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Structural Reliability of Thick FRP Plates subjected to Lateral Pressure Loads

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Abstract

This paper deals with reliability analysis of specially orthotropic plates subjected to transverse lateral pressure loads by using Monte Carlo simulation method. The plates are simply supported around their all edges and have a low short span to plate depth ratio with rectangular plate shapes. Various levels of reliability analyses of the plates are performed within the context of First-Ply-Failure(FPF) analysis such as ply-/laminate-level reliability analyses, failure tree analysis and sensitivity analysis of basic design variables to estimated plate reliabilities. In performing all these levels of reliability analyses, the followings are considered within the Monte Carlo simulation method: (1) input parameters to the strengths of the plates such as applied transverse lateral pressure loads, elastic moduli, geometric including plate thickness and ultimate strength values of the plates are treated as basic design variables following a normal probability distribution; (2) the mechanical responses of the plates are calculated by using simplified higher-order shear deformation theory which can predict the mechanical responses of thick laminated plates accurately; and (3) the limit state equations are derived from polynomial failure criteria for composite materials such as maximum stress, maximum strain, Tsai-Hill, Tsai-Wu and Hoffman.

Keywords: specially orthotropic plates, simplified higher-order shear deformation theory, Monte Carlo simulation method, reliability analysis, sensitivity analysis

Nomenclature

a, b	Laminated plate length and width
E_{11}, E_{22}	Elastic moduli in fibre direction and direction normal to fibre
E_{33}	Elastic modulus in laminated plate thickness direction

$F_i, F_{ij}(i,j=1,2,3,4,5,6)$	Coefficients in polynomial failure criterion
G_{12}	In-plane shear modulus
G_{13}, G_{23}	Shear moduli in laminated plate thickness directions
h	Laminated plate thickness
m, n	Wave numbers
N_i, M_i, P_i	$\int_{-h/2}^{h/2} \sigma_i(1,z,z^3)dz \quad (i=1,2,6)$
$P_s(=\Re)/P_f$	Probability of survival or reliability/Probability of failure
q	Transverse lateral pressure load
$Q_1, R_1/Q_2, R_2$	$\int_{-h/2}^{h/2} \sigma_5(1,z^2)dz/ \int_{-h/2}^{h/2} \sigma_4(1,z^2)dz$
$R, S \quad and \quad T$	Shear strengths in , and laminated plate planes
u, v, w	In-plane and transverse displacements
u^0, v^0, w^0	In-plane and transverse displacements at middle-plane
$X_{T,C}$	Tensile/Compressive lamina normal strengths in x direction
$Y_{T,C}$	Tensile/Compressive lamina normal strengths in y direction
x, y, z	Laminated plate Cartesian coordinates
$\sigma_i(i=1,2,3,4,5,6)$	Stresses in engineering notation
μ_X, σ_X	Mean and standard deviation of basic design variables X
ν_{12}, ν_{21}	In-plane Poisson's ratios
ψ_x, ψ_y	Rotations of normals to mid-plane about y and x
$\xi_x, \xi_y, \phi_x, \phi_y$	Pure functions of laminated plate in-plane dimensions

1 Introduction

The use of fibre reinforced plastic (FRP) composite materials in advanced engineering applications has expanded rapidly and created a need for improved failure analysis capabilities. The mechanical properties of such materials show a high degree of variability due to the heterogeneous make-up of constituent elements and the subjective nature of primarily manual based production processes. As a result, designing structures made from FRP composite materials using conventional failure analysis capabilities based on pure deterministic approaches often leads to unrealistic and conservative safety margins. To overcome this drawback, during the last decade, a number of composite mechanics researchers applied probabilistic concepts, which resulted in structural reliability techniques, in order to achieve improved failure analysis capabilities for the design of FRP composite structures. However, their contributions to the development of rational design procedures for FRP composite structures are limited to cases such as laminates under in-plane loading or simple composite beams under 3-/4-point bending. Therefore, when designers or constructors seek a probabilistic concept based improved failure analysis capability for the design of FRP composite structures subjected to bending, which is a particular concern to marine designers, there is little evidence of systematic work. Previously, two of the authors have attempted to introduce reliability concepts in the strength analysis of large, plated FRP structures within the context of the FPF analysis(Jeong and Shenoi 1998a, Jeong and Shenoi 1998b, Jeong and Shenoi 2000a). The first of these(Jeong and Shenoi 1998a) covered reliability estimation of the strength of mid-plane symmetric laminated plates using Monte Carlo simulation method. The second paper(Jeong and Shenoi 1998b) explores the use of Hasofer-Lind's safety index approach to specially orthotropic laminated plates. In both cases, basic design variables were assumed to follow a normal probability distribution. The third paper(Jeong and Shenoi 2000a) performed the reliability analysis of general laminated plates using Monte Carlo simulation method (Especially, this paper employed a non-normal probability distribution for basic design variables and performed sensitivity analysis of basic design variables). Developed reliability analysis procedures from these papers were used to interpret the experimental results of FRP laminated plates for civil and marine constructions performed by Moy et al.(1996), and were compared with the EUROCOMP Design Code procedures(Clarke 1996) to verify the design procedures presented in the EUROCOMP Design Code, see Jeong and Shenoi(2000a). It should be mentioned that, in all these papers, the laminated plate theories were based on the Kirchhoff's hypothesis, which neglects both transverse shear and normal deformations, due to the high short span to plate depth ratios ($\geq$ 20) of the proposed plates. This paper is an extension of the previous work by considering theoretical application of the developed reliability analysis procedures to thick FRP laminated plates. Here, the strengths of simply supported thick specially orthotropic plates are calculated by applying Monte Carlo simulation method to the FPF of the plates. Transverse lateral pressure loads, elastic moduli, geometric including plate thickness and ultimate strength values of the plates are considered as basic design variables and they are pseudo-randomly generated according to their pre-described probability distribution by using statistical sampling techniques. The generated basic design variables are used in simplified higher-order shear deformation theory, which can take into account both transverse shear and normal deformations, to calculate the maximum stress components of the plates. Maximum stress, maximum strain, Tsai-Hill's, Tsai-Wu's and Hoffman's failure criteria are used to develop the limit state equations. By substituting the calculated maximum stresses with generated ultimate strength basic design variables into the developed limit state equations, the probability of failure of the plates are estimated. Sensitivity analysis is also performed to view the influence of each basic design variable with respect to the estimated reliability of the plates.

2 Laminated Composite Plate Theory and Failure Criteria

Reddy(1984) developed simplified higher-order shear deformation theory for laminated anisotropic plates that can take into account not only transverse shear strains, but also parabolic variations of the transverse shear strains with respect to the laminate thickness direction. This enabled the theory to calculate transverse shear stress components without considering shear correction factors. In developing the theory, he used the displacement field equations developed by Levinson(1980).

$$u = u^0(x,y) + z\psi_x(x,y) + z^2\xi_x(x,y) + Z^3\phi_x(x,y) \tag{1}$$

$$v = v^0(x,y) + z\psi_y(x,y) + z^2\xi_y(x,y) + Z^3\phi_y(x,y) \tag{2}$$

$$w = w^0(x,y) \tag{3}$$

By applying the condition that the transverse shear stresses, and therefore the corresponding transverse shear strains, vanish on the top and bottom of the laminated plates and are nonzero elsewhere in (1) to (3), he obtained the following modified displacement field equations,

$$u = u^0(x,y) + z\left[\psi_x(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\psi_x(x,y) + \frac{\partial w^0(x,y)}{\partial x}\right)\right] \tag{4}$$

$$v = v^0(x,y) + z\left[\psi_y(x,y) - \frac{4}{3}\left(\frac{z}{h}\right)^2\left(\psi_y(x,y) + \frac{\partial w^0(x,y)}{\partial y}\right)\right] \tag{5}$$

$$w = w^0(x,y) \tag{6}$$

The principle of virtual displacements was used to obtain equilibrium equations pertinent to both the displacement field equations, (4) to (6), and usual stress-strain constitutive equations. 5 equilibrium equations in the domain of the laminate mid-plane were derived for 5 displacement coefficient terms as follows,

$$\delta u : \frac{\partial N_1}{\partial x} + \frac{\partial N_6}{\partial y} = 0 \tag{7}$$

$$\delta v : \frac{\partial N_6}{\partial x} + \frac{\partial N_2}{\partial y} = 0 \tag{8}$$

$$\delta w \cdot \frac{\partial Q_1}{\partial x} + \frac{\partial Q_2}{\partial y} + q - \frac{4}{h^2}\left(\frac{\partial R_1}{\partial x} + \frac{\partial R_2}{\partial y}\right) + \frac{4}{3h^2}\left(\frac{\partial^2 P_1}{\partial x^2} + 2\frac{\partial^2 P_6}{\partial x \partial y} + \frac{\partial^2 P_2}{\partial y^2}\right) = 0 \tag{9}$$

$$\delta \psi_x : \frac{\partial M_1}{\partial x} + \frac{\partial M_6}{\partial y} - Q_1 + \frac{4}{h^2}R_1 + \frac{4}{3h^2}\left(\frac{\partial P_1}{\partial x} + \frac{\partial P_6}{\partial y}\right) = 0 \tag{10}$$

$$\delta \psi_y : \frac{\partial M_6}{\partial x} + \frac{\partial M_2}{\partial y} - Q_2 + \frac{4}{h^2}R_2 + \frac{4}{3h^2}\left(\frac{\partial P_6}{\partial x} + \frac{\partial P_2}{\partial y}\right) = 0 \tag{11}$$

Exact Navier-type solution procedure was used to solve the equilibrium equations of the laminated plates. The following doubly infinite half range sine series solutions were assumed for w, ψ_x, ψ_y satisfying simply supported boundary conditions,

$$w = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} W_{mn} \sin\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{12}$$

$$\psi_x = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} X_{mn} \cos\frac{m\pi x}{a} \sin\frac{n\pi y}{b} \tag{13}$$

$$\psi_y = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} Y_{mn} \sin\frac{m\pi x}{a} \cos\frac{n\pi y}{b} \tag{14}$$

The coefficients, W_{mn}, X_{mn}, Y_{mn}, in these assumed solutions were determined by substituting (9) to (11) and stress resultant equations for a symmetric laminated plate into the assumed solutions, (12) to (14).

A general representation of failure envelope for the laminates can be expressed by the tensor polynomial failure criterion.

$$\begin{aligned} &F_1\sigma_1 + F_2\sigma_2 + F_3\sigma_3 + 2F_{12}\sigma_1\sigma_2 + 2F_{13}\sigma_1\sigma_3 + 2F_{23}\sigma_2\sigma_3 \\ &+F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{33}\sigma_3^2 + F_{44}\sigma_4^2 + F_{55}\sigma_5^2 + F_{66}\sigma_6^2 + \cdots \geq 1 \end{aligned} \tag{15}$$

The coefficients F_i and F_{ij} are the functions of unidirectional lamina strengths and defined in Table 1 for maximum stress, maximum strain, Tsai-Hill's, Tsai-Wu's and Hoffman's failure criteria(Reddy and Pandey 1987, Turvey and Osman 1989). It should be mentioned that in Tsai-Hill's failure criterion, when σ_1 and σ_2 are tensile, X and Y are defined as X_T and Y_T. Similarly, when σ_1 and σ_2 are compressive, X and Y are defined as X_C and Y_C, respectively.

3 Monte Carlo Simulation Method in Reliability Analysis

The Monte Carlo simulation method(Ayyub and Haldar 1984, Mansour 1990) is considered as one of the most powerful tools for analysing complex problems and is used to predict the future behaviour of a system involving basic design variables of known or pre-described probability distributions. A set of values of the basic design variables can be generated according to their corresponding probability distributions by using statistical sampling techniques. The generated basic design variables are treated as a sample of experimental observations and used in the limit state equation or safety margin for the system to obtain a simulated solution. As the generation of the basic design variables is repeated, more simulated solutions can be determined. Statistical analysis of the simulated solutions is then performed. The results obtained from the Monte Carlo simulation method depend on the number of the generated basic design variables used. This means that the results are not exact; the accuracy increases as the number of basic design variables and the number of simulation cycles increase. Basic design variable generation from a particular probability distribution involves the use of pseudo-random numbers. A brief summary of the Monte Carlo simulation method in the context of reliability estimation is presented below.

The limit state equation or safety margin for a structural member can be defined by using strength and load parameters.

$$M = STRENGTH - LOAD = g(x_1, x_2, \cdots, x_n) \tag{16}$$

Where, $x_i(i = 1, \cdots, n)$ denote n basic design variables, and $g(.)$ denotes functional relationship between the n basic design variables and failure (or survival) of the structural member. From (16), the failure of the structural member is defined when $M \leq 0$, and the survival of the structural member is defined when $M > 0$, respectively. Therefore, the probability of failure of the structural member can be calculated by performing the following integration over the region where $M \leq 0$.

$$P_f = \int\int, \cdots, \int f_x(x_1, \cdots, x_n) dx_1, \cdots, dx_n \tag{17}$$

Where, f_x is the joint density function of $x_1, \cdots, x_n$. From (17), the probability of survival of the structural member can be calculated as follows.

$$P_s = 1 - P_f \tag{18}$$

The evaluation of the limit state equation can be performed based upon the generated basic design variables. If $g(.) \leq 0$, then failure is known to occur. The estimation of the probability of failure, P_f can be obtained by using the following relation:

$$P_f = \frac{N_f}{N} \tag{19}$$

Where, N_f is the number of simulation cycles when $g(.) \leq 0$, and N is the total number of simulation cycles. Statistical accuracy of the estimated probability of failure can be measured by using the coefficient of variation.

$$V_X(P_f) \cong \frac{\sqrt{Var(P_f)}}{P_f} \tag{20}$$

Where, the variance of P_f can be defined as follows.

$$Var(P_f) \cong \frac{(1-P_f)P_f}{N} \tag{21}$$

Furthermore, percent error of the estimated probability of failure can be obtained by approximating a binomial probability distribution with a normal probability distribution as follows(Ang and Tang 1984).

$$\%Error = 200\sqrt{\frac{1-P_f}{N \times P_f}} \tag{22}$$

There is a 95% chance that the percent error in the estimated probability of failure will be less than that defined by (22).

4 Adopted Procedure and Methodology

The procedure as illustrated in Figure 1 consists of four individual modules. These modules correspond to: (1) selection of basic design variables and their generation according to pre-described probability distributions using pseudo-random number generators; (2) calculation of the maximum stress components of laminated plates; (3) development of the limit state equations using the polynomial failure criteria and assessment of the developed limit state equations; and (4) statistical analysis with respect to results from the limit state equations to estimate the probability of failure of the laminated plates. A brief explanation of each module follows. In module-1, transverse lateral pressure loads, elastic moduli, geometric including plate thickness and ultimate strength values of the plates are selected and then treated as basic design variables. A normal probability distribution is used to describe the variable nature of these variables. Statistical characteristics of the basic design variables are defined from material property tests in terms of mean values and coefficients of variance (COV). Probability distribution parameters such as mean and standard deviation of a normal probability distribution are then estimated. Using these estimated parameters, the basic design variables are pseudo-randomly generated using statistical sampling techniques. In module-2, simplified higher-order shear deformation theory is used to calculate the maximum stress components of the simply supported thick specially orthotropic plates. The generated basic design variables corresponding to transverse lateral pressure loads, elastic moduli and geometric including thickness values of the plates are used as input values for the plate theory. In module-3, maximum stress, maximum strain, Tsai-Hill's, Tsai-Wu's and Hoffman's failure criteria are used to develop the limit state equations for the probabilistic strength analysis of the plates. Calculated maximum stress components of the plates from module-2, and generated basic design variables corresponding to the ultimate strength values of the plates from module-1 are substituted into the developed limit state equations. The failure or survival state of the plates is then defined from the results of the limit state equations. Finally, in module-4, statistical analysis with respect to the results from module-3 is performed to estimate the probability of failure of the plates and its coefficients of variation to find out the statistical accuracy of the results.

5 Statistical Characteristics of Basic Design Variables

The material used in the plates exemplified this paper is T300 carbon/epoxy (DMS2224, 350F cure). The pertinent material properties have been determined by conducting a series of material property tests under ambient room temperature(KIMM 1996). The test methods used in the experiment are listed in Table 2. The tests pertained to mechanical properties in tension, compression, shear and inter-laminar shear. In each case, a total of 35 test specimens was used to determine mean values and coefficients of variance (COV) of material properties. Thus, from the collected material property means and coefficients of variance, the parameters of a normal probability distribution can be determined. Full statistical characteristics of the basic design variables from the material property tests are listed in Table 3. The adoption of the statistical characteristics of the transverse lateral pressure loads is not as simple as that of material properties. It can be done by adopting an arbitrary load, a specific design load or a failure load through analytical failure analysis such as the FPF analysis or Last-Ply-Failure (LPF) analysis. In this paper, the statistical characteristics of the transverse lateral pressure loads are determined from the FPF analysis using previously explained simplified higher-order shear deformation theory and polynomial failure criteria. To validate the present FPF analysis formulation, the present FPF analysis results are compared with those of Turvey and Osman(1989). For a comparison purpose, the details of the material properties and geometric values of the simply supported specially orthotropic plate in Turvey and Osman are borrowed. Their plate has $[0^o/90^o]_s$ stacking sequence and a short span to plate depth ratio of 250. It should be mentioned that Turvey and Osman employed Classical Laminated Plate Theory (CLPT) in their FPF analysis, because they only considered thin laminated plates (the short span to plate depth ratios varied from 62.5 up to 500.0). However, their FPF analysis results can be compared with the present FPF analysis results, because of the negligence of the transverse shear deformations in case of thin laminated plates. The comparison is shown in Table 4. Through this comparison, it can be mentioned that the present FPF analysis formulation is validated successfully by obtaining an excellent agreement between them. In this paper, simply supported specially orthotropic plates having stacking sequence of $[0^o/90^o/0^o/90^o]_s$ are considered as numerical examples. The plates have a short span to plate depth ratio of 4.0 and aspect ratios of 1.0, 1.5 and 2.0. Hence, the determination of the statistical characteristics of the transverse lateral pressure loads for these proposed plates are accomplished from the present FPF analysis formulation in conjunction with the coefficient of variance of test specimen failure loads as shown in Table 5. Loads 1 $\sim$ 5 represent the FPF loads from maximum stress, maximum strain, Tsai-Hill's, Tsai-Wu's and Hoffman's failure criteria, respectively. The mean values of Loads 1 $\sim$ 5 are given from the FPF analysis and they are assumed to follow a normal probability distribution.

6 Numerical Examples and Discussion

Reliability analysis of the plates is performed to investigate the following three categories:

a) probabilistic failures at ply-level and, hence, probabilistic ply failure sequences.

b) probabilistic strength distributions of the plates (probabilistic failures at laminate-level).

c) sensitivity of each basic design variable to the estimated reliability of the plates.

6.1 Ply-level Analysis

The purpose of this analysis is to define the probability of failure at each ply in the plates. As applied transverse lateral pressure loads to the plates, the average loads of Loads 1 $\sim$ 5 in different plate aspect ratios are used. Statistical characteristics of these averaged loads are then obtained using the given COV values, see Table 5. Number of simulation cycles is increased from 1,000 to 10.000, and the probabilities of failure of all the plies in the plates are estimated with respect to each number of simulation cycles. The coefficients of variation are calculated to measure the statistical accuracy of the estimated probabilities of failure at the plies. The results reveal that among the estimated ply probabilistic failures, ply-8 which is in the outmost tension has the significant probability of failure. The other plies have negligible probabilistic failures from practical engineering design viewpoints or are judged to be totally safe under the given transverse lateral pressure loads. These patterns are almost identical regardless of the different limit state equations used and the different plate aspect ratios considered (This is the reason why the results for ply-8 are only ones shown in Figure 2). The estimated probabilistic failures at ply-8 are different according to the plate aspect ratios and the limit state equations, see Figure 2-(a), (c) and (e). The values lie between a minimum of 26.97% and a maximum of 76.15% for the square plate, and between a minimum of 32.45% and a maximum of 67.39% for the plate having plate aspect ratio of 1.5, and between a minimum of 37.69% and a maximum of 61.09% for the plate having plate aspect ratio of 2.0. Maximum stress limit state equation is the most conservative one by estimating the highest probabilistic failure and Tsai-Wu limit state equation is the most optimistic one by estimating the lowest probabilistic failure for the plates having plate aspect ratios of 1.0 and 1.5. However, in case of the plate having plate aspect ratio of 2.0, Tsai-Hill limit state equation is the most conservative one. Statistical accuracy of these estimated probabilistic failures at the plies is increased as simulation cycles increase by showing the convergence of the coefficients of variation, which are shown in Figure 2-(b), (d) and (f). Generally speaking, the ply-level analysis predicts the probability of failures at the outmost 0^o plies and indicates that the plies in tension will have more chance of failure than those in compression. The results also show that there is a modeling uncertainty due to the use of the different limit state equations. The ply-level reliability analysis results can be used to construct failure trees of the plies in the plates. Since all the plies except ply-8 have negligible probabilistic failures, it is necessary to consider the extreme mean values of the basic design variables corresponding to transverse lateral pressure loads with the same coefficient of variance for generating ply failure sequences accurately. This consideration is simply done by increasing the applied transverse lateral pressure loads by 50%. The estimated ply failure sequences of the plates are shown in Table 6. From this analysis, it can be observed that the failures of plies always start at ply-8 and develop from the outside plies to the middle plies of the plates. Although the transverse lateral pressure loads are increased by 50%, the middle plies, ply-3, 4 and 5, have 0% of failures. It is interesting to note that maximum strain limit state equation predicts slightly different ply failure sequences for the plates having aspect ratios of 1.5 and 2.0, and the same observation can be applied to Tsai-Hill limit state equation for the plate having aspect ratio of 2.0. This analysis is performed based on 10,000 simulation cycles. It should be mentioned that these probabilistic ply failure sequences are generated within the context of the FPF analysis.

6.2 Probabilistic Strength Distributions of the Plates

Probabilistic strength distributions of the plates are estimated by considering transverse lateral pressure load basic design variables as deterministic values. In ply-level analysis, the maximum load carrying plies are identified. Deterministic loads are increased for these plies from a small load which produces 0% of failures until a load which yields 100% of failures. The analysis results are shown in Figure 3. From Figure 3, the following observations are made. Maximum stress limit state equation predicts the minimum probabilistic strength distribution in both low and high probability of failure regions (i.e. 5% and 95% probabilities of failure correspond to low and high probability of failure regions) for the square plate, see Figure 3-(a). However, as the plate aspect ratio increases to 1.5 and 2.0, both maximum stress and Tsai-Hill limit state equations predict the minimum probabilistic strength distribution in both low and high probability of failure regions, see Figure 3-(b) and (c). On the other hand, maximum strain and Tsai-Wu limit state equations predict the maximum probabilistic strength distribution in low and high probability of failure regions respectively regardless of the plate aspect ratios. As the plate aspect ratios are increased from 1.0 to 2.0, not only the probabilistic strength distributions start to appear at the smaller transverse lateral pressure loads, but they also get distributed more closely to each other than before while keeping the same order of probabilistic strength distributions. Similar to the ply-level reliability analysis, a modeling uncertainty due to the use of the different limit state equations is also observed in this analysis. From this analysis, a design load which has a specific probability of occurrence for the plates can be deduced. For instance, if designers want to get 90% of reliability design load for a simply supported square $[0^o/90^o/0^o/90^o]_s$, which has the present statistical characteristics of material properties and geometry, then from Figure 3-(a) this approximates to 43.0 - 52.5 MPa design load range will be achieved. In all these cases, the probabilistic strength distributions are generated based on 1,000 simulation cycles with respect to the each deterministic transverse lateral pressure load. Percent error can be obtained through (22); When 50% of reliability is concerned, the value for the probabilistic strength distributions in Figure 3 is 6.3%. Thus, it is 95% likely that the actual probabilistic strength distributions will be within 0.5 ± 0.032.

6.3 Sensitivity Analysis of Basic Design Variables

The sensitivity of each basic design variable to the estimated reliabilities of the plates is investigated by changing the mean of only one basic design variable systematically. For instance, if the sensitivity of E_{11} is to be investigated, then the plate reliabilities are estimated with respect to non-dimensional values, $(E_{11} - iE_{11})/E_{11} (i = \pm0.2, \pm0.4, \pm0.6, \pm0.8)$ with the same coefficient of variance. The results are shown in Figure 4 for the square plate, Figure 5 for the plate having plate aspect ratio of 1.5 and Figure 6 for the plate having plate aspect ratio of 2.0, respectively. From these Figures it can be noticed that the sensitivity of each basic design variable to the estimated plate reliabilities is not affected by the consideration of different plate aspect ratios. However, it can be mentioned that the sensitivities of $G_{12}, G_{13}, G_{23}, X_T$ and Y_C are getting less varied as the plate aspect ratios are increased. When maximum stress, maximum strain and Tsai-Hill limit state equations are used, E_{11}, E_{22} and Y_T affect the estimated plate reliabilities most, while when Tsai-Wu and Hoffman limit state equations are used, E_{11}, E_{22}, X_C and Y_T affect the estimated plate reliabilities most. R, S and T basic design variables have constant sensitivities to the estimated plate reliabilities regardless of the use of different limit state equations and different plate aspect ratios. In general, $E_{11}, G_{12}, G_{13}, G_{23}, X_T$ and Y_T have positive sensitivities and E_{22}, X_C and Y_C have negative sensitivities to the estimated plate reliabilities, respectively. Throughout these analyses, 1,000 simulation cycles are used. Thus, the percent error related to this analysis can be calculated from (22).

7 Concluding Remarks

Previously developed reliability analysis procedures based on the Monte Carlo simulation method [Jeong and Shenoi, 1998a, 2000a] have been applied to the strength of rectangular thick simply supported $[0^o/90^o/0^o/90^o]_s$ plates. Thus, this paper looked at the theoretical application of the successfully developed reliability analysis procedures to thick FRP laminated plates under lateral pressure loads. Ply-/laminate-level reliability analyses including failure tree analysis and sensitivity analysis of basic design variables with respect to the estimated plate reliabilities are performed. Numerical applications indicated that the use of the present failure analysis capabilities could provide rational solutions to the design of laminated composite plates. It should be mentioned that all the reliability analysis results in this paper are estimated within the context of the FPF analysis. However if the same levels of reliability analyses are to be performed on FRP laminated plates within the context of progressive failure analysis, then the present reliability analysis procedures have to be modified to take into account large deflection of the plates, material property degradations and an effective structural reliability method which can reduce computational cost dramatically, see Jeong and Shenoi(1999, 2000a).

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Table 1: The coefficients F_i and F_{ij} values for laminated plate failure criteria

Failure criteria and coefficients	Maximum stress failure criterion	Hoffman's Failure criterion	Maximum strain failure criterion	Tsai-Hill's Failure criterion	Tsai-Wu's failure criterion
F_1	$X_t^{-1} - X_c^{-1}$	$X_t^{-1} - X_c^{-1}$	$(X_t^{-1} - X_c^{-1}) - \upsilon_{TL}(Y_t^{-1} - Y_c^{-1})$	$\cdot$	$X_t^{-1} - X_c^{-1}$
F_2	$Y_t^{-1} - Y_c^{-1}$	$Y_t^{-1} - Y_c^{-1}$	$-\upsilon_{TL} E_L E_T^{-1}(X_t^{-1} - X_c^{-1}) + (Y_t^{-1} - Y_c^{-1})$	$\cdot$	$Y_t^{-1} - Y_c^{-1}$
F_{12}	$-\frac{1}{2} F_1 F_2$	$-\frac{1}{2}(\frac{1}{X_t X_c} + \frac{1}{Y_t Y_c})$	$-\upsilon_{TL} E_L E_T^{-1}(X_t X_c)^{-1} - \frac{1}{2}(1 + \upsilon_{TL}^2 E_L E_T^{-1})$ $\times(X_t^{-1} - X_c^{-1})(Y_t^{-1} - Y_c^{-1}) - \upsilon_{TL}(Y_t Y_c)^{-1}$	$-\frac{1}{2} X^{-2}$	$-\frac{1}{2}(\sqrt{X_t X_c Y_t Y_c})^{-1}$
F_{11}	$(X_t X_c)^{-1}$	$(X_t X_c)^{-1}$	$(X_t X_c)^{-1} + \upsilon_{TL}(X_t^{-1} - X_c^{-1})$ $\times(Y_t^{-1} - Y_c^{-1}) + \upsilon_{TL}^2 (Y_t Y_c)^{-1}$	X^{-2}	$(X_t X_c)^{-1}$
F_{22}	$(Y_t Y_c)^{-1}$	$(Y_t Y_c)^{-1}$	$\upsilon_{TL}^2 E_L^2 E_T^{-2}(X_t X_c)^{-1} + \upsilon_{TL} E_L E_T^{-1}$ $\times(X_t^{-1} - X_c^{-1})(Y_t^{-1} - Y_c^{-1}) + (Y_t Y_c)^{-1}$	Y^{-2}	$(Y_t Y_c)^{-1}$
F_{44}	R^{-2}	R^{-2}	R^{-2}	R^{-2}	R^{-2}
F_{55}	S^{-2}	S^{-2}	S^{-2}	S^{-2}	S^{-2}
F_{66}	T^{-2}	T^{-2}	T^{-2}	T^{-2}	T^{-2}

Table 2: Material test methods

Tests	Test Methods
Tension	ASTM D3039
Compression	2TRI
Shear	ASTM D4255-83 B
Inter-laminar shear	MDC-J1980A

Table 3: *Statistical characteristics of the basic design variables*

(A normal probability distribution is assumed throughout)

Basic design variables	Mean values	COV
E_{11}	133.92 GPa	0.04618
E_{22}	8.84 GPa	0.03959
G_{12}	4.45 GPa	0.03146
G_{13}	4.45 GPa	0.03146
G_{23}	2.78 GPa	0.03129
v_{12}	0.336	0.05952
v_{21}	0.022	0.05952
X_T	1787.0 MPa	0.14270
X_C	1185.0 MPa	0.13502
Y_T	58.36 MPa	0.11138
Y_C	249.96 MPa	0.07437
R	92.60 MPa	0.04125
S	106.93 MPa	0.02434
T	106.93 MPa	0.02434
a	600.0 mm	0.00735
b	600.0 mm	0.00735

Table 4: Validation of the FPF analysis formulation

Failure criteria	Non-dimensional FPF load $(qb^4E_{22}^{-1}h^{-4})$		Non-dimensional FPF deflection (wh^{-1})	
	Turvey and Osman	Present	Turvey and Osman	Present
Maximum stress	11295.7	11299.2	49.43	49.42
Maximum strain	12141 0	12147.9	53.13	53.14
Tsai-Hill	11367.8	11355.6	49.75	49.67
Tsai-Wu	11881.2	11885.1	51.99	51.99
Hoffman	11310.6	11314.3	49.50	49.49

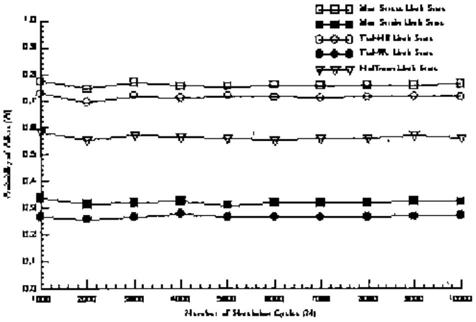
Table 5: Statistical characteristics of the transverse lateral pressure load basic design variables

Basic design variables	Mean values(MPa) for various A.R.(aspect ratio)			COV
	Plate A.R. of 1.0	Plate A.R. of 1.5	Plate A.R. of 2.0	
Load-1	50.61	31.81	27.19	0.11917
Load-2	61.42	36.31	29.70	0.11917
Load-3	51.75	31.98	27.16	0.11917
Load-4	64.04	37.34	30.23	0.11917
Load-5	55.30	33.90	28.41	0.11917
Average	56.62	34.27	28.54	0.11917

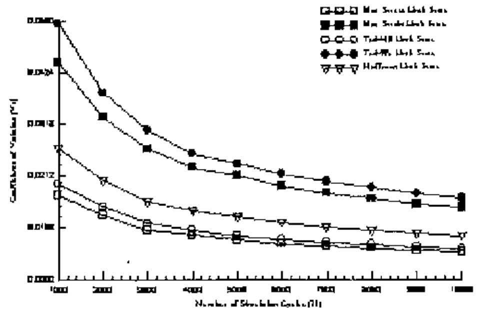
Table 6: Failure tree analysis results

Limit state equations	Play failure sequences		
	Plate A.R. of 1.0	Plate A.R. of 1.5	Plate A.R. of 2.0
Maximum Stress	$8 \rightarrow 7 \rightarrow 1 \rightarrow 2 \rightarrow 6 \rightarrow 3, 4, 5$	$8 \rightarrow 7 \rightarrow 2 \rightarrow 1 \rightarrow 6 \rightarrow 3, 4, 5$	$8 \rightarrow 7 \rightarrow 2 \rightarrow 6 \rightarrow 1, 3, 4, 5$
Maximum Strain	$8 \rightarrow 7 \rightarrow 1 \rightarrow 2 \rightarrow 3, 4, 5, 6$	$8 \rightarrow 2 \rightarrow 7 \rightarrow 1 \rightarrow 3, 4, 5, 6$	$8 \rightarrow 2 \rightarrow 7 \rightarrow 1, 3, 4, 5, 6$
Tsai-Hill	$8 \rightarrow 7 \rightarrow 1 \rightarrow 2 \rightarrow 6 \rightarrow 3, 4, 5$	$8 \rightarrow 7 \rightarrow 2 \rightarrow 1 \rightarrow 6 \rightarrow 3, 4, 5$	$8 \rightarrow 2 \rightarrow 7 \rightarrow 6 \rightarrow 1, 3, 4, 5$
Tsai-Wu	$8 \rightarrow 7 \rightarrow 1, 2, 3, 4, 5, 6$	$8 \rightarrow 7 \rightarrow 1, 2, 3, 4, 5, 6$	$8 \rightarrow 7 \rightarrow 2 \rightarrow 1, 3, 4, 5, 6$
Hoffman	$8 \rightarrow 7 \rightarrow 1 \rightarrow 6 \rightarrow 2, 3, 4, 5$	$8 \rightarrow 7 \rightarrow 2 \rightarrow 6 \rightarrow 1, 3, 4, 5$	$8 \rightarrow 7 \rightarrow 2 \rightarrow 6 \rightarrow 1, 3, 4, 5$

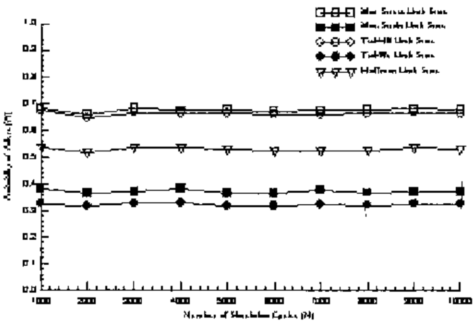
Figure 1: Flowchart for the reliability analysis of laminated plates using Monte Carlo simulation method



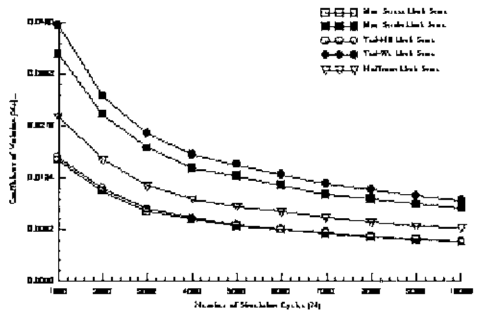
(a)



(b)



(c)



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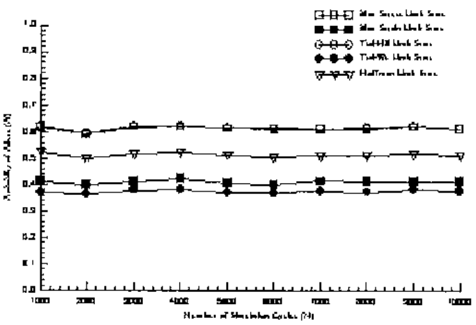
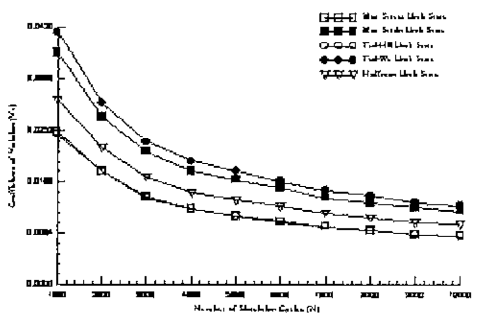
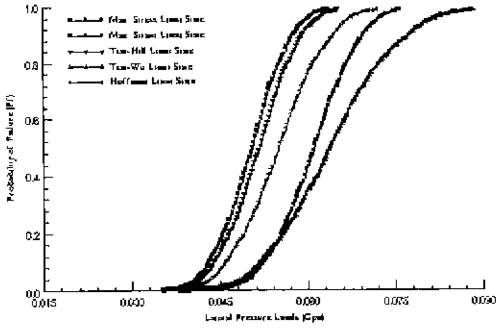


(e)

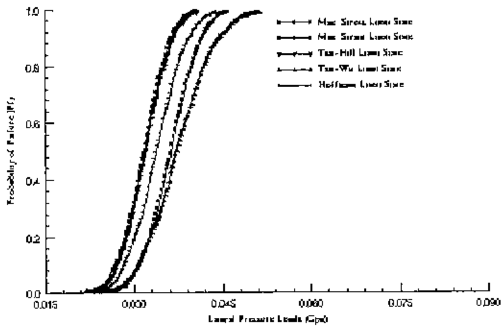


(f)

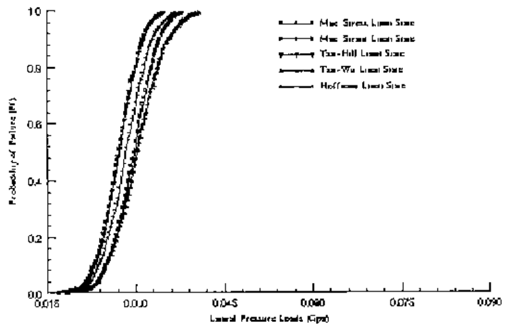
Figure 2: Probabilistic failure analysis at 8-ply of simply supported $[0^0/90^0/0^0/90^0]_s$ splates and their coefficients of variation: figures (a) and (b) for plate aspect ratio of 1.0; figures (c) and (d) for plate aspect ratio of 1.5; and figures (e) and (f) for plate aspect ratio of 2.0



(a)



(b)



(c)

Figure 3: Probabilistic strength distributions of simply supported $[0^0/90^0/0^0/90^0]_s$ plates: figures (a), (b) and (c) for plate aspect ratios of 1.0, 1.5 and 2.0, respectively

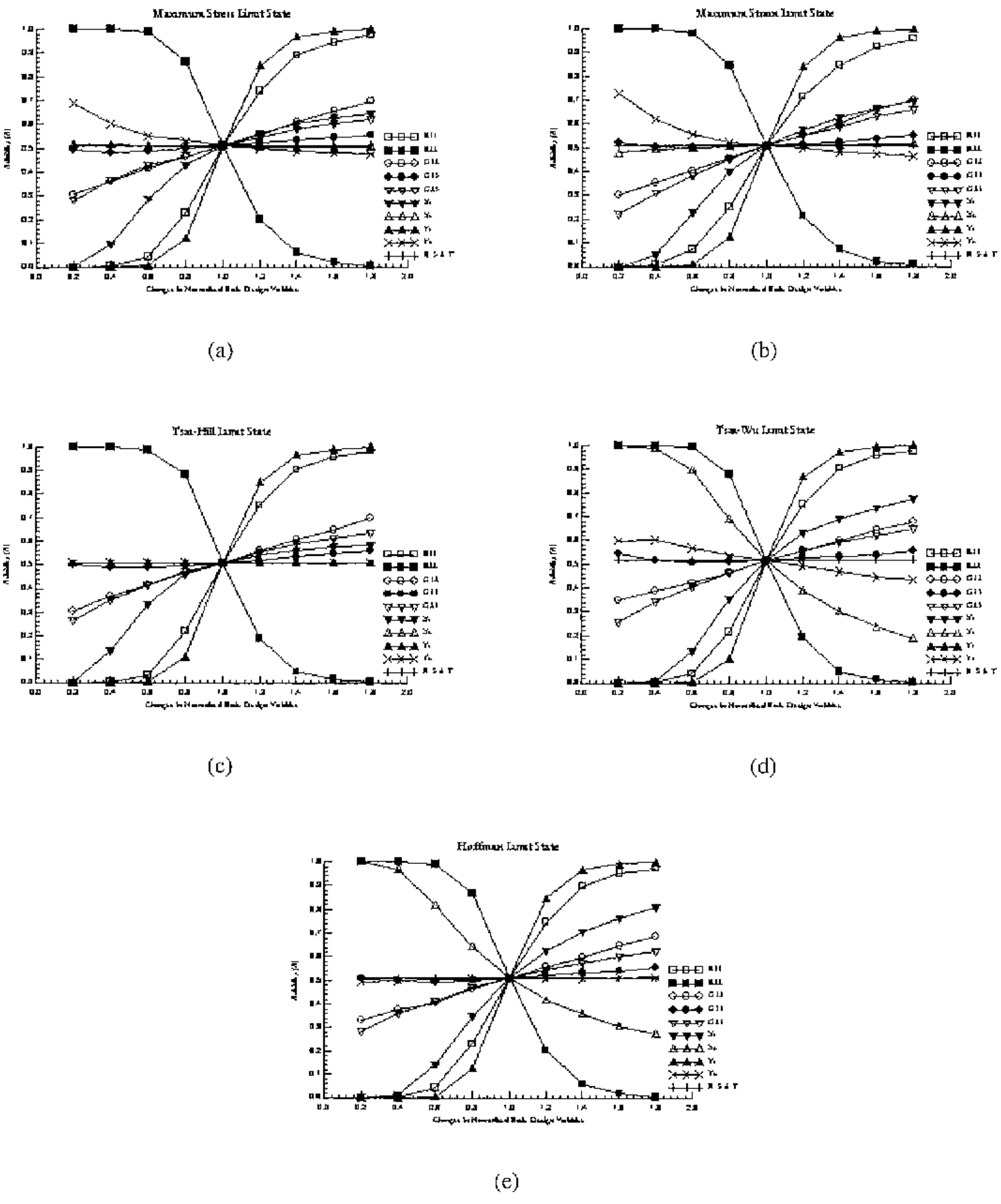
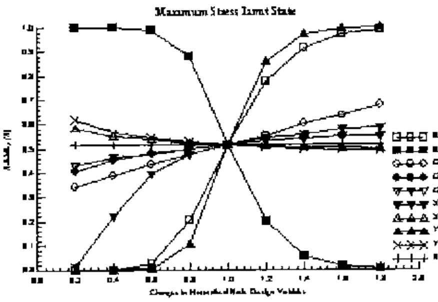
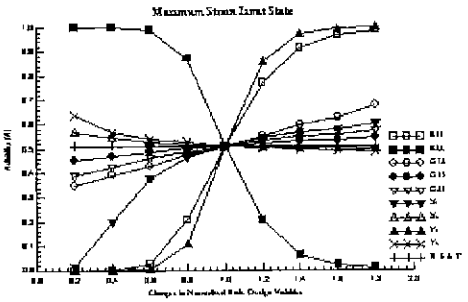


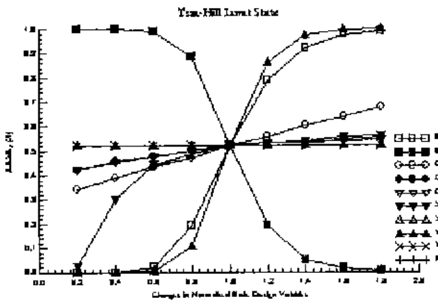
Figure 4: Sensitivity analysis of the basic design variables of the simply supported square plate of lay-up $[0^0/90^0/0^0/90^0]_s$



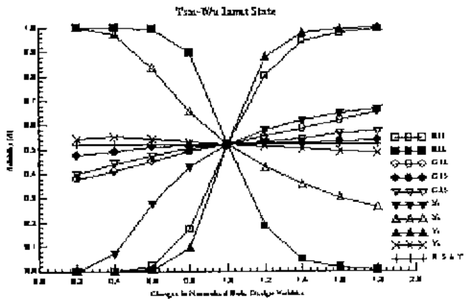
(a)



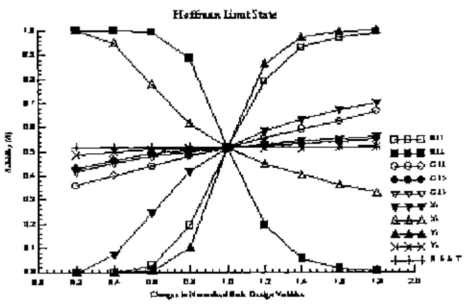
(b)



(c)



(d)



(e)

Figure 5: Sensitivity analysis of the basic design variables of the simply supported plate of lay-up $[0^0/90^0/0^0/90^0]_s$ having plate aspect ratio 1.5

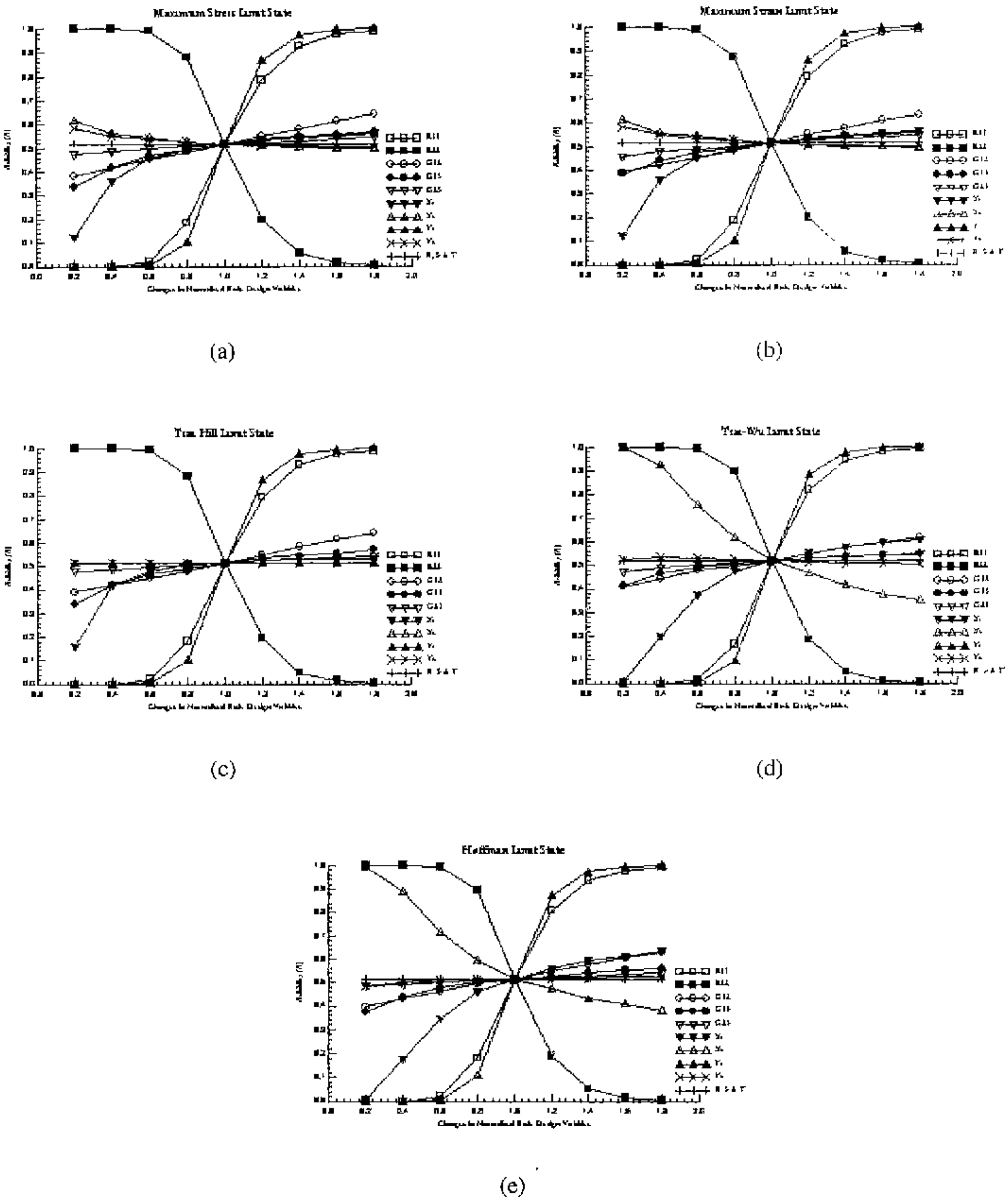


Figure 6: Sensitivity analysis of the basic design variables of the simply supported plate of lay-up $[0^0/90^0/0^0/90^0]_s$ having plate aspect ratio 2.0