

# Passification of Nonlinear Systems via Dynamic Output Feedback<sup>1)</sup>

Young-Ik Son, Hyungbo Shim and, Jin-Heon Seo

**Abstract** - The relative degree one and weakly minimum-phase conditions have been major obstacles for passification of the given system. In this paper, a dynamic output feedback passifier which can remove the obstacles is presented. The proposed method does not require any modification of the given output except just adding a new term. Therefore, the scheme is more suitable for output feedback passification.

## 1. Introduction

Consider a smooth nonlinear system of the form

$$(P) : \begin{cases} \dot{x} = f(x) + g(x)u \\ y = h(x) \end{cases} \quad (1)$$

where  $x \in R^n$  is the state,  $u \in R$  is the control input and  $y \in R$  is the output.

The system (1) is said to be  $C^k$ -passive if there exists a  $C^k$  nonnegative real-valued function  $V(x)$ ,  $V(0) = 0$ , such that  $\forall x(0) = x_0 \in R^n, \forall t \geq 0$ , the following *dissipation inequality(DI)* holds:

$$V(x(t)) - V(x_0) \leq \int_0^t u^T(\tau)y(\tau) d\tau \quad (2)$$

It was shown in [2, Th. 4.10] that, under some mild assumptions, the system (1) is globally *state* feedback equivalent to a  $C^2$ -passive system with a positive definite storage function if and only if the system (1) has relative degree one and is globally weakly minimum-phase. In particular, the system is put into the global normal form

$$\begin{cases} \dot{z} = q_0(z) + q_1(z, y) \\ \dot{y} = b(z, y) + a(z, y)u \end{cases} \quad (3)$$

where  $a(z, y)$  is nonsingular for any  $(z, y) \in R^n$  and the origin of  $\dot{z} = q_0(z)$  is stable with a Lyapunov function  $W(z)$ . Then, by a regular feedback

$$u = -a^{-1}(z, y)(b(z, y) + L_{q_1}^T W(z, y)) + a^{-1}(z, y)r \quad (4)$$

the closed-loop system becomes passive from  $r$  to  $y$ . (Indeed, it can be shown by taking derivative of a storage function  $W(z) + \frac{1}{2}y^T y$ .) Moreover, a solution to the problem of passification via *output* feedback has also proposed in [5].

However, when the given system has the relative degree greater than one or the system is not weakly minimum-phase, it is impossible to make the system passive by a feedback because the condition, relative degree one and weakly minimum-phase, is not only sufficient but also necessary for feedback passification with the given output function  $h(x)$  [2]. Therefore, the class of systems is severely restricted to which the passification approach can be applied.

To circumvent this structural obstacles many researches including recursive step-by-step constructions in [10] have been made recently (see also [8], [9] and references therein). However, in those previous results only one of the obstacles to feedback passification is concerned and the given output  $h(x)$  is totally ignored. Moreover, because all the existing methodologies are approached by the state feedback, it seems almost impossible to apply them to the output feedback passification.

The purpose of this paper is to study the methodology which removes the obstacles to passification. The proposed method is to construct a new system to be interconnected parallel with the given system and to render the composite system passive by a feedback law. Although the design of the additional system is not an easy task in general, the advantage of our method is that it does not require any modification of the given output function  $h(x)$  except adding a new term. We utilize this advantage for the output feedback passification inspired by Jiang and Hill [5].

**Notations** : A function is said to be  $C^k$  when it is con-

1) This work was supported by the Brain Korea 21 Project.  
 Manuscript Received January 12, 2000 ; Accepted May 6, 2000.  
 Young I. Son (School of Electrical Engineering, Seoul National University, Seoul, Korea)  
 Hyungbo Shim (Automation and Systems Research Institute, Seoul National University, Seoul, Korea/Center for Control Engineering & Computation, University of California, Santa Barbara, USA)  
 Jin H. Seo (School of Electrical Engineering, Seoul National University, Seoul, Korea)

tinuously differentiable  $k$  times, and a smooth function implies  $C^\infty$  function.  $V: R^n \rightarrow R$  is said to be *positive definite* if  $V(0) = 0$  and  $V(x) > 0$  for all  $x \neq 0$ , and *proper* if, for any  $a \in R$ , the set  $V^{-1}([0, a]) = \{x \in R^n: 0 \leq V(x) \leq a\}$  is compact. Lie derivative of a vector field  $f$  with respect to  $V$  is defined to be  $L_f V(x) = dV(x) \cdot f(x)$  where  $dV(x)$  is the gradient of the function  $V$ .

**Acronyms :** CT = Coordinate Transformation; SISO = Single-Input Single-Output; ZD = Zero Dynamics.

## 2. Main Idea

We are interested in the passification of the SISO system (1) of which only the output  $y$  is available. When the given system (1) has relative degree one and is weakly minimum-phase, the passification is achieved by state feedback in [2] and by output feedback in [5], respectively. A simple interpretation of the notion of relative degree  $\rho$  is exactly equal to the number of times one has to differentiate the output  $y(t)$  at time  $t = t_0$  in order to have the value  $u(t_0)$  of the input explicitly appearing [4]. To deal with the case that relative degree is greater than one or is not even well-defined, we specifically assume the following.

**Assumption 1.** For the system (1),  $L_g h(x) = 0, \forall x \in R^n$ .

By Assumption 1, the standard feedback passification is not possible with the given  $y$  (see [2]). Instead, consider a new SISO system  $(V)$  of the form

$$(V): \dot{\eta} = k(\eta) + m(\eta)u, \quad y_\eta = l(\eta) \quad (5)$$

where  $\eta \in R^p$  and suppose the system has relative degree one, that is, for the system (5)

$$L_m l(\eta) \neq 0, \quad \forall \eta \in R^p \quad (6)$$

Then the parallel interconnection of two systems,  $(P)$  and  $(V)$  (see Fig. 1), constitutes a new system of the form

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{\eta} \end{pmatrix} = \begin{pmatrix} f(x) \\ k(\eta) \end{pmatrix} + \begin{pmatrix} g(x) \\ m(\eta) \end{pmatrix} u \\ \bar{y} = h(x) + l(\eta) \end{cases} \quad (7)$$

which also has relative degree one by (6).

Now suppose the system (7) is put into the following global normal form with a suitable CT [1],

$$\begin{aligned} \dot{z} &= q_0(z) + q_1(z, \bar{y})\bar{y}, \quad z \in R^{n+p-1} \\ \bar{y} &= b(z, \bar{y}) + a(z, \bar{y})u \end{aligned} \quad (8)$$

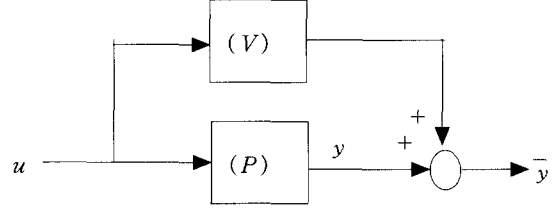


Fig. 1 Composite System

Since  $\bar{y} = L_f h(x) + L_k l(\eta) + L_m l(\eta)u$ ,  $a(z, \bar{y})$  is the  $(z, \bar{y})$ -coordinate expression of  $L_m l(\eta)$  and does not vanish by (6).

When the composite system is weakly minimum-phase (i.e., there exists a  $C^k$  positive definite and proper function  $W(z)$ ,  $k \geq 2$ , such that  $L_{q_0} W(z) \leq 0 \quad \forall z \in R^{n+p-1}$ ), the system can be rendered passive by output feedback under the following assumptions. These results are adopted from [5].

**Assumption 2.** There exist two smooth functions  $a_1(z) > 0$  and  $a_0(\bar{y})$  so that  $a(z, \bar{y}) = a_1(z)a_0(\bar{y})$  for each pair  $(z, \bar{y})$ . Moreover, assume that the  $a_0(\bar{y})$  is globally invertible for all  $\bar{y} \in R$ .

**Assumption 3.** There exist two nonnegative functions  $\phi_1$  and  $\phi_2$  such that, for all  $(z, \bar{y})$

$$\begin{aligned} |L_{q_1} W(z, \bar{y}) + a_1^{-1}(z)b(z, \bar{y}) + \\ \frac{1}{2} \frac{\partial}{\partial \bar{y}} a_1^{-1}(z, \bar{y})\bar{y}| \leq \phi_1(\bar{y})|\bar{y}| + \phi_2(\bar{y})|L_{q_0} W(z)|^{\frac{1}{2}} \end{aligned} \quad (9)$$

**Theorem 1.** Under Assumptions 1-3, the system (7) is rendered passive from  $r$  to  $\bar{y}$  by the output feedback

$$u = a_0^{-1}(\bar{y})(-\varepsilon\bar{y} - \bar{y}\phi_1(\bar{y}) - \bar{y}\phi_2^2(\bar{y}) + r) \quad (10)$$

with an arbitrary constant  $\varepsilon \geq 0$ , provided that the ZD of the system (7) are stable.

**proof.** By the analysis of the proof [5, Prop. 4.1], the time derivative of a storage function  $V = W(z) + \frac{1}{2} a_1^{-1}(z) \bar{y}^2$  gets an inequality which leads to the passivity of the closed-loop system (8) and (10) with the new input  $r$ . Indeed,

$$\begin{aligned} \dot{V}(z, \bar{y}) &= L_{q_0} W + L_{q_1} W \cdot \bar{y} + \frac{1}{2} \frac{\partial}{\partial \bar{y}} a_1^{-1}(z, \bar{y}) \bar{y}^2 + a_1^{-1}(z) \bar{y}(b + a_1(z)a_0(\bar{y})u) \\ &= L_{q_0} W + \bar{y}(L_{q_0} W + \frac{1}{2} \frac{\partial}{\partial \bar{y}} a_1^{-1}(z, \bar{y}) \bar{y} + a_1^{-1}(z)b) + \bar{y}a_0(\bar{y})u \end{aligned} \quad (11)$$

With the inequality (9), the application of output feedback (10) and Schwarz's inequality give the following ine-

qualities.

$$\begin{aligned} \dot{V}(z, \bar{y}) &\leq L_{q_0}W + |\bar{y}|\phi_2(\bar{y})|L_{q_0}W|^{\frac{1}{2}} - |\bar{y}|^2\phi_2^2(\bar{y}) - \varepsilon\bar{y}^2 + \bar{y}r \\ &\leq \frac{3}{4}L_{q_0}W - \varepsilon\bar{y}^2 + \bar{y}r \end{aligned} \quad (12)$$

This completes the proof.  $\square$

Thus the overall controller is composed of the dynamic part of (5) and the regular feedback (10).

**Remark 1.** Once the given system is passified by a feedback, a simple additional feedback of the output renders the system asymptotically stable under detectability condition (see e.g. [10]). If the system is rendered (state) strictly passive with positive definite storage function, the additional feedback is no more required [6, Lemma 10.6]. If the system (7) is globally minimum phase, the system is also globally asymptotically stabilized by (10) with  $\varepsilon > 0$  and  $r = 0$ . That is, the system is asymptotically stabilized as well as passified by output feedback only.

### 3. A Closer Look in Special Cases

In this section the main idea is looked into more closely for two specialized systems. One is the system which has the input vector field  $g(x) = Gx + B$  and a linear system is the other. Moreover,  $m(\eta)$  and  $l(\eta)$  in the system (5) are supposed to be  $m(\eta) = M = [1 \ 0 \ \dots \ 0]^T$  and  $l(\eta) = L\eta = [1 \ 0 \ \dots \ 0]\eta = \eta_1$ , respectively. For these special cases, Assumption 2 and the relative degree condition (6) are always satisfied and Assumption 3 is relaxed in some sense. Furthermore, we provide an explicit form of CT to global normal form.

#### A. Special Nonlinear System Case

Consider the following interconnected system

$$\begin{cases} \begin{pmatrix} \dot{x} \\ \dot{\eta}_1 \\ \dot{\eta}_2 \end{pmatrix} = \begin{pmatrix} f(x) \\ k_1(\eta) \\ k_2(\eta) \end{pmatrix} + \begin{pmatrix} Gx + B \\ 1 \\ 0 \end{pmatrix} u \\ \dot{\bar{y}} = h(x) + \eta_1 \end{cases} \quad (13)$$

where  $x \in R^n$ ,  $\eta_1 \in R$  and  $\eta_2 \in R^{p-1}$ . Suppose the  $x$ -dynamics of (13) satisfy Assumption 1.

By a pre-CT,  $\begin{pmatrix} x \\ \eta_1 \end{pmatrix} \rightarrow \begin{pmatrix} \bar{x} \\ \eta_2 \end{pmatrix}$ , the above system (13) becomes

$$(C_b) : \begin{cases} \dot{\bar{x}} = f(\bar{x}) + (G\bar{x} + B)u \\ \dot{\bar{y}} = k_1(\bar{y} - h(\bar{x}), \eta_2) + L_f h(\bar{x}) + u \\ \dot{\eta}_2 = k_2(\bar{y} - h(\bar{x}), \eta_2) \end{cases} \quad (14)$$

Since the vector field  $(Gx + B)$  is complete and the system (14) has relative degree one, Proposition 9.1.1 in [4] can be applied in order to find a suitable CT which transforms (14) into global normal form.

**Remark 2.** The proposed CT in [4, Prop. 9.1.1] characterizes the ZD submanifold of order  $n+p-1$  which, in general, is not diffeomorphic to  $R^{n+p-1}$ . (See Remark 9.1.1 in [4].) However, since we took  $m(\eta) = [1 \ 0 \ \dots \ 0]^T$  and  $l(\eta) = \eta_1$ , our ZD submanifold is diffeomorphic to  $R^{n+p-1}$ . That is, we can take  $n+p-1$  coordinate functions out of  $n+p$  elements proposed in [4, Prop. 9.1.1].

In fact, by letting  $\chi \doteq \begin{bmatrix} Gx + B \\ 1 \\ 0 \end{bmatrix}$ , the CT is the flow

$$\Phi_{\bar{y}}^{\chi} \begin{pmatrix} \bar{x} \\ \bar{y} \\ \eta_2 \end{pmatrix} = \begin{pmatrix} \bar{x} + \int_0^{\bar{y}} (Gx(\tau) + B) d\tau \\ \bar{y} - \bar{y} \\ \eta_2 \end{pmatrix}. \quad \text{In the CT, the}$$

second element of new coordinate is always zero, which is ignored.

Under Assumption 1, the proposed CT is<sup>2)</sup>

$$\begin{cases} \dot{\xi} = e^{-G\bar{y}}x + \left( \int_0^{\bar{y}} e^{G\tau} d\tau \right) \cdot B = x + \left( \int_0^{\bar{y}} e^{G\tau} d\tau \right) (Gx + B) \\ \dot{z} = \eta_2 \\ \dot{\bar{y}} = \bar{y} + \eta_1 \end{cases}$$

with the inverse :  $x = e^{G\bar{y}}\xi + \left( \int_0^{\bar{y}} e^{G\tau} d\tau \right) \cdot B$ . Using this CT, the system (13) is rewritten as the following normal form

$$\begin{cases} \dot{\xi} = q_{01}(\xi, z) + q_{11}(\xi, z, \bar{y})\bar{y} \\ \dot{z} = q_{02}(\xi, z) + q_{12}(\xi, z, \bar{y})\bar{y} \\ \dot{\bar{y}} = k_1(\bar{y}, \xi, z) + L_f h(\bar{y}, \xi) + u \end{cases} \quad (15)$$

of which the ZD are

$$\begin{cases} \dot{\xi} = f(\xi) - (G\xi + B)(k_1(-h(\xi), z) + L_f h(\xi)) \doteq q_{01}(\xi, z) \\ \dot{z} = k_2(-h(\xi), z) \doteq q_{02}(\xi, z) \end{cases} \quad (16)$$

We should note that the input vector field of system (15) is one and this implies the Assumption 2 is satisfied.

**Corollary 1.** Suppose there exist  $k_1$  and  $k_2$  such that the system (15) is globally weakly minimum-phase. Moreover, assume that there exist two nonnegative functions  $\phi_1$  and  $\phi_2$  such that, for all  $(\xi, z, \bar{y})$

2) With the help of  $\frac{\partial}{\partial x} \Phi_t^f(x) f(x) + \frac{\partial}{\partial t} \Phi_t^f(x) \equiv 0$  where  $\Phi_t^f(x)$  is flow of  $\dot{x} = f(x)$  (see [6, p.96, Ex.2.46]), the input  $u$  does not appear in  $(\xi, z)$ -dynamics at our desire.

$$|L_1 h + L_{q_0} W(\xi, z, \bar{y})| \leq \phi_1(z, \bar{y})|\bar{y}| + \phi_2(z, \bar{y})|L_{q_0} W(\xi, z)|^{\frac{1}{2}},$$

where  $q_0 = [q_{01}^T \ q_{02}^T]^T$  and  $q_1 = [q_{11}^T \ q_{12}^T]^T$ . Then, global output feedback passification of the system (15) is achieved.

**proof.** By applying the following output feedback law

$$u = -\varepsilon \bar{y} - \bar{y} \phi_1(z, \bar{y}) - \bar{y} \phi_2^2(z, \bar{y}) + r \quad (17)$$

with an arbitrary constant  $\varepsilon \geq 0$ , the closed-loop system (15)-(17) is rendered passive from  $r$  to  $\bar{y}$ . It can be shown by taking the time derivative of a storage function  $W(\xi, z) + \frac{1}{2} \bar{y}^2$ .  $\square$

Comparing with Assumption 3, we can see that these  $\phi_1(z, \bar{y})$ ,  $\phi_2(z, \bar{y})$  contain  $z$  in Corollary 1. Thus these are some milder than the functions  $\phi_1(\bar{y})$ ,  $\phi_2(\bar{y})$  in Assumption 3.

The second special case is linear system.

### B. Linear System Case

Consider a linear SISO system

$$\dot{x} = Fx + Gu, \quad y = Hx \quad (18)$$

of which  $HG = 0$ . Thus, Assumption 1 is satisfied.

In the linear system case, Assumptions 2-3 are satisfied automatically when the system has relative degree one and is minimum phase. This implies that the output feedback passification is always possible for such linear system. (See [3, Remark 4.1].) Therefore, in the linear system case, if we construct an additional linear system such that the interconnected system has relative degree one and is minimum phase, the output feedback passification problem is solved.

For a new SISO linear system of the form

$$\dot{\eta} = K\eta + Mu, \quad y_\eta = L\eta, \quad (19)$$

we suppose  $M = [1 \ 0 \ \dots \ 0]^T$  and  $L = [1 \ 0 \ \dots \ 0]$  without loss of generality.<sup>3)</sup>

The parallel interconnection with (18) and (19) constitutes the following linear system

$$(C): \begin{cases} \begin{pmatrix} \dot{x} \\ \dot{\eta}_1 \\ \dot{\eta}_2 \\ \dot{y} \end{pmatrix} = \begin{pmatrix} F & 0 & 0 \\ 0 & K_{11} & K_{12} \\ 0 & K_{21} & K_{22} \end{pmatrix} \begin{pmatrix} x \\ \eta_1 \\ \eta_2 \end{pmatrix} + \begin{pmatrix} G \\ 1 \\ 0 \end{pmatrix} u \\ y = Hx + \eta_1 \end{cases} \quad (20)$$

Having a relative degree one, the composite system (20) is

transformed, by a CT of  $\begin{pmatrix} \xi \\ y \end{pmatrix} = \begin{bmatrix} I-GH & -G & 0 \\ H & I & 0 \\ 0 & 0 & I \end{bmatrix} \begin{pmatrix} x \\ \eta_1 \\ \eta_2 \end{pmatrix}$ , to the normal form of which the ZD are

$$\begin{pmatrix} \dot{\xi} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} F - GHF + GK_{11}H & -GK_{12} \\ -K_{21}H & K_{22} \end{bmatrix} \begin{pmatrix} \xi \\ z \end{pmatrix} \quad (21)$$

Finally, if we find a matrix  $K = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}$  such that the system matrix of (21) is Hurwitz, then the problem of output passification is solved.

## 4. An Example

Because the "passivity" has been widely used to analyze stability of a general class of interconnected nonlinear systems (see, e.g., [10]), we apply the proposed methodology to the stabilization of a system. We can see that the result is related to the low-order dynamic output stabilizing controller problem for nonlinear systems. As mentioned in Remark 1, the passification and stabilization of the following system is achieved at the same time via dynamic output feedback.

Consider a nonlinear SISO system of the form

$$(P_{ex}): \begin{cases} \dot{x}_1 = -x_1 - 2x_2 + x_3 + u \\ \dot{x}_2 = 0.5 \sin x_2 + x_3 \\ \dot{x}_3 = -2x_2 + u \end{cases}, \quad y = x_2. \quad (22)$$

Since the system (22) has relative degree 2, (22) satisfies Assumption 1.

If a SISO system  $(V_{ex}): \dot{\eta} = u, \quad y_\eta = \eta$  is connected parallel to the system (22), the composite system has relative degree one. The new output  $\bar{y} = y + \eta$ . Thus, by the CT proposed in the previous section

$$\begin{pmatrix} \xi_1 \\ \xi_2 \\ \xi_3 \\ \bar{y} \end{pmatrix} = \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ y \end{pmatrix},$$

the following normal form<sup>4)</sup> is obtained.

$$\begin{pmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \\ \dot{\bar{y}} \end{pmatrix} = \begin{pmatrix} -\xi_1 - 2\xi_2 - 0.5 \sin \xi_2 \\ 0.5 \sin \xi_2 + \xi_3 \\ -2\xi_2 - 0.5 \sin \xi_2 - \xi_3 \\ 0.5 \sin \xi_2 + \xi_3 \end{pmatrix} + \begin{pmatrix} -1 \\ 1 \\ -1 \\ 1 \end{pmatrix} \bar{y} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} u. \quad (23)$$

Moreover, with a Lyapunov function

3) This is obtained by a similarity transformation.

4) Note that  $z_2 = x_2 = y$ .

$W(\xi) = \frac{1}{2} \xi^T \begin{pmatrix} 1 & 0 & -1 \\ 0 & 7.25 & 2.25 \\ -1 & 2.25 & 4.25 \end{pmatrix} \xi$ , it can be shown that the

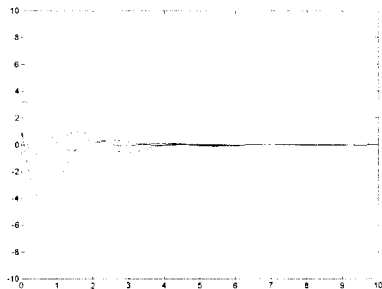
above system (23) has asymptotically stable ZD and satisfies Assumptions 2-3. Hence, by the following dynamic output feedback controller

$$u = a(y, \eta) = -5y - 0.5 \sin y - 2\bar{y} + r, \quad (24)$$

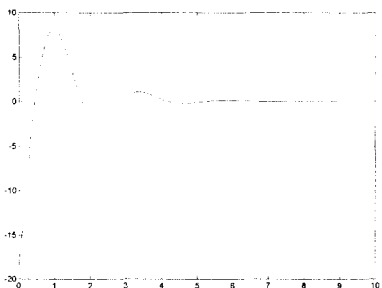
the strict passivity of the closed-loop system (22) and (24) from  $r$  to  $\bar{y}$  is obtained. In addition, the closed-loop system is globally asymptotically stable with  $r = 0$ . This is illustrated by the following simulation result (Fig. 2). In the simulation, an initial state  $[1 \ \pi \ 1]^T$  is used.

## 5. Conclusion

The problem of passification via dynamic output feedback is studied for nonlinear systems which do not satisfy the condition of relative degree one and weakly minimum-phase. The proposed methodology does not search for a new output as in [8] or [10], but makes a parallel interconnection with a new system to be designed. Therefore, the composite system's output is just the sum of the given output and an additional term. This gives some



(a) Controlled state  $\xi$



(b) Control Input  $u$

Fig. 2 Simulation Result

advantages for the *output* feedback passification. Under some conditions as in [5], the dynamic output feedback passification is achieved.

Further research should be performed for a systematic construction of the additional system which renders the whole system has relative degree one and is weakly minimum-phase.

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**Young-Ik Son** received the B. S. and M. S. degrees in Electrical Engineering from the Seoul National University, in 1995 and 1997, respectively. He is currently working toward the Ph. D. degree in the School of Electrical Engineering, Seoul National University, Seoul, Korea. His research interests include nonlinear systems control

theory, passive systems theory and the observer design for nonlinear systems.



clude the nonlinear observer, the analysis and control of nonlinear systems.

**Hyungbo Shim** received the B.S., M.S., and Ph.D. degrees from School of Electrical Engineering at Seoul National University, Korea, in 1993, 1995 and 2000, respectively. He is currently working at Automation and Systems Research Institute, Seoul National University as a postdoctoral researcher. His research interests include



1989 in the Department of Engineering at Texas Tech University, Lubbock. Since 1989, he has been with the School of Electrical Engineering at Seoul National University, Seoul, Korea, where he is currently a Professor. His research interests include nonlinear systems theory, large scale systems control, missile guidance problem using differential game theory and infinite dimensional system theory.

**Jin-Heon Seo** was born in Seoul, Korea, in 1952. He received the B. S. and M. S. degrees in Electrical Engineering from the Seoul National University, in 1978 and 1980, and the Ph. D. degree in electrical Engineering from University of California, Los Angeles, in 1985. He served as an Assistant Professor from 1985 to