International Journal of Ocean Engineering and Technology, Vol. 3, No.1, pp 49~55, April 2000 (ISSN 1229-1668)

Improvement on the Free Spanning Analysis of Offshore Pipelines

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(Received 13 November 2000, accepted 8 April 2000)

ABSTRACT. *Improvement was made on the free span analysis of the offshore pipelines. The effect of axial force (both tension and compressive force) can be explicitly applied to the current design code. The closed form solutions of beam-column equation were derived for the typical boundary conditions. The solutions can be used to find the natural frequencies of the span using the energy balance concept. The results can be applied to the current design code and will result more realistic calculation of free span lengths of offshore pipelines.*

KEY WORDS: Offshore Pipeline, Vortex Shedding, Allowable Free Span Length, Axial Force

1. Introduction

For a safe operation of offshore pipeline during and after installation, the free span lengths should be maintained within the allowable lengths which are determined during the design stage. Free spans may be caused by seabed unevenness and change of seabed topology such as scouring or sand waves. Once a free span longer than the allowable span length occurs, the free span may suffer the vortex-induced vibration (VIV) and consequently suffer the fatigue damages on the pipe.

The vortex shedding phenomenon results two kinds of periodic forces on a cylindrical pipe. A pipe will start to oscillate in-line when the flow velocity is low. As the flow velocity increases, vortices are shed alternately. As the flow velocity increases further, the cross-flow oscillation begins to occur and the vortex shedding frequency may approach to the natural frequency of the pipe span. Amplified responses due to the resonance between the vortex shedding frequency and natural frequency of the free span may cause a fatigue damage. Thus, the determination of the critical allowable span lengths under the various environments becomes an important part of a pipeline design.

During the extensive study of the vortex-induced vibration, it was found that the current design code is insufficient to include all the aspect of design, installation, and operation. The axial force (tension or compression) has significant effect to determine the critical allowable span lengths. Therefore, this study suggests some improvement on the current design code regarding the vortex-induced vibration and free span analysis.

2. Parameters of Vortex Shedding Analysis

Strouhal number, S_t

The vortex shedding frequency depends mainly on the diameter of cylindrical pipe and the velocity of the flow. It is dimensionless Strouhal number which is given by :

$$S_t = \frac{f_s D}{U} \tag{2.1}$$

where, f_s vortex shedding frequency

U : flow velocity normal to the pipe axis

D : diameter of the pipe

Reynolds number, R_e

The process of the vortex shedding changes with the Reynolds number, R_e, is given by

$$R_e = \frac{UD}{\nu} \tag{2.2}$$

where, ν . kinematic viscosity of fluid

Vortex shedding is well organized at sub-critical ($300 < R_e < 3\times10^5$) and trans-critical ($R_e > 3.5\times10^6$) ranges. At the critical range ($3\times10^5 < R_e < 3.5\times10^6$), the vortex shedding is disorganized and vortex-induced motion is insignificant.[1)]

Stability parameter, K_s

It has been demonstrated that the stability parameter determines uniquely the maximum amplitude of vibrations. [2)] Stability parameter K_s is defined as .

$$K_s = \frac{2m_e\delta}{\rho D^2} \tag{2.3}$$

where, δ : logarithmic decrement of damping

ρ : mass density of surrounding water

m_e : effective mass per unit length includes structural mass, added mass and the mass of any fluid contained within the pipe.

Reduced velocity, V_r

For determination of the velocity ranges of the occurrence of VIV, the Reduced velocity, V_r is used. V_r is defined as

$$V_r = \frac{U}{f_n D} \tag{2.4}$$

where, f_n : natural frequency of the pipe

Symmetric vortices are shed when V_r falls between 1.0 and 2.2. When V_r exceeds 2.2, vortices are shed alternately. According to the DnV code, in-line vortex shedding may occur when $1.0 < V_r < 3.5$. As flow velocity increases further, the cross-flow oscillation occurs and lock-in happens when V_r is about 5.0.

3. Allowable Span Length by DnV Code

3.1 Allowable Span Lengths

Allowable span lengths are governed by code limitations regarding maximum allowable stresses and by the onset of vortex shedding criteria. The allowable pipe span lengths are chosen as the lesser lengths found from the span length criteria :

- ANSI code: allowable stress (static).
- DnV code: onset of VIV (dynamic).

Most of the allowable span lengths are governed by VIV. Thus, this study is dealing with only the allowable span due to the vortex shedding.

3.2 Calculation of Allowable Span Lengths

In-line motion

According to the DnV code, in-line vortex shedding may occur when $1.0 < V_r < 3.5$ and $K_s < 1.8$. Once the range of the in-line oscillations has been established based on the upper and lower bound values of V_r, the corresponding pipe spans can be determined. Fig. A.3 in DnV code is used to obtain the lower bound value of V_r based on K_s for the onset of in-line motion.[3] The upper bound value of V_r is set at 3.5 The natural frequency of the pipe can then be calculated using the aforementioned relationship. The allowable pipe spans (lower bound values) are then computed by solving for the span length L in the following formula.

$$L = \left(\frac{EI}{m_e}\right)^{.25}\left(\frac{CV_r D}{2\pi U}\right)^{.5} \tag{3.1}$$

where, E . modulus of elasticity
I : moment of inertia
C : end boundary coefficient

Cross-flow motion

For cross-flow oscillations, vortex shedding may occur when $K_s < 16$ and V_r falls within the range as determined in Fig. A.5 in DnV code. The allowable pipe spans are then determined same procedure with Eq. (3.1) and cross-flow V_r

3.3 Maximum Amplitude of Vibration

In-line motion

Once the stability parameter is determined, the maximum amplitude of motion can be directly obtained from Fig. A.4 in DnV code.

Cross-flow motion

The maximum amplitude of cross-flow motion can be determined by Fig. A.6 in DnV code. The mode shape parameters are required to get the maximum amplitude of cross-flow motion.

Mode shapes

Free transverse vibration of Bernoulli-Euler beam is governed by :

$$\frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right) + \rho A\frac{d^2y}{dt^2} = 0 \tag{3.2}$$

where, y : displacement of the beam
ρA : the mass per unit length of the beam

Assuming a harmonic motion given by

$$y(x,t) = Y(x)\cos(\omega t - \alpha) \tag{3.3}$$

and substitute Eq. (3.3) into Eq. (3.2) to get the eigen-value equation.

$$EI\frac{d^4Y}{dx^4} - \rho A\omega^2 Y = 0 \tag{3.4}$$

On substituting

$$\beta^4 = \frac{\rho A\omega^2}{EI} \tag{3.5}$$

We obtain the fourth-order differential equation

$$\frac{d^4Y}{dx^4} - \beta^4 Y = 0 \tag{3.6}$$

for the vibration of a uniform beam.

The general solution of Eq. (3.6) may be written in the form

$$Y(x) = C_1 \sinh\beta x + C_2 \cosh\beta x + C_3 \sin\beta x + C_4 \cos\beta x \quad (3.7)$$

The four constants and β can be obtained using boundary conditions.

Fixed end boundary condition :

$$Y=0,\ \frac{dY}{dx}=0 \quad (3.8)$$

Pinned end boundary condition :

$$Y=0,\ \frac{d^2Y}{dx^2}=0 \quad (3.9)$$

Free end boundary condition :

$$\frac{d^2Y}{dx^2}=0,\ \frac{d^3Y}{dx^3}=0 \quad (3.10)$$

The mode shape for each boundary condition are as follow.

Fixed-free boundary condition :

$$Y(x) = C\{\cosh\beta x - \cos\beta x - K_r(\sinh\beta x - \sin\beta x)\} \quad (3.11)$$

where, $K_r = \frac{(\cosh\beta L + \cos\beta L)}{(\sinh\beta L + \sin\beta L)}$

Pinned-pinned boundary condition :

$$Y(x) = C\sin\frac{r\pi x}{L} \quad (3.12)$$

Fixed-pinned boundary condition :

$$Y(x) = C\{\cosh\beta x - \cos\beta x - K_r(\sinh\beta x - \sin\beta x)\} \quad (3.13)$$

where, $K_r = \frac{(\cosh\beta L - \cos\beta L)}{(\sinh\beta L - \sin\beta L)}$

Fixed-fixed boundary condition :

The equation of the mode shape is the same as Eq. (3.13), but the different boundary conditions result the different form of the mode shape.

Mode shape parameter, γ

The mode shape parameter, γ is defined as :

$$\gamma = Y_{max}\left\{\frac{\int_0^L Y^2(x)\,dx}{\int_0^L Y^4(x)\,dx}\right\}^{1/2} \quad (3.15)$$

where, Y_{max} : maximum value of the mode shape.

Once the mode shape parameter is obtained from Eq. (3.15), the maximum amplitude of cross-flow motion can be obtained from Fig. A.6 in DnV code.

3.4 Results by DnV Code Approach

A pipe diameter of 324 mm (12-inch) and wall thickness of 16 mm (0.625-inch) was used for the calculations. Table 1 shows mode shape factors for various mode shapes and boundary conditions. Table 2 shows vortex shedding frequencies for in-line and cross-flow directions as flow velocity increases. Fig. 1, Fig. 2, and Table 3 show allowable span lengths and amplitudes of motion for various boundary conditions.

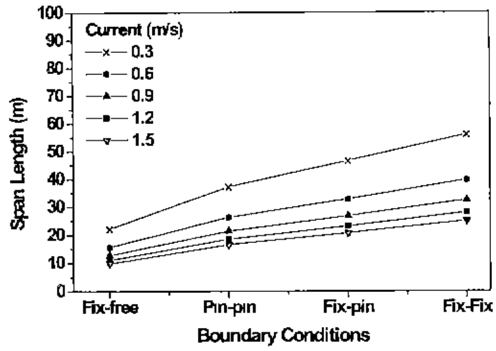

Fig. 1 Allowable Span Lengths(m) by DnV (In-line)

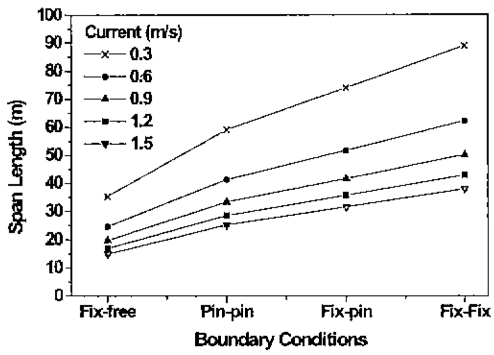

Fig. 2 Allowable Span Lengths(m) by DnV (Cross-flow)

As mentioned in Section 2, the maximum amplitude of vortex-induced vibration are uniquely determined by the stability parameter. This fact indicates that the maximum amplitude is not controlled by the fluid speed. If fluid speed increases, the allowable span length will be reduced and the maximum amplitude remains the same.

4. Effect of Axial Force on Span Analysis

4.1 Effect of Axial Force

The load on the pipeline during the operation is not the same as loading condition during the installation. During the installation, pipeline may have residual tension due to the lay-barge method.

During the operation, the pipeline may have operational load due to the operational pressure and temperature. This operational loading will cause very high compressive force into the part of the pipeline. Therefore, the effect of axial force is studied in this section and applied to the DnV code to calculate the modified allowable span lengths.

Table 1 Mode Shape Factors

B.C.	Mode Number	Mode Shape Factor
Fixed	1	1.3050
	2	1.4987
Free	3	1.5371
	4	1.5634
Pinned	1	1.1547
	2	1.1547
Pinned	3	1.1547
	4	1.1547
Fixed	1	1.1613
	2	1.1934
Pinned	3	1.2057
	4	1.2124
Fixed	1	1.1670
	2	1.1613
Fixed	3	1.1824
	4	1.1934

Table 2 Vortex Shedding Frequencies (Hz)

Current speed (m/s)	0.3	0.6	0.9	1.2	1.5
In-line	0.4804	0.9607	1.4411	1.9215	2.4018
Cross-flow	0.1900	0.3895	0.5997	0.8190	1.0457

Table 3 Amplitude of Motion (m)

B.C.	Fix-Fix	Pin-Pin	Fix-Pin	Fix-Free
In-line	0.025	0.025	0.025	0.025
Cross-flow	0.429	0.378	0.381	0.384

4.2 Governing Equation and Solutions

The axial force may greatly alter the shape of the elastic deflection and their influence on the equilibrium conditions can not be neglected. When the pipeline be subjected to a transverse downward load of $w(x)$ per unit length, and an axial force, N_x. The governing differential equation for a beam under an axial force is :

$$\frac{d^2}{dx^2}\left(EI\frac{d^2y}{dx^2}\right)+N_x\frac{d^2y}{dx^2}=w(x) \tag{4.1}$$

The positive N_x and negative N_x respectively denoting compression and tension. The general solutions of Eq. (4.1) are :

$$N_x<0 : \; y=C_1\sinh\lambda x+C_2\cosh\lambda x + C_3x+C_4+\frac{w(x)x^2}{2N_x} \tag{4.2}$$

$$N_x=0 : \; y=C_1x^3+C_2x^2 + C_3x+C_4+\frac{w(x)x^4}{24EI} \tag{4.3}$$

$$N_x>0 : \; y=C_1\sin\lambda x+C_2\cos\lambda x + C_3x+C_4+\frac{w(x)x^2}{2N_x} \tag{4.4}$$

$$\text{where, } \lambda=\sqrt{\frac{|N_x|}{EI}} \tag{4.5}$$

The coefficients C_1, C_2, C_3, C_4 were determined from various boundary conditions and presented in Table 4.

The boundary conditions are provided by the conditions that the ends of the beam are

Fix end condition :

$$y=0,\; \frac{dy}{dx}=0 \tag{4.6}$$

Pinned end condition :

$$y=0,\; \frac{d^2y}{dx^2}=0 \tag{4.7}$$

Free end condition :

$$\frac{d^2y}{dx^2}=0,\; EI\frac{d^3y}{dx^3}+N_x\frac{dy}{dx}=0 \tag{4.8}$$

When N_x approaches to zero, the deflections of Eqs. (4.2) and (4.4) converge to Eq. (4.3.) If N_x is positive and λ approaches to a critical value, then deflections of the beams increases indefinitely and yields Euler's buckling load (P_E) for the beams.

4.3 Natural Frequencies by an Energy Method

To obtain the fundamental frequency, the Rayleigh method can be applied. This method assumes that the maximum potential energy of the system is equal to its maximum kinetic energy. The fundamental frequency can be obtained by equating the potential and kinetic energy.[5)]

$$\omega_n^2=\frac{g\sum_{i=1}^{n}m_iy_i}{\sum_{i=1}^{n}m_iy_i^2} \tag{4.13}$$

Table 4 Coefficients in Beam-Column equation

B.C.	N_x	C_1	C_2	C_3	C_4
Fix - Free	$-$	$\frac{wL}{\lambda N_x}$	$\frac{-w(\lambda L\sinh\lambda L+1)}{\lambda^2 N_x\cosh\lambda L}$	$-\lambda C_1$	$-C_2$
	0	$\frac{-wL}{6EI}$	$\frac{wL^2}{4EI}$	0	0
	+	$\frac{wL}{\lambda N_x}$	$\frac{-w(\lambda L\sin\lambda L-1)}{\lambda^2 N_x\cos\lambda L}$	$-\lambda C_1$	$-C_2$
Pin - Pin	$-$	$\frac{w(\cosh\lambda L-1)}{\lambda^2 N_x\sinh\lambda L}$	$\frac{-w}{\lambda^2 N_x}$	$\frac{-wL}{2N_x}$	$-C_2$
	0	$\frac{-wL}{12EI}$	0	$\frac{wL^3}{24EI}$	0
	+	$\frac{-w(\cos\lambda L-1)}{\lambda^2 N_x\sin\lambda L}$	$\frac{w}{\lambda^2 N_x}$	$\frac{-wL}{2N_x}$	$-C_2$
Fix - Pin	$-$	$\frac{w(-2\cosh\lambda L+\cosh\lambda L\cdot(\lambda L)^2+2)}{2N_x\lambda^2(\cosh\lambda L\cdot\lambda L-\sinh\lambda L)}$	$\frac{-w(-2\sinh\lambda L+\sinh\lambda L\cdot(\lambda L)^2+2\lambda L)}{2N_x\lambda^2(\cosh\lambda L\cdot\lambda L-\sinh\lambda L)}$	$-\lambda C_1$	$-C_2$
	0	$\frac{-5wL}{48EI}$	$\frac{wL^2}{16EI}$	0	0
	+	$\frac{w(2\cos\lambda L+\cos\lambda L\cdot(\lambda L)^2-2)}{2N_x\lambda^2(\cos\lambda L\cdot\lambda L-\sin\lambda L}$	$\frac{-w(2\sin\lambda L+\sin\lambda L\cdot(\lambda L)^2-2\lambda L)}{2N_x\lambda^2(\cos\lambda L\cdot\lambda L-\sin\lambda L)}$	$-\lambda C_1$	$-C_2$
Fix - Fix	$-$	$\frac{wL(-2\cosh\lambda L+2+\lambda L\sinh\lambda L)}{2\lambda N_x(2+\lambda L\sinh\lambda L-2\cosh\lambda L)}$	$\frac{-wL(-2\sinh\lambda L+\lambda L+\lambda L\cosh\lambda L)}{2\lambda N_x(2+\lambda L\sinh\lambda L-2\cosh\lambda L)}$	$-\lambda C_1$	$-C_2$
	0	$\frac{-wL}{12EI}$	$\frac{wL^2}{24EI}$	0	0
	+	$\frac{wL(2\cos\lambda L-2+\lambda L\sin\lambda L)}{2\lambda N_x(-2+\lambda L\sin\lambda L+2\cos\lambda L)}$	$\frac{wL(-2\sin\lambda L+\lambda L+\lambda L\cos\lambda L)}{2\lambda N_x(-2+\lambda L\sin\lambda L+2\cos\lambda L)}$	$-\lambda C_1$	$-C_2$

4.4 Lloyd's Approximate Formula

$$\omega_n = \frac{C}{L^2}\sqrt{\frac{EI}{m_e}}\left(1-\frac{N_x}{P_E}\right)^{5} \qquad \textbf{(4.14)}$$

Eqs. (4.13) and (4.14) give the same natural frequencies of beams if axial load become zero. The natural frequency equation of beams without axial force is as follows :

$$\omega_n = C\sqrt{\frac{EI}{m_e L^4}} \qquad (4.15)$$

C is a constant depending on the boundary conditions and order of natural frequencies.

Fig. 3 shows axial load effect on the natural frequency of the pipe for various boundary condition. The natural frequencies significantly change with the axial load.

The natural frequencies of pipe given by Lloyd are compared with exact solutions and presented in Figs. 4, 5, and 6. The approximate formula given by Lloyd fairly matches with the exact solution except the free-fixed condition.

4.5 Allowable Span Lengths

The exact solutions of the beam equation under the axial load were used to calculate the natural frequency with the Rayleigh method. The natural frequencies obtained by Eq. (4.13) can be used with the current DnV code instead of Eq. (4.14). Thus a more reallistic allowable span lengths can be obtained. For the improved method to calculate the allowable span lengths under the axial force, an iterative method can be applied. Fig. 7 and Fig. 8 show allowable span lengths for various axial load factors and boundary conditions. Axial load factors used in the calculation are the ratio of the axial force and submerged weight of the pipe span. The allowable span lengths increase with increment of tension, and decrease with increment of compression. The allowable span lengths for the cross-flow are more sensetive to the axial forces.

5. Conclusions

If offshore pipelines during the operation and installation could include high axial force, it's effect could not be neglected. The results of the study shows axial load effect on the natural

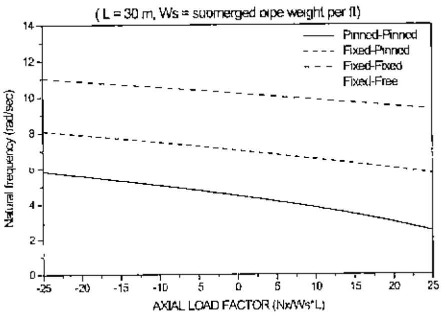

Fig. 3 Effect of the Axial Force on the Natural Frequency

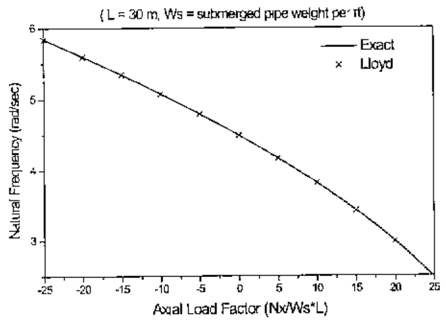

Fig. 4 Effect of the Axial Force on the Natural Frequency for Pinned-pinned B.C.

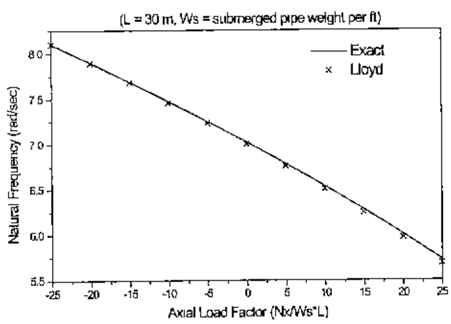

Fig. 5 Effect of the Axial Force on the Natural Frequency Fixed-pinned B.C.

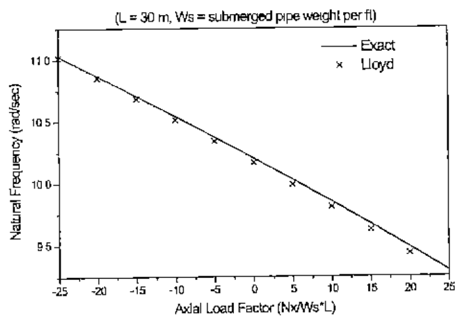

Fig. 6 Effect of the Axial Force on the Natural Frequency for Fixed-fixed B.C.

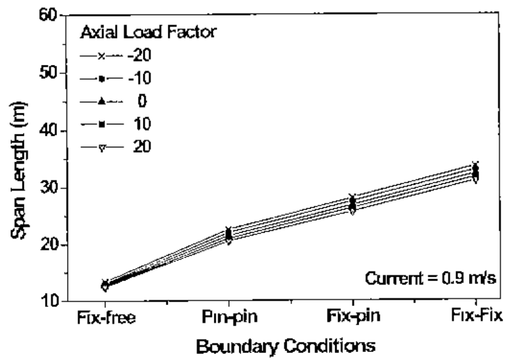

Fig. 7 Allowable Span Lengths(m) with Axial force (In-line)

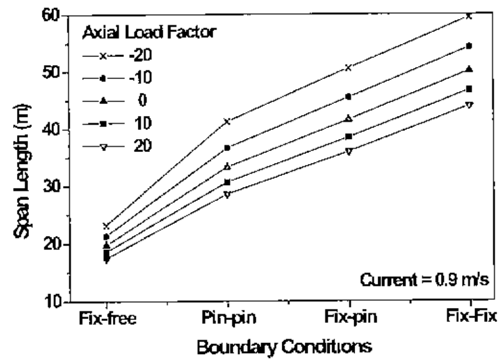

Fig. 8 Allowable Span Lengths(m) with Axial force (Cross-flow)

frequencies and allowable span lengths of the pipeline for various boundary conditions. The natural frequencies significantly change with respect to the axial load.

Exact solutions of beam-column equation are derived for various boundary conditions. The solutions are used to find natural frequencies with energy balance method. This method was compared with Llyod's approximate method. Two methods fairly match.

An improvement was made on the vortex shedding analysis of offshore pipelines. It can be applied to the current design code and will result more accurate calculation of free span lengths of offshore pipelines.

Acknowledgement

The present study was supproted by the Research Institute of Industrial Technology, Pusan National University.

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