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Parametric Analysis of Slamming Forces: Compressible and Incompressible Phases

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Abstract

The slamming force occurring in the free fall impact of cylindrical bodies on the water surface is analyzed in both compressible and incompressible stages. In the compressible phase the hydrodynamic analysis is carried on by the acoustic approximation, obtaining a closed form expression for the maximum impact force. The incompressible analysis is approached through an unsteady boundary element method to compute the free surface evolution and the slamming force on the body. A similar behavior seems to characterize the maximum slamming force versus a dimensionless mass parameter.

Keywords: slamming, acoustic approximation, boundary element method

1 Introduction

The free fall impact of two dimensional bodies on the water surface is analyzed and particular attention is addressed to the arising hydrodynamic forces. Air cushion and surface tension effects are neglected: the former may be important for very low deadrise angles, while the latter affects the solution only on the smallest scales. As usually done, the flow is assumed to be irrotational and the fluid inviscid: this allows the velocity field to be described in terms of a velocity potential ϕ.

Two different phases in the water entry process may be identified (Korobkin and Pukhnachov 1988). In a very early stage of the impact, the flow is dominated by compressibility effects where small particle displacements are observed and pressure waves are radiated into the fluid. Successively, due to the entry velocity reduction, a second stage follows in which large particle displacements are produced and compressibility effects can be definitively neglected. In particular the formation of a water jet at the edge of the free surface is observed.

In the first stage the acoustic approximation(Skalak and Feit 1966) is employed to recover the pressure field on the body, while, in the incompressible phase, the unsteady flow field is computed by means of a boundary element method (Zhao and Faltinsen 1993). In both cases the coupling between hydrodynamic forces and the equation of the rigid body motion is used to compute the actual drop velocity. The increase in wetted length produces a growth of the slamming force. On the other hand, the drop velocity decay causes a reduction in the slamming pressure. The two combined effects yields a maximum in the slamming force whose behavior versus characteristic dimensionless parameters is studied.

2 Compressible Stage

The slamming force evolution on a cylindrical body with the y axis parallel to the water surface, is here investigated. A general relationship between the hydrodynamic force and the body's shape sectional equation is determined. When the wedge and the circular cylinder cases are considered, the analytical expression of the corresponding maximum impact force is determined.

It is assumed that the acoustic approximation is able to describe the pressure field on the body surface, as specified in (Skalak and Feit 1966). For a blunt shaped body this is generally possible if the initial drop velocity V_0 is small compared with the speed of sound in water c. Under this conditions, if the fluid is at rest before the impact, the potential velocity ϕ satisfies the following equation:

$$\nabla^2\phi(x,t) - \frac{1}{C^2}\frac{\partial^2\phi(x,t)}{\partial t^2} = 0, \qquad \frac{\partial\phi}{\partial z} = V_0 \;\; on \;\; S_B, \quad \phi = 0 \;\; on \;\; S_F,$$

where S_B is the wetted body surfaces, S_F the free surface and z the axis normal to the water plane directed downward. In the same hypotheses, the pressure and force respectively are (Skalak and Feit 1966):

$$p = -\rho\frac{\partial\phi}{\partial t}, \qquad F = \rho c \iint \frac{\partial\phi}{\partial z} d\xi d\eta$$

where ρ is the water density and ξ, η are cartesian coordinates of integration defined along the water plane. The integral is performed over the water surface region affected by the perturbation induced by the body entry.

Moreover, an edge Mach number $M_e = V_e/c$ larger than one is assumed, where V_e is the edge velocity, *i.e.* the velocity of the boundary line of the contact area S_B. When $M_e > 1$ the boundary moves faster than the perturbation waves and the previous integral must be calculated only on S_B where the integrand equals the body velocity. Finally, in this phase, both gravity force and the hydrostatic contribution are neglected.

Let the sectional body shape be described by the cartesian equation $x = f(z)$. The force per unit length along the y-axis is then:

$$F' = 2\rho c \int_0^{b(t)} \dot{\zeta} d\xi = 2\rho c\dot{\zeta} b(t) = 2\rho c\dot{\zeta} f(\zeta)$$

where ζ is the depth of the cylinder and $b(t)$ the distance of the boundary lines of S_B from the plane $x = 0$. The nonlinear equation of the body motion is then:

$$m'\ddot{\zeta} + 2\rho c\dot{\zeta} f(\zeta) = 0, \quad \longrightarrow \quad \dot{\zeta} = V_0 - \frac{2\rho c}{m'}\int f(\zeta)d\zeta \tag{1}$$

where m' is the body mass per unit length and the velocity is obtained by eliminating the time dependence. When using this relationship, the impact force can be expressed in term of the depth:

$$F'(\zeta) = 2\rho c\left(V_0 - \frac{2\rho c}{m'}\int f(\zeta)d\zeta\right) f(\zeta) \tag{2}$$

A relative maximum in the hydrodynamic force occurs when the force derivative equals zero. Therefore an ordinary equation is recovered whose solution gives $\zeta_{F\ max}$ that substituted into (2) provides the maximum value $F'_{max} = F'(\zeta_{F\ max})$. However it must be noticed that this analysis only holds when the edge Mach number is larger than one. This restraint leads to:

$$\frac{df}{dz}(\zeta)\left(M_0 - \frac{2\rho}{m'}\int f(\zeta)d\zeta\right) > 1 \tag{3}$$

Therefore, the maximum depth $\zeta_{\lim}$ below which the developed analysis is still valid is the solution of the equation corresponding to the inequality (3).

As a final remark, the evaluation of the effects of the impact force on the body velocity is of interest. In other words it is interesting to check whether the work done by the hydrodynamic force is significant or not with respect to the initial kinetic energy of the body. To this aim the following reduction factor is introduced:

$$f_\nu = \frac{\Delta V}{V_0} = \frac{V_0 - \dot{\zeta}|_{\zeta_{\lim}}}{V_0} = \frac{2\rho c}{m' V_0}\int_0^{\zeta_{\lim}} f(\zeta)d\zeta \tag{4}$$

2.1 Wedge

In this case the sectional equation is $x = f(z) = z/\tan\beta$, where β is the deadrise angle. After some mathematics equation (2) provides the maximum impact force and the slamming coefficient C_s:

$$F'_{max} = \frac{4}{3}V_0\sqrt{\frac{p_0 m'}{3\tan\beta}},\ \ p_0 = \rho V_0 c, \quad \longrightarrow \quad C_s = \frac{F'_{max}}{\frac{1}{2}\rho V_0^2 L} = \frac{8}{3}\sqrt{\frac{\mu}{3\tan\beta M_0}},\ \ \mu = \frac{m'}{\rho L^2}$$

where L is a characteristic length of the wedge (*e.g.* that defined in the following section). In particular this equation suggests a square law dependence of the slamming coefficient on the mass parameter μ. By using (4), the reduction factor takes the form: $f_\nu = 1 - \tan\beta/M_0$. Therefore the velocity reduction in the compressible impact stage is only important when the ratio between the tangent of the deadrise angle and the entry Mach number is small. However, keeping in mind that M_0 must be much smaller than one when the acoustic approximation is used, only when very small deadrise angles are considered a significant velocity reduction occurs.

2.2 Circular Cylinder

In this case the sectional equation is $x = \sqrt{2Rz - z^2}$, where R is the circle section radius. After some mathematics, the equation corresponding to (2) is:

$$C_s(\xi) = \frac{F'}{\frac{1}{2}\rho V_0^2 R} = \frac{4}{M_0}\left\{1 - \frac{2}{\mu M_0}\left[\cos^{-1}(1-\xi) - (1-\xi)\sqrt{2\xi - \xi^2}\right]\right\}\sqrt{2\xi - \xi^2},$$
$$\xi = \frac{\zeta}{R},\ \ \mu = \frac{m'}{\rho R^2}$$

whose maximum occurs in correspondence of :

$$\frac{(2\xi - \xi^2)^{3/2}}{1-\xi}\left[\cos^{-1}(1-\xi) - (1-\xi)\sqrt{2\xi - \xi^2}\right] = \frac{\mu M_0}{2} \tag{5}$$

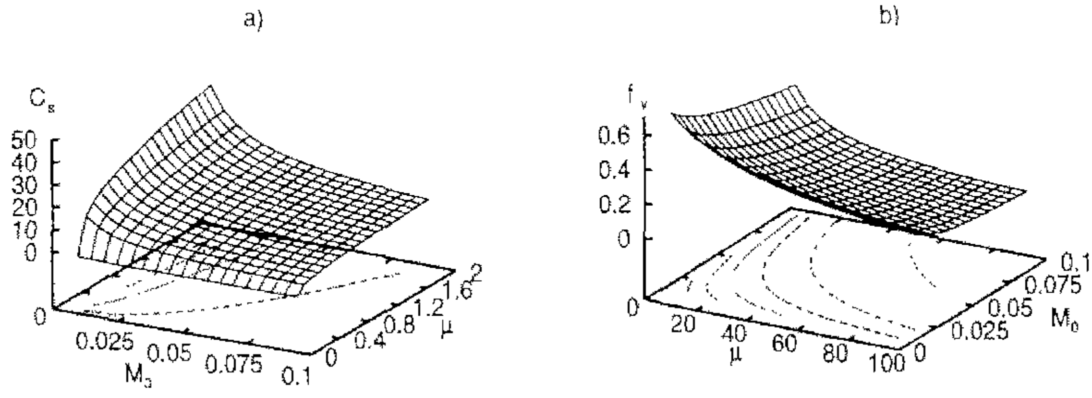

Figure 1: a) Slamming coefficient versus mass parameter and entry Mach number.
b) Velocity reduction factor versus mass parameter and entry Mach number.

The combined solution of the previous equations leads to the maximum slamming coefficient as a function of the two dimensionless parameters μ and M_0. Results, plotted in Figure la, show that high values of the slamming coefficient arise especially for small entry velocity Mach numbers and large mass parameters.

Finally the velocity reduction factor is recovered by using (4) that, after some mathematics, becomes: $f_\nu = 1 - \frac{\sqrt{2\xi_{lim} - \xi_{lim}^2}}{(1-\xi_{lim})}$. The plot of f_ν is sketched in Figure 1b: high values of the velocity reduction factor occur when small values of μ are considered.

3 Incompressible Stage

When the edge Mach number is much smaller than one, the incompressible approximation holds. In this case the velocity potential ϕ, satisfying the Laplace equation in the fluid domain, is solved by means of a boundary element formulation. In any point x inside the domain the velocity potential is:

$$\phi(x) = \int_{S_B \bigcup S_F} \left[\frac{\partial \phi}{\partial \nu}(y) G(x-y) - \phi(y) \frac{\partial G}{\partial \nu}(x-y) \right] dS(y) \tag{6}$$

where G is the two dimensional Green's function of the Laplace operator and ν is the inward unit normal vector. At each time step the normal derivative of ϕ, accordingly to the impermeability condition, is assigned on S_B while, on the free surface, the velocity potential is provided by the unsteady Bernoulli's equation:

$$\frac{D\phi}{Dt} = gz + \frac{|\nabla \phi|^2}{2} \tag{7}$$

According to Zhao and Faltinsen(1993), the gravity term in using (6) will not be considered in the following neglecting its effects on the dynamic evolution of the free surface. The velocity potential on S_B and its normal derivative on S_F are determined by solving the integral equation obtained by applying (5) on the boundary of the fluid domain $S_B \bigcup S_F$.

The problem is numerically solved by using a procedure similar to that proposed by Zhao and Faltinsen(1993). In the following only a brief description of the scheme and main differences with respect to the original one are discussed. The boundary of the fluid domain is discretized by segments on which the velocity potential and its normal derivative are assumed to be constant. A second order Runge-Kutta scheme is employed for time integration of the free surface evolution and the associated potential. The free surface is updated by moving the midpoint of each panel and then using a cubic spline to reconstruct the vertices distribution. For the sake of accuracy, at each time step, the panel distribution on the free surface in highly curved regions is refined.

The formation of the water jet at the edge of S_F is associated with the velocity singularity that needs a suitable procedure in the frame of the numerical scheme. Actually, during the first time steps the problem exhibits a weak velocity singularity, and the free surface is assumed to intersect the body. Successively, a thin water jet develops, characterized by strong velocity gradient normal to the body contour. When the distance of the midpoint of the first free surface panel from S_B becomes smaller than a cut-off length, the first panel is replaced by a segment orthogonal to the body surface. On this element a linear variation of the velocity potential is assumed as in Zhao and Faltinsen(1993). During time integration, whenever the angle between the second panel on the free surface and the body contour becomes smaller than a limiting angle of $2°$, it is excluded and the linear element is moved back. Once the velocity potential along the body surface is known, the dynamic pressure distribution is computed by the unsteady Bernoulli's equation, neglecting the gravity term.

To validate the numerical procedure, a constant drop velocity $V_0 = 2m/s$ is assigned to a wedge with a deadrise angle $\alpha = 10°$. In Figure 2a free surface configurations at several instant are shown. Analytical approaches to this problem show the existence of similarity solution: numerical scheme appears to be able to reproduce this behavior.

In order to simulate the free fall of the wedge, the equation of the rigid body motion is integrated to provide the actual drop velocity. The equation of the rigid body motion reads:

$$\frac{m'}{\rho}\ddot{\zeta} = -\frac{F'}{\rho} + g\left(\frac{m'}{\rho} - \Pi\right) \tag{8}$$

where Π is the body volume below the undisturbed free surface. The last contribution in (7) accounts for the balance between the body weight and the hydrostatic restoring force and is the only contribution for $V_0 \to 0$. In Figure 2b the time histories of the drop velocity is shown for several values of the mass parameter. This is defined by assuming as characteristic length the ratio V_0^2/g. In the first stage, due to gravity effects, the body accelerates. Successively, due to the combined effects of the slamming and the hydrostatic restoring forces, the drop velocity decays yielding a pressure reduction. In Figure 3a the time histories of the slamming force for the same mass parameters adopted before are shown, along with the line representing the numerical result obtained by Zhao & Faltinsen(1993) for a constant drop velocity (*i.e.* $\mu \to \infty$). It should be remarked that this line is obtained by using the slamming parameter $F'/(\rho V_0^3 t)$ reported in Zhao and Faltinsen(1993) and does not represent an effective time history. A sharply growth characterizes the slamming force at the beginning of the impact. Successively, the modeling of the jet formation induces numerical oscillations that are reduced by applying a proper filter to the time histories. Even though weak perturbations still remain, the occurrence of a maximum in the

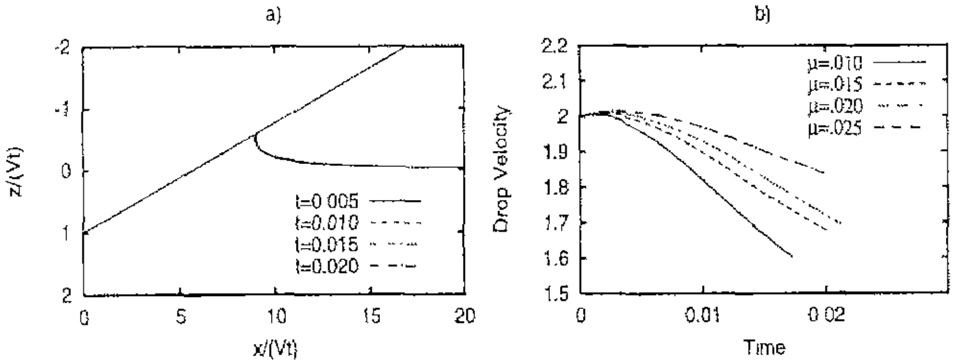

Figure 2: a) Dimensionless free surface elevation at different time steps.
b) Time histories of the drop velocity for several mass parameter.

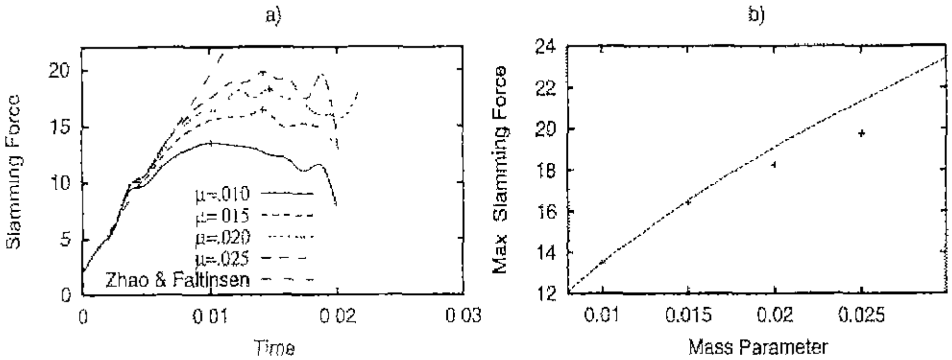

Figure 3: a) Time histories of the slamming force for several mass parameters.
b) Maximum slamming force versus the mass parameter.

slamming force (denoted by a cross) is evident and, as in Vorus(1996), this value grows with the mass parameter.

The behavior of the maximum is shown in Figure 3b versus the mass parameter, along with the square law obtained for the compressible stage. An analogous increasing trend is characteristic for both phases, although the correlation laws are slightly different.

4 Concluding Remarks

The impact of cylindrical bodies on the water surface has been analyzed in both the compressible and incompressible stages. In the former an analytical expression for the maximum of the slamming force is found for the wedge and the circular cylinder. In the latter a numerical simulation of the slamming process is carried out for the wedge and the maximum slamming force is evaluated. In both cases this value exhibits a similar increasing trend versus the mass parameter.

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