (1995) utilized the mixed integer programming.

Our research proposes a supply chain model that includes such factors as market demand uncertainty, product characteristics like different input compositions, capacity limits of suppliers as well as the manufacturer, along with other cost elements. In effect, we are taking into account multiple factors for effective supply chain management: most of these factors have been identified as critical in the literature.

3. A Mathematical Model for Supply Chain Management

Before developing the mathematical model, we elaborate more on the research context for the analytical model.

3.1 A Supply Chain Context

Suppose there is a manufacturing company. The company manufactures (e.g., assembles) and sells k different products using n different kinds of raw materials that are supplied by m suppliers (Figure 1). The decision problem faced by the manufacturer is to determine *how much of each raw material to procure from which supplier in order to maximize its profit during the current decision horizon*: in fact, this is the manufacturer's objective. The problem gets complicated if the market demands for the products are probabilistic, i.e. demand uncertainty is involved. One can find this kind of supply chain setting easily in the real world situation. For instance, the manufacturer can be a computer company that sells assembled computers to its customers, by assembling parts procured from multiple suppliers.

For a complete decision model, we need to consider relevant variables and parameters associated with such factors as supply costs, suppliers' capacity, manufacturer's own production capacity, information regarding market demand for the products, overstocking and under-stocking costs, and the like.

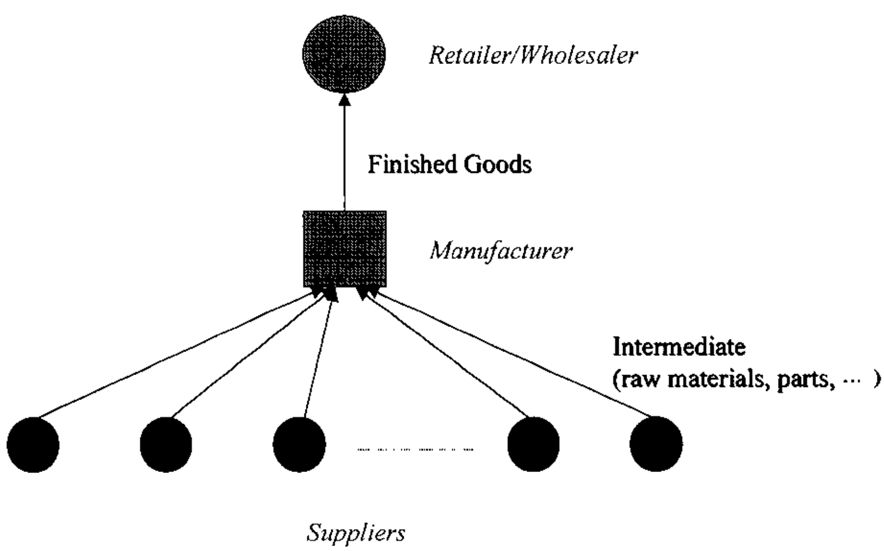


Figure 1. Research context – the supply network in this research

3.2 Variables, Parameters, and Other Notations

Figure 1 shows the supply network context for which we build our analytical model. In order to develop the mathematical model, we continue to elaborate on the supply chain setting laid out in the previous section. As mentioned already, the manufacturer produces y_l units of product l, each of which generates the net sales revenue of r_l, $l=1,\cdots,k$. One unit of product l requires b_{il} of raw material i, $i=1,\cdots,n$. That is, the manufacturer needs n different types of raw materials. There are m suppliers available, who provide the manufacturing company with the needed materials: we denote each supplier with j, $j=1,\cdots,m$. Another key decision variable for the manufacturer is x_{ij}, which represents the amount of raw material i purchased from supplier j. For each unit of raw material i supplied by j, the manufacturer pays c_{ij}.

Assembling one unit of product l inside the manufacturing company consumes 'productive resources' by h_l, which can represent either production cost or necessary space, for instance. We further assume that such resources are limited by Q. In addition, we take into account the suppliers' resource constraints: v_{ij} represents the amount of supplier j's internal resources, that are required to process/produce one unit of raw material i for supplying it to the manufacturer. We assume that such resources owned by supplier j are limited by q_j.

As already mentioned, market demands for the assembled products follow a probability distribution. Let Z_l denote the random variable of the demand for product l. For the purpose of our analysis, we assume that the probability density function of Z_l, $f_{Z_l}(z_l)$, follows a normal distribution with mean ξ_l and standard deviation σ_l (Nahmias and Schmidt 1984). That is, $Z_l \sim N(\xi_l, \sigma_l^2)$.

Suppose z_l is the actual amount of market demand for product l and y_l the amount of product l produced by the manufacturer. If it turns out that the manufacturer produces too little of product l, it is left with an unsatisfied demand of $z_l - y_l$. On the other hand, if it produces too much, it ends up with an amount of $y_l - z_l$ as unsold goods. Each of these cases is undesirable: the first situation involves goodwill costs due to understocking, while the latter overstocking costs. Let u_l represent the unit understocking cost and w_l the unit overstocking cost associated with product l. Definitions of key variables, parameters, and other notations for the research model are summarized in Appendix 1.

Before developing our mathematical model and algorithm, summarizing key assumptions in this paper is in order. First, the decision making process is assumed as follows. At the beginning of the current decision horizon, the manufacturer knows only the stochastic distribution of the market demand for each assembled product without knowing the actual demand quantities. Then, the company has to decide how much of each raw material to order from which supplier(s), conditioned by the constraint on its internal processing capacity. Receiving orders from the manufacturer, suppliers process the ordered amounts of supply, the maximum amounts being constrained by their own capacities. Thus, even before placing orders, the manufacturer has to take into consideration the capacity/resource constraints not only of its own, but also of its suppliers. After obtaining all of the ordered materials from the suppliers, the manufacturing company assembles the products. Now at the end of the current decision horizon, the actual market demand for the products becomes realized. Without loss of generality in the context of our supply network, factors such as lead-time and delivery delay from suppliers to the manufacturer are not considered in our model. Since our current research is more focused on the dynamics among the manufacturer's and suppliers' capacity constraints facing the stochastic market demand, information asymmetry and distortion are not considered here although very important for supply chain management in general.

3.3 Defining the Mathematical Model

With the notations developed above, we can formulate the decision problem, $\underline{\mathrm{P}}$, as follows. Note that in this mathematical model, the manufacturer's objective is to maximize its profit by optimally deciding *how much of each raw material to procure from which supplier during the current decision period.*

$$\underline{\mathrm{P}}: \quad \underset{\substack{y_l, l=1,\cdots,k \\ x_{ij}, i=1,\cdots n, j=1,\cdots m}}{Maximize} \quad \sum_{l=1}^{k} \left\{ \int_{0}^{y_l} \left[r_l z_l - w_l (y_l - z_l) \right] f(z_l) dz_l + \int_{y_l}^{\infty} \left[r_l y_l - u_l (z_l - y_l) \right] f(z_l) dz_l \right\}$$

$$- \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij} \tag{1}$$

$$\textit{Subject to} \quad \sum_{l=1}^{k} b_{il} y_l \le \sum_{j=1}^{m} x_{ij}, \quad i = 1, \cdots, n \tag{2}$$

$$\sum_{i=1}^{n} v_{ij} x_{ij} \le q_j \,, \quad j = 1, \cdots, m \tag{3}$$

$$\sum_{l=1}^{k} h_l y_l \le Q \tag{4}$$

$$x_{ij} \ge 0 \,, \quad y_l \ge 0 \ \text{ for } \ i = 1, \cdots, n \,,$$
$$j = 1, \cdots, m \,, \quad l = 1, \cdots, k \tag{5}$$

The objective function in (1) consists of three parts. The first part is the expected value of the profit from product l when the realized demand of the product is less than the actual quantity produced, whereas the second part represents the expected profit when the manufacturing company underproduced product l. The first two parts are summed over the entire products. The last part of the objective function represents the total costs the manufacturing company pays to its suppliers for the raw materials. Thus, it is subtracted from the sum of the net profits.

The first set of constraints, (2), relates to the resource requirements for producing and/or assembling the products. Constraints in (3) show the capacity limitations faced by the suppliers. Finally, the third constraint in (4) captures the manufacturer's own capacity limitation. In addition, we have the nonnegativity constraints in (5).

3.4 Characterizing the Solution

In theory, we can solve P by using the Karush-Kuhn-Tucker (KKT) conditions. However, as it turns out, it is practically inefficient to try to get a closed solution of the problem by applying the KKT conditions in earnest. The primary motivation for us to identify the KKT conditions is *not* to find out the optimal solution, *but* to gain valuable 'mathematical insights,' with which we can improve our analytical algorithm. We will be able to do that by thoroughly examining the dynamic relationship among key parameters and variables (or, characterizing the optimal solution), which is revealed by the KKT conditions.

Keeping in mind the motivation or reason why we first look at the KKT solution, we derive the first order necessary conditions for an optimal solution. We need to establish the Lagrangian as follows:

$$L = \sum_{l=1}^{k}\left\{\int_{0}^{y_l}[r_l z_l - w_l(y_l - z_l)]f_{Z_l}(z_l)dz_l + \int_{y_l}^{\infty}[r_l y_l - u_l(z_l - y_l)]f_{Z_l}(z_l)dz_l\right\} - \sum_{i=1}^{n}\sum_{j=1}^{m}c_{ij}x_{ij}$$
$$+\sum_{i=1}^{n}\lambda_i\left(\sum_{j=1}^{m}x_{ij} - \sum_{l=1}^{k}b_{il}y_l\right) + \sum_{j=1}^{m}\mu_j\left(q_j - \sum_{i=1}^{n}v_{ij}x_{ij}\right) + \eta\left(Q - \sum_{l=1}^{k}h_l y_l\right), \tag{6}$$

where λ_i is the Lagrangian multiplier for raw material i, μ_j for supplier j, and η for the manufacturer. With L defined above, we can obtain the following set of first order conditions for optimality. Assuming $Z_l(z) = 0$ for $z < 0$,

$$\frac{\partial L}{\partial y_l} = -w_l F(y_l) + (r_l + u_l)(1 - F(y_l)) - \sum_i \lambda_i b_{il} - \eta h_l \le 0 \text{ and } y_l \ge 0 \tag{7}$$

By rearranging (7), we obtain (8):

$$\frac{r_l + u_l - \sum_i \lambda_i b_{il} - \eta h_l}{r_l + u_l + w_l} \le F(y_l) \tag{8}$$

Thus, the first KKT condition for the optimality is

$$\left[\left\{r_l + u_l - \sum_i \lambda_i b_{il} - \eta h_l\right\} - (w_l + r_l + u_l)F(y_l)\right]y_l = 0. \tag{9}$$

Condition (9) implies that if $y_l > 0$, then $\frac{r_l + u_l - \sum_i \lambda_i b_{il} - \eta h_l}{r_l + u_l + w_l} = F(y_l)$ (10)

If (10) holds, we can evaluate y_l as follows:

$$y_l = F^{-1}\left(\frac{r_l + u_l - \sum_i \lambda_i b_{il} - \eta h_l}{r_l + u_l + w_l}\right) \tag{11}$$

Another key first order condition is obtained by partially differentiating L with regard to x_{ij}, i.e.

$$\frac{\partial L}{\partial x_{ij}} = -c_{ij} + \lambda_i - \mu_j v_{ij} \le 0, \quad x_{ij} \ge 0, \quad (\lambda_i - c_{ij} - \mu_j v_{ij})x_{ij} = 0. \tag{12}$$

When $x_{ij} > 0$, it must hold that $\lambda_i = c_{ij} + \mu_j v_{ij}$.

Before continuing to derive the remaining parts of necessary conditions, we would like to characterize the optimal solutions based on (11) and (12).

Observation 1: Characterizing y_l^*. In order for (11) to be meaningful, it needs to hold that $r_l + u_l \geq \sum_i \lambda_i b_{il} + \eta h_l$. (11b)

Referring to Appendix 1 for relevant definitions, we can posit that $r_l + u_l$ represents the incremental benefit the manufacturer could earn by producing one more unit of y_l, *assuming the additional unit is sold in the market.* That is, by selling the additional unit, the company will receive r_l from the customer, and save u_l because of avoiding the stock-out for the additional unit. The right hand side of (11b) denotes the marginal cost to produce the additional unit. Since λ_i is the unit marginal price of material i, $\sum_i \lambda_i b_{il}$ represents the total cost of resources needed to produce one unit of y_l. In addition, the manufacturer has to consume h_l of its own internal resources (e.g., production process and/or plant space) whose unit marginal value is η. Therefore, we can conclude the right hand side of (11b) is the total cost, both *external and internal*, to produce one unit of y_l.

In essence, (11b) postulates that unless the marginal value of y_l is larger than the marginal cost, the manufacturer should not produce the last unit of y_l. Finally, y_l^* is determined after the adjustment related with the overstocking cost, w_l, is done as in the denominator of (11). We can see that as the overstocking cost increases, y_l^* decreases, i.e., $\frac{\partial y_l^*}{\partial w_l} < 0$. Similarly, we can easily show that $\frac{\partial y_l^*}{\partial r_l} > 0$, $\frac{\partial y_l^*}{\partial u_l} > 0$, $\frac{\partial y_l^*}{\partial b_{il}} < 0$, and $\frac{\partial y_l^*}{\partial h_l} < 0$: the interpretations are consistent with the observation above.

Observation 2: Characterizing x_{ij}^*. As shown in (1)-(5), the manufacturer's optimal solution is affected by *not only* its own production decision *but also* its suppliers' cost structure and production capacity. From (12), we have another necessary conditions that must hold for an optimal solution, i.e., $\lambda_i = c_{ij} + \mu_j v_{ij}$, given that the supply i supplied by the supplier j has a positive economic value.

The condition $\lambda_i = c_{ij} + \mu_j v_{ij}$ ensures that at the optimal solution, the manufacturer procures x_{ij} from the suppliers so that the supplier j's marginal supply cost for a unit of supply i becomes equal to the manufacturer's marginal value of supply i, for $j=1,\ldots,m$. Recall that v_{ij} is the supplier j's internal cost of resources needed to produce a unit of supply i, μ_j the marginal price of the internal resources, and c_{ij} the manufacturer's procurement cost for supply i from supplier j. In effect, the manufacturing company's production decision becomes intricately connected with the suppliers' production constraints.

Now we continue to derive the remaining necessary conditions associated with λ_i, μ_j, and η as follows.

For λ_i, $\frac{\partial L}{\partial \lambda_i} = \sum_j x_{ij} - \sum_l b_{il} y_l \geq 0$, $\lambda_i \geq 0$, $\lambda_i\left(\sum_j x_{ij} - \sum_l b_{il} y_l\right) = 0$ (13)

If $\lambda_i > 0$, $\sum_j x_{ij} = \sum_l b_{il} y_l$.

For μ_j, $\frac{\partial L}{\partial \mu_j} = q_j - \sum_i v_{ij} x_{ij} \geq 0$, $\mu_j \geq 0$, $\mu_j\left(q_j - \sum_i v_{ij} x_{ij}\right) = 0$ (14)

If $\mu_j > 0$, $q_j = \sum_i v_{ij} x_{ij}$.

Finally, for η, $\frac{\partial L}{\partial \eta} = Q - \sum_{l=1}^{k} h_l y_l \geq 0$, $\eta \geq 0$, $\eta\left(Q - \sum_{l=1}^{k} h_l y_l\right) = 0$ (15)

If $\eta > 0$, $Q - \sum_{l=1}^{k} h_l y_l = 0$.

Equations (7)-(15) constitute the KKT first order conditions for the optimal solution.

As pointed out already, we should be able to solve $\underline{\mathrm{P}}$ so that we derive a set of optimal values of the decision variables and Lagrangian multipliers that satisfy the first order conditions. However, doing so could involve so much computational effort that it takes an enormous amount of time and cost even with a highly capable computer.

If we carefully look at each of the necessary conditions, we can see that each needs to satisfy a type of complementary slackness condition. For instance, from (13), we have $\frac{\partial L}{\partial \lambda_i} \geq 0$, $\lambda_i \geq 0$, and $\lambda_i\left(\sum_j x_{ij} - \sum_l b_{il} y_l\right) = 0$. Therefore, for (13) we need to think about two separate cases, $\frac{\partial L}{\partial \lambda_i} = 0$ or $\lambda_i = 0$. Considering the number of equations we have to take into account for our problem is $k + m \times n + n + m + 1$, we must evaluate $2^{k+mn+n+m+1}$ cases simultaneously. Assuming the problem has a moderate size of $k = m = n = 10$, the total number of cases to assess becomes $2^{131} \cong 10^{39}$. Thus, it will take quite a long time for even a reasonably efficient computer to solve this size of problem.

3.5 Problem Partition and a Mathematical Algorithm

Because of this potentially excessive requirement of computation, we had better seek for a simpler way to solve the problem. One possibility is to partition the original problem into smaller, more manageable, ones. It is possible because $\underline{P}$ has a special structure. That is, we can separate $\underline{P}$ into two, $\underline{P}_1$ and $\underline{P}_2$, as follows, assuming that we obtain y_l^* s in the first part, $\underline{P}_1$.

$$\underline{P}_1: \underset{y_l, l=1,\ldots,k}{Maximize} \quad \sum_{l=1}^{k} \left\{ \int_0^{y_l} [r_l z_l - w_l(y_l - z_l)] f(z_l) dz_l + \int_{y_l}^{\infty} [r_l y_l - u_l(z_l - y_l)] f(z_l) dz_l \right\}$$

$$Subject\ to \quad \sum_{l=1}^{k} h_l y_l \le Q$$

$$y_l \ge 0 \text{ for } l = 1, \ldots, k.$$

By solving $\underline{P}_1$, we obtain y_l^* s. We can use these y_l^* s in constructing the second part of the original problem, i.e., $\underline{P}_2$.

$$\underline{P}_2: \underset{x_{ij}}{Minimize} \quad \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

$$Subject\ to \quad \sum_{l=1}^{k} b_{il} y_l^* \le \sum_{j=1}^{m} x_{ij}, \qquad i = 1, \cdots, n$$

$$\sum_{i=1}^{n} v_{ij} x_{ij} \le q_j, \qquad j = 1, \cdots, m$$

$$x_{ij} \ge 0 \text{ for } \qquad i = 1, \cdots, n, \quad j = 1, \cdots, m$$

The special structure of the original problem makes it possible to work with such a partition. The solution procedure is as follows: first, we solve $\underline{P}_1$ and obtain y_l^* s, which are used in the constraints of $\underline{P}_2$. Once we attain y_l^* s and plug them in $\underline{P}_2$, the second part of the problem, i.e., $\underline{P}_2$, becomes just an ordinary linear programming problem. As long as feasible, solving $\underline{P}_2$ given y_l^* s should be straightforward. Even when $\underline{P}_2$ appears to be infeasible, we can eventually obtain optimal solutions satisfying both $\underline{P}_1$ and $\underline{P}_2$, by adjusting Q in $\underline{P}_1$ so that we get smaller y_l^* s. This procedure is described in Figure 2.

However, the solution derived through this algorithm is an approximation at best. Recall the original problem, $\underline{P}$, and its constraints in (2)~(5). The truly optimal solution has to consider the revenue and cost parts in the objective function simultaneously, not sequentially as in our partitioning approach: the optimal amount of production, i.e., y_l^*, should depend on not only the revenue, but also the material cost side. Thus, partitioning the whole problem into the revenue and the cost-related ones may not be as accurate as the holistic method. Nevertheless, we think our approach engenders a good approximation: it should be true, especially when the limit of suppliers' capacity is not reached, that is, the constraint (3) is not binding. As shown later in our numerical examples, this is not an unusual requirement. But, one must be more cautious in interpreting the algorithm's implications when the constraint (3) becomes binding.

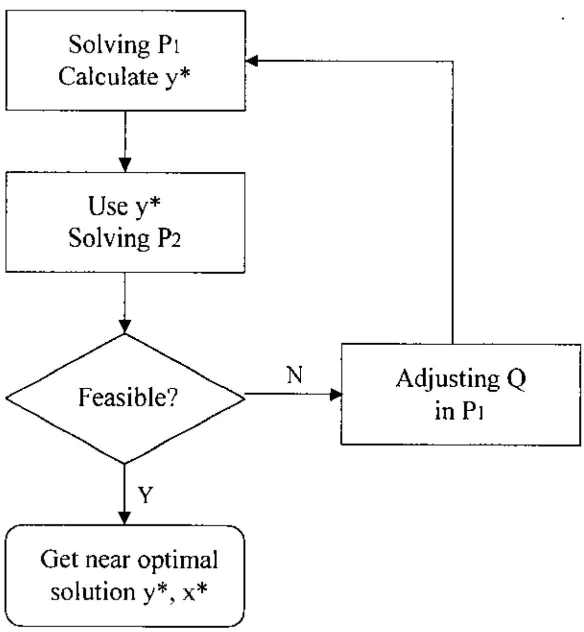


Figure 2: The mathematical algorithm

Before proceeding to the numerical examples, we elaborate more on our numerical analysis technique solving $\underline{P}_1$. First, we employ the most widely-used method, *Newton-Raphson method,* when solving the nonlinear equations. In order to simplify the solution process, when deriving the KKT conditions for $\underline{P}_1$, we implicitly impose $\sum_{l=1}^{k} h_l y_l \approx Q$. Imposing this enables us to reach the *near-optimal* solution much faster. This method should not distort the solution significantly as long as Q is a valuable resource so that the firm tries to utilize it as much as possible.

Once we solve $\underline{P}_1$, the rest of the algorithm progresses straightforward since $\underline{P}_2$ is just an ordinary LP problem.

We recapitulate the entire solution procedure as follows.

Step 1. *Partition* $\underline{P}$ *into two,* $\underline{P}_1$ *and* $\underline{P}_2$.

Step 2. *Use the Newton-Raphson method to solve* $\underline{P}_1$ *and calculate* y_l^*.

Step 3. *With* y_l^*, *solve* $\underline{P}_2$. *If* $\underline{P}_2$ *is infeasible, go to Step 4. Otherwise go to Step 5.*

Step 4. *Adjust* Q. *Reduce it by a predetermined magnitude. Go to Step 2.*

Step 5. *Calculate* x^*.

Step 6. *Report an optimal solution* (y_l^*, x^*).

4. Numerical Examples and Analysis

This section shows how we can utilize the mathematical algorithm laid out in the previous section. In order to demonstrate the capability of the algorithm, we first present an example with multiple products, raw materials, and suppliers. Then, we focus on a real world case so as to highlight the practical applicability of our model, and thus draw managerial insights as well.

Using the mathematical algorithm developed in the previous section, we can present many different types of numerical examples by changing the parameter values. For the purpose of our paper, however, we concentrate on the following relationships:

(i) *(how the optimal value of the objective function changes as the manufacturer's capacity* (Q) *changes;*

(ii) *manufacturing company's procurement amount from each supplier given a particular* Q *;*

(iii) *total material (procurement) cost of each product;*

(iv) *production quantity of each product given a particular* Q *;*

(v) *how the change in a product's demand uncertainty affects the other products' production quantities;*

(vi) *how the change in a supplier's capacity affects the manufacturer's procurement amounts from other suppliers.*

4.1 Numerical Examples

Applying the mathematical algorithm, we present numerical examples of a supply network consisting of 5 products, 5 suppliers, and 5 different raw material types. For the following numerical examples, we use the parameter values in Table 1 and Appendix 2.

Table 1. Product-related Parameters

l	*1*	*2*	*3*	*4*	*5*
r_l	\$150	200	220	230	250
h_l	1	2	2	2	3
μ_l	200	160	180	160	200
σ_l	80	60	70	60	80
u_l	\$100	90	50	90	150
w_l	\$60	40	20	10	100

First, we calculate the optimal objective values by varying the manufacturer's production capacity, Q. Figure 3 shows that at about $Q = 2150$, the largest optimal value is obtained, and after that point the objective value remains the same.

For the remaining examples, thus, we are mainly concerned with the capacity range of $Q \leq 2150$. Figure 4 shows the amounts of the manufacturing company's procurement from the suppliers, given $Q = 2100$. It is interesting to observe that each raw material is procured from one supplier only. It is consistent with the industry practice: manufacturing companies select the most efficient (e.g., low-cost) supplier among many for one type of supply when there are no other qualitative considerations such as strategic importance, long-term relationship building, and the like (Rosenblatt, et al. 1998). But, as we will see later in this section, this pattern of 'one supplier for one material type' will change as the suppliers' capacity limit is reached.

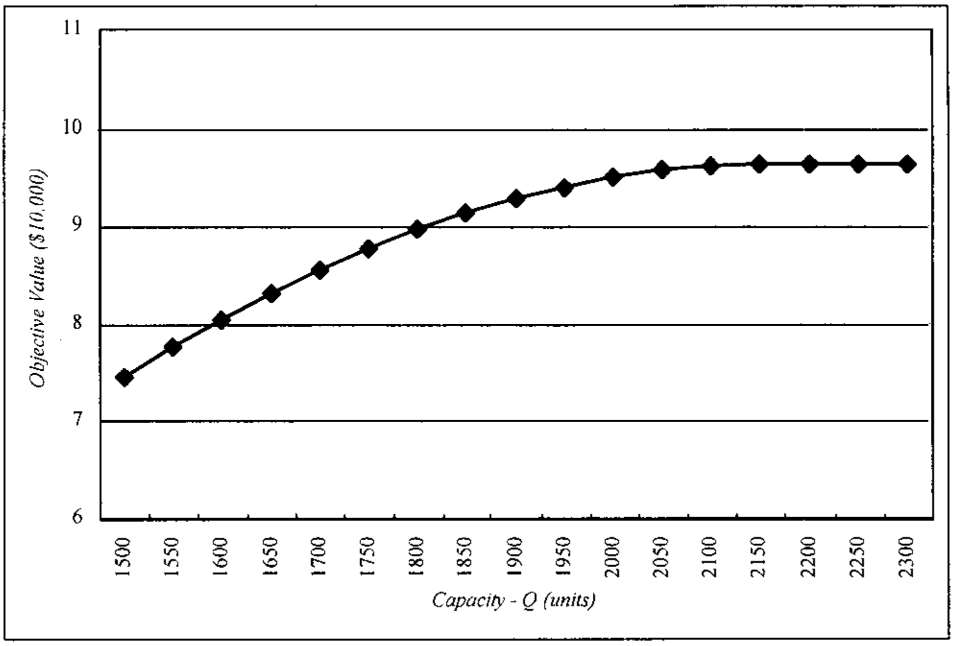
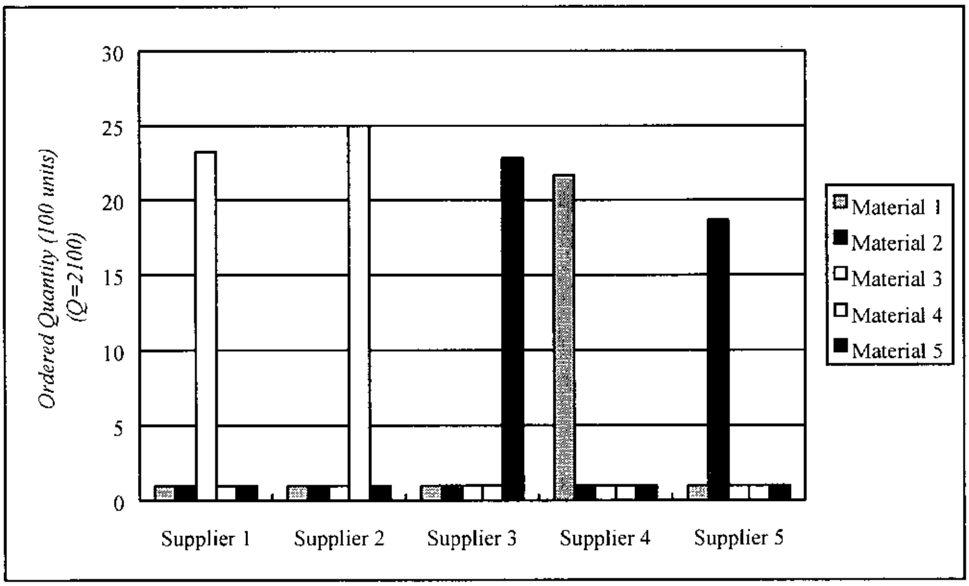


Figure 3: Manufacturing Company's Net Profit as a Function of Capacity, Q



Note : For the simple representation (for better contrast), we use the short bars (with the height of 1) to represent the case that 'none' is procured.

Figure 4. Supply Quantities from Suppliers

	Unit Revenue (r_l)	*Internal Resource*	*Total Supply*	*Revenue-Cost*
y_1	\$150	1	\$47	3.19
y_2	\$200	2	\$51	3.92
y_3	\$220	2	\$53	4.15
y_4	\$230	2	\$56	4.11
y_5	\$250	3	\$68	3.68

Note: The 'total supply cost' in this table is a simple summation of the raw material costs, and different from the 'actual total cost' of the product. The 'total cost' of a product should also include costs related with production capacity and associated opportunity costs. Here we simply focus on the ratio between a product's revenue and the cost of necessary raw materials.

Figure 5. Material Cost for Each Product

Using Figure 4 and tables in Appendix 2, we can calculate the total cost of materials supplied by the suppliers for each product. Figure 5 tabulates the total supply (material) cost along with the unit revenue for each product. The ratio of revenue to supply cost can be regarded as the measure of each product's relative profitability: the higher the ratio, the more profitable. For instance, if we just look at the revenue-cost ratio, product 3 is the most profitable. But, we also need to take into account the internal resource requirement denoted by h_l: although product 1 seems least profitable, it can still be preferred once its low value of h_1 is considered.

Figure 6 shows how the production amount of each product varies as the manufacturer's capacity changes. First, we can see that product 1 is produced in the largest amount, whereas product 2 in the smallest amount. This is somewhat surprising should we just consider the revenue-cost ratio in Figure 5. But, as we pointed out already, product 1 consumes the least amount of the manufacturer's capacity, Q. Thus, if the marginal value of Q is very high, the total production cost (including raw material cost and the opportunity cost of the manufacturer's capacity) of product 1 can be cheaper than others.'

This explanation seems plausible since the increasing rate of product 1's production amount is flatter than other products' as Q increases: an increase in Q implies that the marginal value of the manufacturer's production capacity decreases. On the other hand, the increasing rates of production of product 3 and product 5 are steeper than others: we conclude that as Q increases, it becomes more attractive to produce product 3 and 5 which consume more manufacturer's capacity, although more profitable according to their revenue-cost ratios.

It is interesting to see that product 5 is produced more than product 3 and 4.

The revenue-cost ratio clearly indicates that product 3 and 4 are more profitable than product 5, and also product 5 consumes more internal production resources than the others do (i.e., $h_5 = 3$ while $h_3 = 2$ and $h_4 = 2$). In order to grasp this counterintuitive phenomenon, we need to consider the demand characteristics: the average demand for product 5 is higher than those for the other two, i.e., $\mu_5 = 200$ whereas $\mu_3 = 180$ and $\mu_4 = 160$.

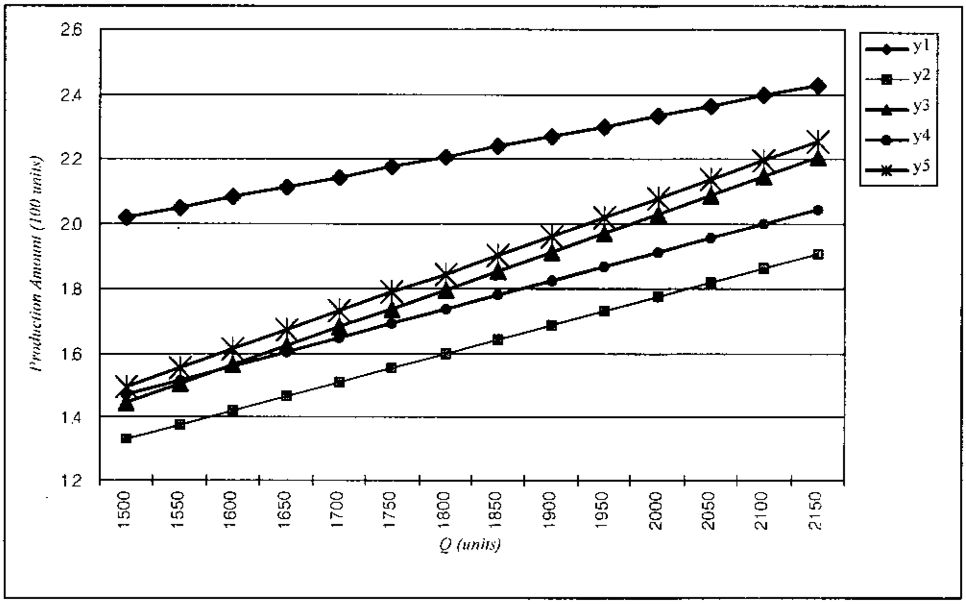


Figure 6. Each Product's Production Amount as a Function of Q

In order to see how the demand uncertainty represented by its standard deviation affects the production decision, we vary the standard deviation of product 4 and show the optimal production amounts of the products in Figure 7. It is already expected that the product whose demand uncertainty increases (e.g., y_4) is produced less: but, it is intriguing to observe that its effect on other product's production amount differs across the products. It seems that product 3 and 5 are those that receive the most benefit from the increasing demand uncertainty of product 4. We can infer that the situation observed in Figure 7 is qualitatively similar with that in Figure 6: the fact that the production amount of product 4 decreases due to its increased uncertainty is equivalent to that the portion of Q available to other products increases. Now, as the effective amount of Q increases, we can expect the manufacturing company to produce product 3 and 5 more than others as in Figure 6.

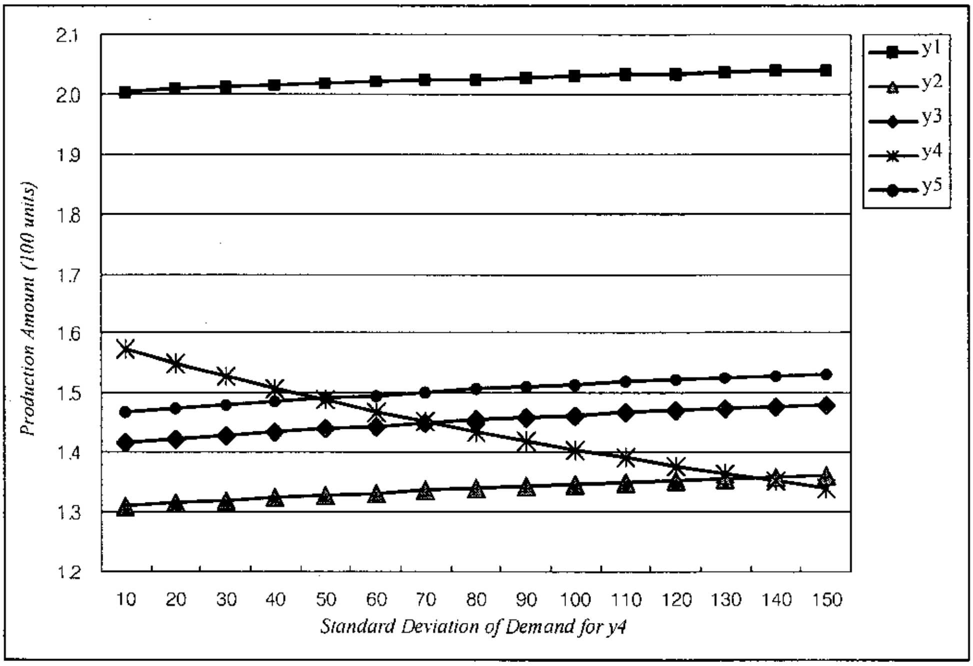


Figure 7. Each Product's Production Amount as y_4's Demand Uncertainty Varies

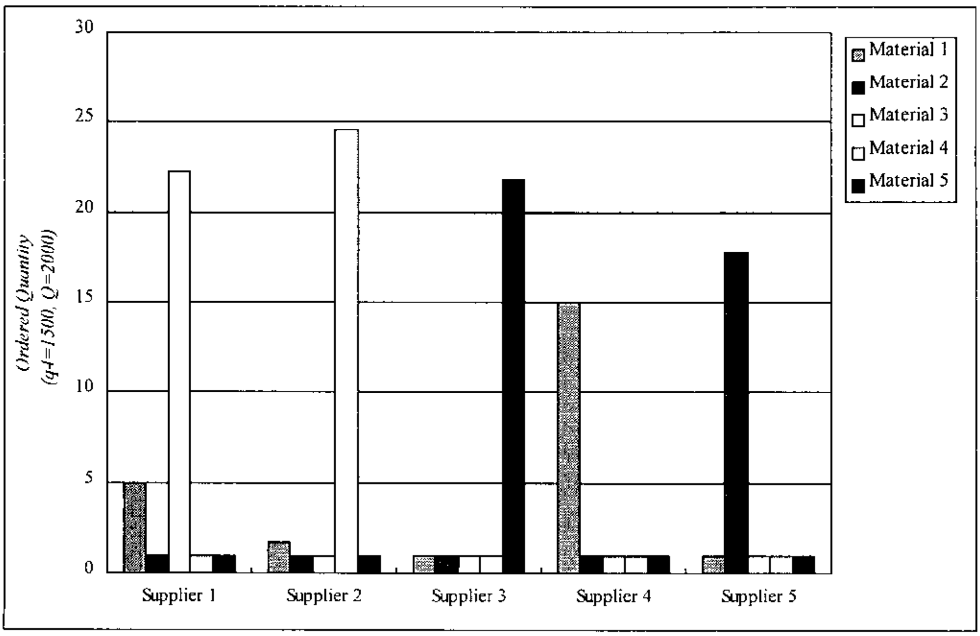


Figure 8. Supply Quantities from Suppliers as Supplier 4's capacity Changes

When discussing Figure 4, we mentioned that the manufacturing company tends to choose a supplier for each particular supply. But, this kind of restrictive selection seems to disappear as a supplier's capacity decreases. Figure 8 shows how the reduction in supplier 4's capacity affects the manufacturer's procurement from other suppliers. Because of its capacity limit, the supplier is unable to satisfy all of the requirements from the manufacturer. As a result, the manufacturer has to transact with additional suppliers (e.g., supplier 1 and 2) for the particular supply (e.g., material 1) despite possible disadvantages such as high cost and poor quality.

4.2 A Case Study

In order to highlight the interaction between important parameters in the real world setting, we use the data from the case study on a computer manufacturing company. This company assembles and sells two models of computer, *Model C* and *Model P*. We conducted a field study for this company and estimated necessary data as follows. Input requirements for the products are in Table 2: we are focusing only on the components in which the two models differ. We can see that one unit of *Model C* consists of three components, one *Celeron*, one *Motherboard*, and one 4.3MB *HDD* (hard disk drive), while one unit of *Model P* needs one *Pentium II*, one *Motherboard*, and 6.4MB *HDD*.

Table 2. Product Specification – Input Requirements

Supplier	Component				
	Celeron (1)	*Pentium II* (2)	*Motherboard* (3)	*HDD4.3* (4)	*HDD6.4* (5)
Model C ($l = 1$)	1		1	1	
Model P ($l = 2$)		1	1		1

We summarize the supply cost information in Table 3. There are four suppliers supplying components to the assembler. *Intel* supplies three components, *Samsung* two components, and others only one component at the prices given in the table. Finally, based on the company's historical data, we estimate the product information as in Table 4: data such as sales revenue, mean and standard deviation of market demand, understocking and overstocking costs, capacity requirements are estimated for each product, based on the previous 2 years' sales data.

Table 3. Supply Costs

Supplier (j)	Component – Supply (i)				
	Celleron (1)	*Pentium II* (2)	*Motherboard* (3)	*HDD4.3* (4)	*HDD6.4* (5)
Intel (1)	115	285	155		
SOYO (2)			135		
LG (3)			147		
Samsung (4)				171	181

Table 4. Product Information

Product	r_l	h_l	μ_l	σ_l	u_l	w_l
$l=1$	525	80	25.1	3.972	10	100
$l=2$	720	80	25.15	3.747	12	170

Outcomes of the numerical analysis for the case are presented in Figure 9 to 12. We can suggest similar interpretations given for the previous numerical examples in Figure 3 to 8. Figure 9 is comparable with Figure 3: the most desirable production capacity for this computer assembler is about 3400. Figure 10 shows the product mix between product 1 and product 2 as the manufacturer's assembly capacity, Q, changes: as the capacity increases, the manufacturer increases the production of product 1 faster than that of product 2.

In Figure 11, we can see that as the market demand uncertainty for product 2 increases, the production amount of product 1 increases whereas that of product 2 decreases as we already observed in Figure 7. Finally, Figure 12 shows that the assembler's total profit consistently decreases as the market uncertainty of product 2 increases.

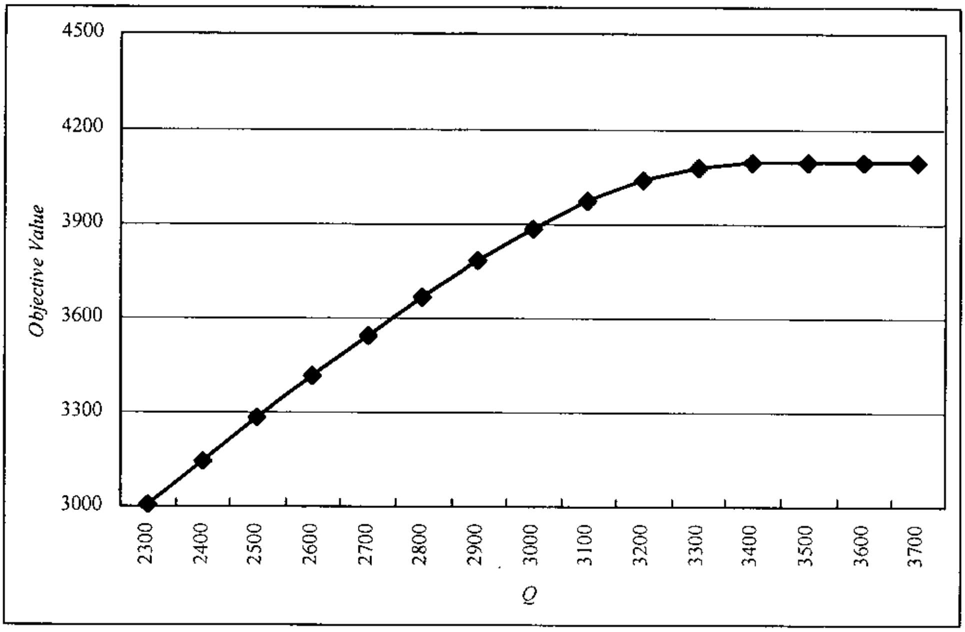


Figure 9. Computer Assembler's Net Profit as a Function of Capacity, Q

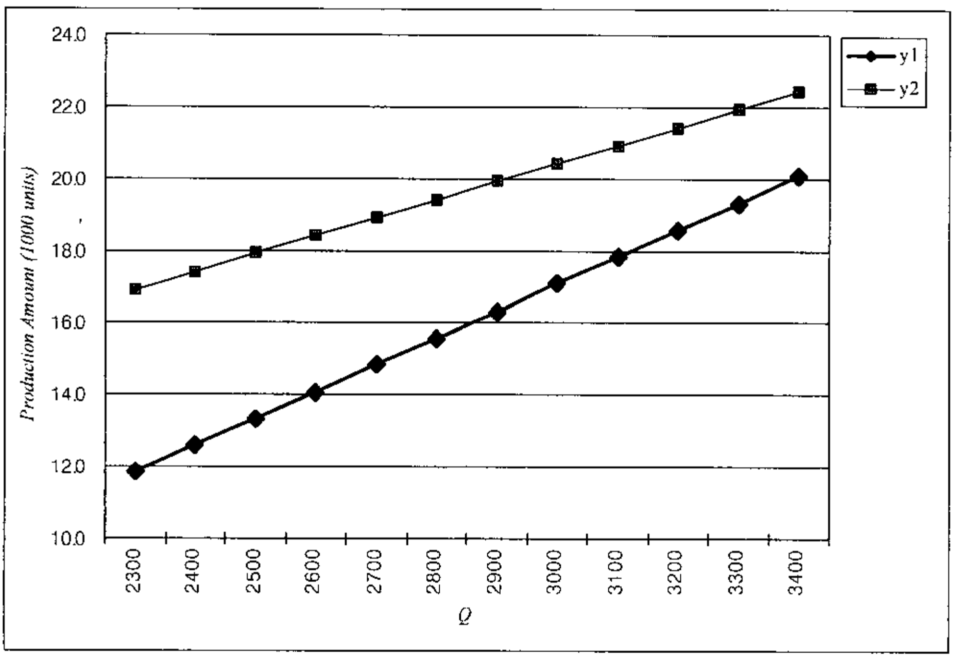


Figure 10. Each Product's Production Amount as a Function of Q

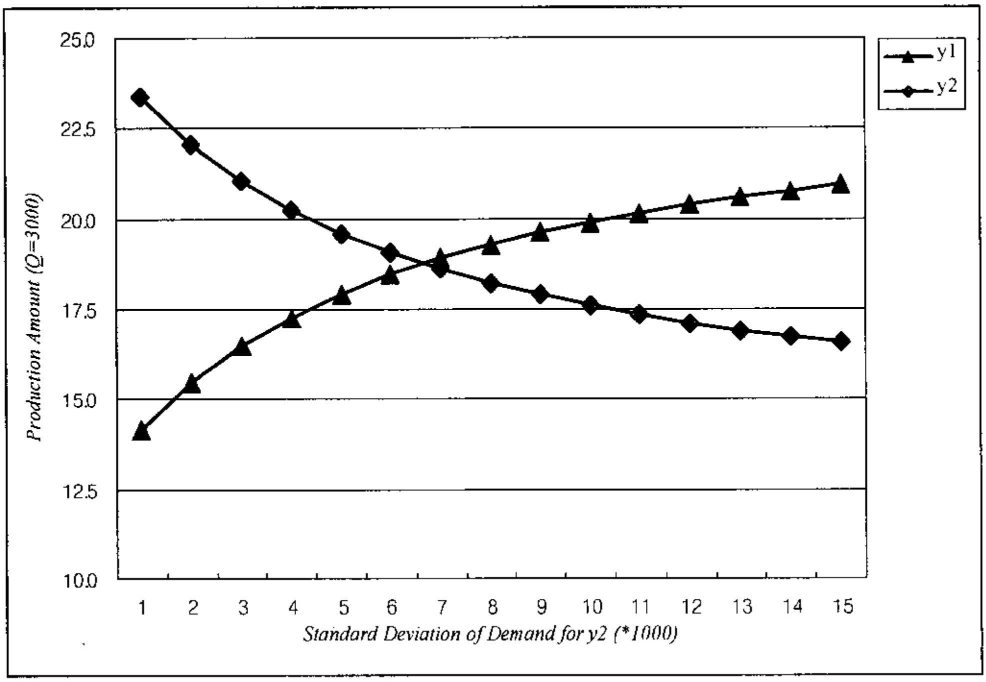


Figure 11. Each Product's Production Amount as y_2's SD Varies

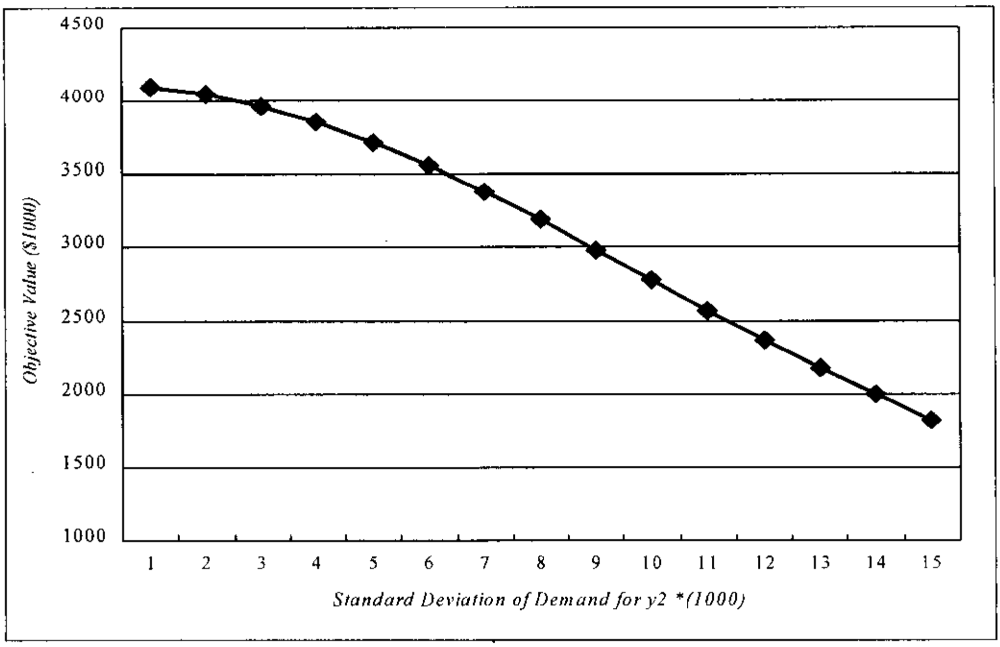


Figure 12. Computer Assembler's Net Profit as a Function of SD of y_2

5. Managerial Implications and Discussion

In this paper, we have developed a mathematical algorithm to solve a supply chain management problem faced by a manufacturing company that assembles and sells multiple products using materials from several suppliers. In order to show the utility of this algorithm, we presented numerical examples using an example set of parameters and data from a real world case study.

The result of our research indicates that the manufacturing company has to reach an optimal supply decision by taking into account such key factors as its production capacity and understocking and overstocking costs, market demand uncertainty, supply costs, and suppliers' capacities.

The manufacturer's procurement decision is dependent on *not only* its own capacity *but also* its suppliers'. We also observed there exist tradeoffs between products as their demand uncertainties change unequally: a product's demand uncertainty has an adverse effect on its optimal production amount. It is not just the manufacturer who has to pay close attention to its supply chain partners, i.e., suppliers. Each supplier also has to consider the manufacturer's capacity (i.e., Q) since that could determine which supplies the manufacturer procures from which supplier.

Based on the results, we can state that in order to optimize the supply chain performance, decisions made by the manufacturing company and its suppliers need to be integrated fully.

We already mentioned some of the limitations in our research. For instance, our approach to partition a complex nonlinear mathematical problem into two more easily solvable ones is an efficient way to solve the kind of problem we are dealing with in this paper. As alluded already, however, the algorithm might work in a slightly less accurate way when the suppliers' capacity limits are tightly binding. Thus, developing a more effective algorithm to alleviate the potential problem is a definite improvement. Although the specific context of our research is more relevant to the case of a single decision period, investigating the case in a more dynamic decision context should be an important extension of our research.

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Appendix 1. Definitions of Variables and Parameters in the Mathematical Model

- $i = 1, \cdots, n$, raw material index
- $j = 1, \cdots, m$, supplier index
- $l = 1, \cdots, k$, product index
- x_{ij} : amount of raw material i from supplier j
- y_l : production quantity of product l
- b_{il} : amount of raw material i required for producing one unit of product l
- v_{ij} : supplier j's cost (e.g., production cost/space) to provide one unit of raw material i to the manufacturer
- q_j : supplier j's resource (budget/space) limit, associated with v_{ij} ; thus, $\sum_{i=1}^{n} v_{ij} x_{ij} \le q_j$ must hold
- h_l : manufacturer's cost (e.g., production cost/space) to produce one unit of product l
- Q : manufacturer's resource (budget/space) limit, associated with product l ; thus, $\sum_{l=1}^{k} h_l y_l \le Q$ must hold
- c_{ij} : unit cost of raw material i from supplier j
- r_l : sales price of product l
- u_l : unit understocking cost of product l
- w_l : unit overstocking cost of product l
- Z_l : random variable for the demand of product l
- $f_{Z_l}(z_l)$: probability density function of Z_l
- λ_i : Lagrangian multiplier for raw material i
- μ_j : Lagrangian multiplier for supplier j
- η : Lagrangian multiplier for the manufacturer

Appendix 2.

Table A1: b_{il}

i l	1	2	3	4	5
1	2	1	3	1	3
2	1	3	2	1	2
3	3	2	1	4	1
4	2	1	2	3	4
5	1	3	2	2	3

Table A2. c_{ij}

i l	1	2	3	4	5
1	$8	8	12	6	15
2	10	15	8	10	5
3	5	7	14	9	8
4	9	5	10	13	8
5	12	9	5	7	6

Table A3. v_{ij}

i	1	2	3	4	5
1	1.5	2	3	1	3
2	2	1	1	3	1
3	2	1.5	1	3	2.5
4	1.5	3	2.5	2	3
5	3	2	3	2	1.5

Table A1: q_j

j	1	2	3	4	5
q_j	10,000	7,500	9,000	6,000	12,500