VORTEX SHEAR VELOCITY AND ITS EROSION IN THE SCOUR HOLE

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Abstract: Scour hole is formed due to the high shear stress of the jet flow at the outlet of a hydraulic structure and vortex erosion occurs in the scour hole. It is important to determine the amount of vortex erosion for the design of bed protection. If the vortex erosion continues and reaches to the hydraulic structure, it causes the deformation of the structure itself. To obtain the amount of the vortex erosion, it is necessary to determine the shear velocity of the line vortex in the scour hole. Average shear velocity in the scour hole was derived by the theory of energy conservation and found to be related to the upstream overflow velocity. The amount of vortex erosion from the scour hole was obtained using entrainment equation for given value of shear velocity. For a design purpose, if the flow velocity at the end of an apron and the properties of bed material are given, the amount of vortex erosion was obtained.

Key Words: scour hole, vortex erosion, bed protection, entrainment, shear velocity, line vortex

1. INTRODUCTION

Streambed erosion at the outlet of a hydraulic structure occurs by the high shear stress of the jet flow, and a resulting large scour hole is formed. Once the scour hole is formed, a line vortex occurs in it and this vortex causes additional erosion problems (Kim and Choe, 2000). Vortex erosion occurs at the bottom surface of the scour hole which is characterized by the entrainment and circulation of bottom sediment with the same direction of the line vortex. The macroturbulence of the line vortex consists of continuous and strong upward flow. It is compared with that formed at the lee-side of a sand ripple in the river whose turbulence is not so strong and intermittent, which supports the existing concept that the local concentration of the suspended sediment is related to the intensity of the vertical component of turbulent vortices (Ikeda and Asaeda, 1983).

Sand particles lifted from the scour hole are circulated with the same direction of the line vortex and deposited contributing the formation of a hump or swept away to the downstream by the main flow.

Granular jump analogous to the hydraulic jump also occurs in the scour hole, since the upstream slope of the scour hole is relatively steep and is connected to a flatter value. The volume fraction of the suspended sediments in the scour hole is relatively high on the steep slope which has the high erosion potential. Momentum transfer is made through the collisions of sediment particles and energy dissipation evidently takes place within the granular jump. It is assumed that the grain-inertia regime on a fluid-granule mixed flow is formed (Savage, 1984).

It is important to determine the amount of vortex erosion in the scour hole for the design of bed protection. If the vortex erosion continues and reaches to the hydraulic structure, it causes the three failure mechanism such as slip circle, liquefaction and internal erosion, and finally will lead to the deformation of the structure itself (Kim and Choe, 2000). To obtain the amount of the vortex erosion, it is necessary to determine the shear velocity of the line vortex in the scour hole.

The main purpose of this study is to derive the mathematical formulation of the vortex shear velocity and the vortex erosion in the scour hole and to verify them through the experiments.

2. AVERAGE SHEAR VELOCITY IN SCOUR HOLE

The average shear velocity in the scour hole is obtained using the energy conservation method between two crests as shown in Fig. 1.

In Fig. 1, H is the water level at the upstream reservoir. H_1 and H_2 are the water levels on the apron and on the hump crest, respectively, u_0 is the upstream average velocity, u is the velocity of the line vortex in the scour hole, u_{max} is the maximum velocity of the line vortex L is the length along the bed of the scour hole between H_1 and H_2. Difference between the rates of the energy influx, F_i into and the energy efflux, F_o out of the considered area is equal to the change of the kinetic vortex energy, E_v generated in the scour hole, the energy loss, E_l per unit time due to the skin friction along the bed of the scour hole and the energy loss, E_t per unit time due to the turbulent fluctuation in the scour hole. Thus,

$$F_i - F_o = \partial(E_v + E_t)/\partial t + E_l / time \quad (1)$$

$$F_i = \rho\, qgH = \rho\, qg(H_1 + u_0^2/2g) \quad (2)$$

$$F_o = \rho\, qg\,(H_2 + V_2^2/2g) \quad (3)$$

$$\frac{\partial E_v}{\partial t} = \frac{\partial}{\partial t}\left[\frac{1}{V/Q}\int\frac{u^2}{2}dA\right] \quad (4)$$

$$E_l / time = \tau / \rho \cdot L u_{max} \quad (5)$$

where q is the fluid discharge per unit width, Q is the fluid discharge, V is the volume of the line vortex, V_2 is the average velocity on the crest hump and A is the area occupied by the line vortex. Energy dissipation in the scour hole is done mainly through vortices and hence the en-

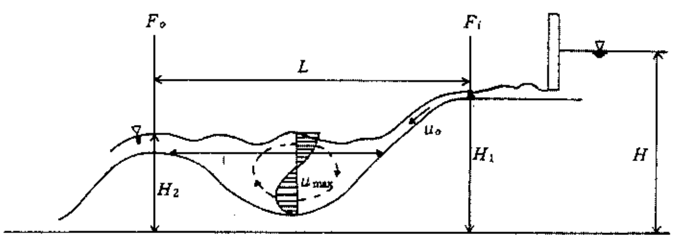

Fig. 1. Definition Sketch for Energy Conservation Method

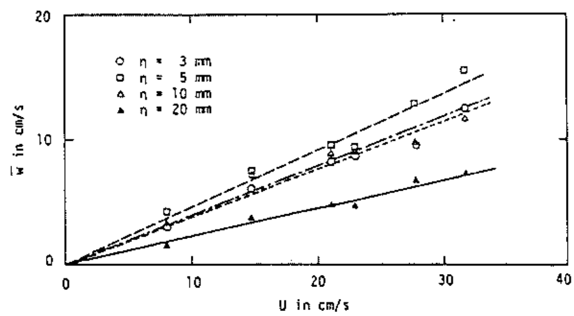

Fig.2. Relationships Between $\overline{W}$ and U (Ikeda and Asaeda, 1983)

ergy loss due to turbulence fluctuation can be neglected. Thus,

$$\rho q g(H_1+\frac{u_0^2}{2g})-\rho q g(H_2+\frac{u_0^2}{2g})=\frac{\rho}{V/Q}\int\frac{u^2}{2}dA+\frac{\pi}{\rho}Lu_{max}$$

$$\frac{\tau}{\rho}(=u_*^2)=\frac{\rho q g(H_1+\frac{u_0^2}{2g}-H_2-\frac{V_2^2}{2g})-\rho q\int\frac{u^2}{2}\frac{dA}{A}}{Lu_{max}} \quad (6)$$

Eq. (6) is the mathematical form of the average shear velocity in the scour hole, u_*, but is rather complicated. If it can be estimated using the outer parameter, it will be very convenient.

According to Ikeda and Asaeda (1983), there is a linear relationship between the mean value of a vertical component of turbulent velocity, $\overline{W}$ and the longitudinal average velocity, U as described by

$$\overline{W}=\beta U \quad (7)$$

where β is the proportionality coefficient whose value is 0.23~0.46. Fig. 2 shows this relationships. Value of the shear velocity was found to be of the same magnitude as the vertical velocity fluctuation near the trailing ridge of the sand ripple (Ikeda and Asaeda, 1983). Thus,

$$\frac{u_*^2}{U^2}=\frac{W'^2}{(\overline{W}/\beta)^2}=\beta^2\cdot\frac{W'^2}{\overline{W}^2}=\beta^2\left(\frac{W-\overline{W}}{\overline{W}}\right)^2 \quad (8)$$

From the measurements over the rippled bed, Ikeda and Asaeda(1983) obtained the following result:

$$\left(\frac{W-\overline{W}}{\overline{W}}\right)^2 \fallingdotseq 0.0324 \quad (9)$$

Therefore, eq. (8) becomes as,

$$\frac{u_*^2}{U^2}=0.0017\sim0.0068 \quad (10)$$

Based on the above results, the relation between the upstream average velocity, u_o and the average shear velocity, u_* in the scour hole is considered as,

$$u_*=\alpha_2\cdot u_o \quad (11)$$

where α_2 is the proportionality coefficient.

3. VORTEX EROSION BY LINE VORTEX

The amount of suspension from the scour hole of the sand bed per unit width and unit time is given by (Asaeda et al. 1989),

$$q_s = \frac{2}{3}\rho K\sqrt{\frac{6}{\pi}(s+1)}u_*\int_{\xi_1}^{\xi_2}\int_{\eta_o}^{\infty}\sqrt{\frac{\pi}{4}C_L\eta - \frac{\pi}{8}C_D\xi_o^2S_1(\xi)}$$
$$\times\frac{1}{\sqrt{2\pi}}\exp\frac{(-\eta^2)}{2}d\eta d\xi \tag{12}$$

assuming that the suspension occurs from the area between ξ_1 and ξ_2 of the scour hole. K is the ratio of number of entrained particles to the total number of particles on the sand bed, u_* is the shear velocity in the scour hole and s is the submerged specific gravity of sand particles. η is the standardized variable of the lift force and η_o corresponds to the critical stage when sand particles are initiated, which is represented by,

$$\eta_o = \frac{\pi}{8}C_DS_1(\xi)\frac{4\xi_o^2}{\pi C_L} \tag{13}$$

where C_D is the drag coefficient, $S_l(\zeta)$ is the slope of the scour hole at ζ, ζ is the distance along the scour hole, ζ_o is the ratio of a particle fall velocity, w to the shear velocity, u_* and C_L is the lift coefficient.

Substituting eq. (11) into eq. (12), The amount of the vortex erosion in the scour hole is obtained as,

$$q_s = \frac{2}{3}\rho K\sqrt{\frac{6}{\pi}(s+1)\alpha_2u_o}\int_{\xi_1}^{\xi_2}\int_{\eta_o}^{\infty}\sqrt{\frac{\pi}{4}C_L\eta - \frac{\pi}{8}C_D\xi_o^2S_1(\xi)}$$
$$\times\frac{1}{\sqrt{2\pi}}\exp\frac{(-\eta^2)}{2}d\eta d\xi \tag{14}$$

The dimensionless volumetric vortex erosion, E_s is considered as the amount of the vortex erosion divided by the amount of sediment deposits into the scour hole,

$$E_s = \frac{q_s}{swl_{12}S_1(\xi)} \tag{15}$$

where l_{12} is the length between ζ_1 and ζ_2 of the scour hole.

Asaeda et al. (1989) found that the amount of the vortex erosion was related to the dimensionless shear velocity, ζ_o ($=w/u_*$) and the particle Reynolds number, R_P given by,

$$R_p = \frac{d\sqrt{sgd}}{\nu} \tag{16}$$

where d is the particle diameter and ν is the kinematic viscosity.

According to Akiyama and Fukushima (1968), the effect of R_P on ζ_o changes with the square root of R_P as shown in Fig. 3.

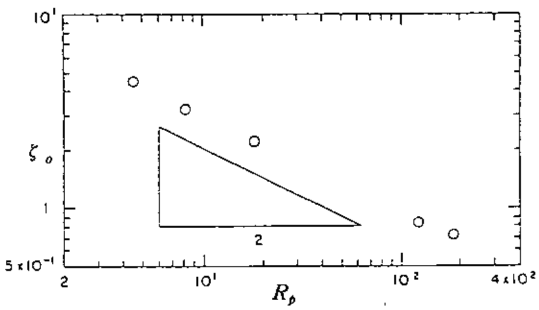

Fig. 3. Dimensionles Shear Velocity as Function of Particle Reynolds Number(Akiyama and Fukushima, 1968)

Therefore, it is expected that the dimensionless volumetric vortex erosion is the function of a similarity variable, z given by,

$$z = (u_* / w) R_P^{0.5} \qquad (17)$$

4. EXPERIMENTS IN FIXED AND MOVABLE BED MODEL

4.1 Experiments in Fixed Bed Model

In order to measure the velocity field of the line vortex, the artificial scour hole as shown in Fig. 1 was made to simulate the actual scour hole using cement mortar inside the recirculating channel of 0.3m wide and 3m long (Kim, 1992). The profile of the artificial scour hole corresponds to those of the actual scour hole when large macroturbulence occurred at movable bed conditions. Transparent scale was attached to the side wall of the channel. As a tracer method, aluminum particles mixed with a neutral detergent were put into the channel at the upstream end. Photos were taken using a motor driven camera with a vertical light sheet by a slide projector in dark condition. Photo development was done under the condition of ISO up to 3200 in order to obtain clear trajectory. These photos were used for obtaining digitized velocity vector as shown in Fig. 4 (Kim and Choe, 2000).

In Fig. 4, the time sequential photos using a motor driven camera were obtained. These photos were copied and enlarged using a copy machine. The digitized velocity vector was made input into a monitor using a digitizer. The velocity field of the line vortex was measured from the length of the trajectory of aluminum particle divided by exposure time. The slit width of a vertical light sheet and the exposure time of the motor driven camera were adjusted considering concentration and velocity of aluminum particles. By trial and error, the slit width was determined as 0.5~0.7mm and the exposure time was as 1/15~1/30 second. Variation of the profile of the scour hole according as scouring proceeds was simulated step by step and the same experiments were repeated.

By the above procedure, the area of line vortex, A and the velocity, u and $u_{\max}$ were obtained, and hence the friction velocity, u_* was calculated using eq. (6).

4.2 Experiments in Movable Bed Model

To measure the amounts of vortex erosion, a series of experiments was done. Fig. 5 shows the experimental arrangement.

Sand, the median diameter of which is 0.5mm was put at the downstream part of an apron inside the same recirculating channel as in the fixed bed model. The side wall of the channel was made of glass. A transparent scale was attached to the side wall for quantitative measurement of flushing process. Since the length

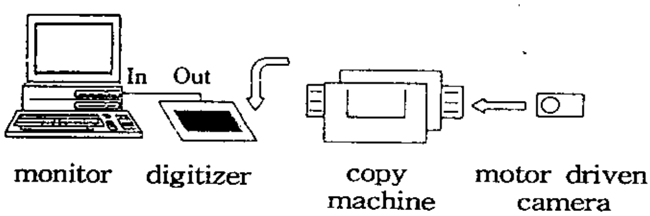

Fig. 4. Data System for Flow Trajectory (Kim and Choe, 2000)

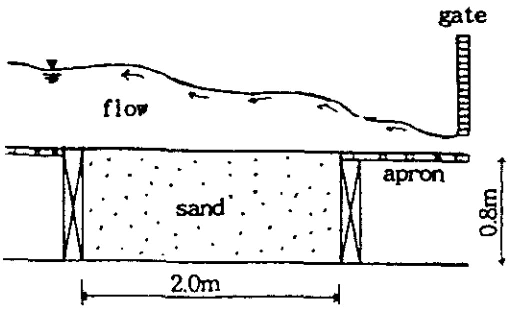

Fig. 5. Experimental Arrangement in Movable Bed Model

Table 1. Summary of Experimental Results in Fixed Bed Model

Run No.	u_o(cm/s)	H_1(cm)	H_2(cm)	V_2(cm/s)	H_2(cm)	L(cm)	u_*(cm)
R1	75	10.4	7.7	70.5	25.2	41.14	4.35
R2	105	10.6	7.9	73.5	31.5	41.14	4.83
R3	120	11.1	8.2	80.0	33.1	41.14	5.65
R4	132	11.4	8.4	83.4	32.6	41.14	6.36
R5	145	11.8	8.7	87.0	35.2	41.14	7.01
R6	154	12.2	9.2	90.6	37.3	41.14	8.91
R7	81	11.5	7.7	84.3	31.2	37.20	4.05
R8	122	12.0	8.2	90.3	34.3	37.20	6.15

between the upstream end of the channel and the location of a gate was relatively short, a thick screen was installed at the entrance of the channel.

A video camera and a time-lapsed video tape recorder were used to measure the time variation of a vortex erosion. The test section was illuminated by strong light for clean view. Measurements through a video recording were carried out every 10 seconds in order to estimate the rates of a vortex erosion and, in turn, to investigate the relationship between the inflow discharge and these rates.

An electro-optical concentration meter was used to measure the sediment concentration in the state of the vortex erosion (Kim and Kim, 1999). The analog signals obtained from the electro-optical concentration meter were converted into digital ones, and then collected in the computer.

5. EXPERIMENTAL RESULTS AND DISCUSSION

Measured values of parameters in eq. (6) are shown in Table 1, and the relationships between u_* and u_o are shown in Fig. 6.

It can be seen that reasonable linearity is revealed in Fig. 6, where α_2 = 0.046~0.058. Thus, if the upstream average velocity is given, the mean shear velocity can be obtained using Fig. 6.

Fig. 7 shows the rates of the vortex erosion as function of similarity variable using eqs. (15) and (17). In Fig. 7, Theoretical curves of Akiyama and Fukushima (1986), Itakura and Kishi (1980), and Engelund and Fredsoe (1985) were

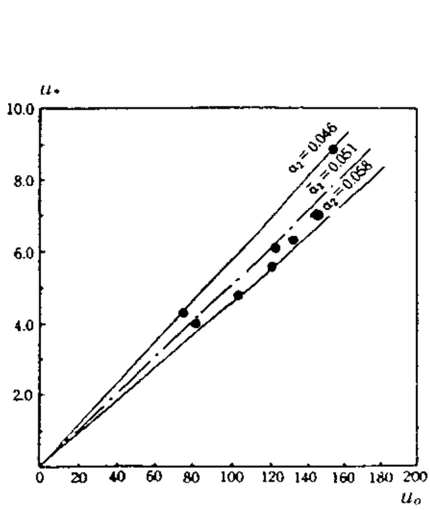

Fig. 6. Relationships between u_0 and u_*

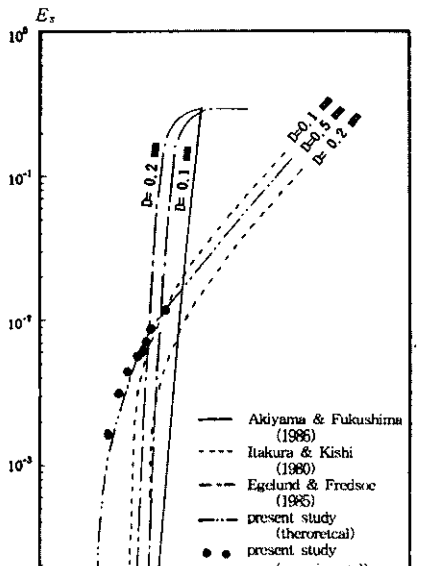

Fig. 7. Reate of Vortex Erosion as Function of Similarity Variabel

also plotted. These values were revised according to eqs. (15) and (17).

It seems that the steep slope of the relationships between the entrainment rate, E_s and the similarity variable, z is strongly related to the explosive nature of suspension phenomenon. Relationships of E_s and z in this study take similar type with those of Itakura and Kishi (1980). The reason is that they determined these in the state of high shear stress, while others etermined those at the rippled bed.

At higher values of the shear velocity, entrainment rate in this study is lower than that of others. It seems that high concentration of suspended sediments acts to limit the increase in as shear velocity increases further. The entrainment rate at lower values is larger than that of others. Physical explanation for this behaviour is that the macroturbulence in turbulent vortices formed in a scour hole, which determines the entrainment mechanism, produces a continuous and strong upward flow. It is compared with turbulence formed at the lee-side of a sand ripple whose upward flow is not so strong and intermittent.

6. CONCLUSIONS

Vortex erosion and the large scour hole occur due to the macroturbulence of the line vortex which produces continuous and strong upward flow different from the conventional vortex formed in the lee-side of the sand ripple. Average shear velocity in the scour hole was derived by the theory of energy conservation and found to be related to the upstream overflow velocity.

For given value of shear velocity, the amount of vortex erosion from the scour hole was obtained using the entrainment equation. Comparisons of erosion rates with others showed larger value at the low shear velocity due to macroturbulence which produces continuous and strong upward flow, and showed smaller value at the high shear velocity due to high concentration of suspended sediments which acts to limit the increase in entrainment as shear velocity increases further.

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REFERENCES

Akiyama, J., and Fukushima, Y. (1986). "Entrainment of noncohesive bed sediment into suspension." *Proceedings 3rd International Symposium on River Sedimentation*, pp. 804-813.

Asaeda, T., Nakai, M., Shyam, K. M., and Tamai, N. (1989). "Sediment entrainment in channel with rippled bed." *Journal of Hydraulic Engineering*, ASCE, Vol. 115, No. 3, pp. 327-339.

Engelund, D. M. and Fredsoe, E. A.(1985). "Observations of rapidly flowing granular-fluid materials." *Journal of Fluid Mechanics*, Vol. 143, pp. 357-380.

Ikeda, S., and Asaeda, T. (1983). "Sediment suspension with rippled bed." *Journal of Hydraulic Engineering*, ASCE, Vol. 109, No. 3, pp. 409-423.

Itakura, T., and Kishi, T. (1980). "Open channel flow with suspended sediments." *Journal of Hydraulic Engineering*, ASCE, Vol. 106, No. 8, pp. 1345-1352.

Kim, J. H. (1992). "Experimental study on vortex characteristics at downstream part of hydraulic structure." *Journal of Agricultural Engineering*, KSAE, vol. 34. pp. 151-163.

Kim, J. H. and Choe, J. W. (2000). "Vortex structure in the scour hole by gate opening of hydraulic structure." *Water Engineering Research*, Vol. 1, No. 1, pp. 83-92.

Kim, J. H. and Kim, J. S. (1999). "Sheet erosion in embankment of noncohesive materials due to unsteady slow." *Journal of Civil Engineering*, KSCE, Vol. 3, No. 1, pp. 58-69.

Savage, S. B. (1984). "Gravity flow of cohesionless granular materials in chute and channels." *Journal of Fluid Mechanics*, Vol. 142, pp. 53-66.

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