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On Long-term Prediction Scheme in Ocean Engineering

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ABSTRACT: *This paper proposes a long-term prediction of offshore structures in ocean waves. All short-term statistics is generated by the simulation for all the combinations of significant wave heights and spectral peak periods. The simulation has been tested first on linear system, whose analytic solution is known, to verify if the simulation works accurately. Then the scheme was applied to the nonlinear system. This paper demonstrated that the proposed scheme could be an efficient tool in estimating the response of offshore structures.*

KEY WORDS: Long-term prediction, Simulation, Equation of motion, Significant wave height, Spectral peak period

1. Introduction

In designing offshore structures, the extreme responses of the structure due to ocean waves have to be considered. The long-term prediction has been studied by many researchers(Hoffman and Walden, 1977), (Ochi, 1978), (Haver, 1985). Their approach can be summarized as follows. The prediction of responses of offshore structures is generally made in regular waves. The regular wave responses must be then translated to responses in the presence of random ocean waves. The short term and long term response analyses are essential to get the accurate estimation of the design load of the waves. The short term responses corresponds to a few hours while the long-term responses are obtained based on the anticipated life of the structure. But the long-term response predictions are usually based on only a few years of wave data. We need to extend the few years of time of extreme responses to the life time of the structure.

In this paper, the author proposes a long-term prediction scheme which is applicable in the filed of ocean engineering. The basic idea of the proposed scheme is that the simulation of the responses of the structure is performed for all combinations of significant wave heights and spectral peak periods in the given wave data. This paper demonstrates that the increasing computation power of computer makes this tremendous amount of computation possible.

2. Wave Data

In order to estimate the long-term probability of responses of offshore structures, first of all, the long-term distribution of waves must be known. The data used in this study is adopted from Haver(Haver 1985). Haver made a 3-year observation on the Northern North Sea in order to produce wave data and presented a tabulation in terms of the number of observations for each pair of significant wave height and spectral peak wave period combination. The measurement was achieved by using a surface following wave rider buoy. The probability of occurrence of a particular pair can be determined from the fraction of its number of observations to the total number of observations. Table 1 represents joint frequency table of H_S and T_P.

The joint probability density function can be written in terms of H_S and T_P as follows

$$f_{H_S T_P}(h, t) = f_{H_S}(h) \cdot f_{T_P \mid H_S}(t \mid h) \qquad (1)$$

where h,t represents a significant wave height and peak period of wave, respectively. H_s and T_P are random variables for significant wave height and spectral peak period. $f_{H_s}(h)$ is the probability density function of significant wave height and $f_{T_P \mid H_S}(t \mid h)$ represents the conditional probability density function of T_P given H_s. The joint probability density function for H_s and T_P can be obtained by fitting the data from the wave data. $f_{H_s}(h)$ can be approximated by the following functional form

$$f_{H_S}(h) = \begin{cases} \dfrac{1}{\sqrt{2\pi}\alpha h} \exp\left\{-\dfrac{(\ln h - \lambda)^2}{2\alpha^2}\right\}, & h \le \eta \\ \beta \dfrac{h^{\beta-1}}{\rho^\beta} \exp\left\{-\left(\dfrac{h}{\rho}\right)^\beta\right\}, & h > \eta \end{cases} \qquad (2)$$

where λ and α^2 represents the mean and variance of $\ln H_S$, respectively We can notice that $f_{H_s}(h)$ is modelled by a log normal distribution for $H_S < \eta$ and is modelled by a Weibull distribution for $H_S > \eta$. Haver proposed the values of the parameter as η=3.27m, λ=0.836, α^2=0.376, ρ=2.822, and β =1.547 . These values are good approximations for the given wave data shown in Table 1. The conditional joint probability

Table 1 Joint frequency data

Hs \ Tp	4.0	4.0-5.0	5.0-6.0	6.0-7.0	7.0-8.0	8.0-9.0	9.0-10.0	10.0-11.0	11.0-12.0	12.0-13.0	13.0-14.0	14.0-15.0	15.0-16.0	16.0-17.0	17.0-18.0	18.0-19.0	19.0-20.0	20.0	Sum
-0.5	1	3	12	17	10	12	5	6	3	1	1								71
0.5-1.0	16	68	121	133	96	91	78	38	24	8	2	1	1						677
1.0-1.5	6	63	151	170	226	171	156	79	67	41	17	4	2	1					1154
1.5-2.0		11	127	230	227	186	168	113	81	64	45	17	3	1	2		1	1	1277
2.0-2.5		2	41	146	216	202	146	128	68	50	33	31	10	5	1	1	1	2	1083
2.5-3.0			11	69	184	204	119	94	106	73	45	29	19	6	4	2		1	966
3.0-3.5				22	92	207	120	102	81	71	47	33	19	6	3				803
3.5-4.0				8	44	162	119	92	57	74	40	22	14	8	3	1			644
4.0-4.5					16	103	114	75	60	43	18	18	10	5	5				467
4.5-5.0				1	3	44	76	45	51	29	27	9	10	10	8	2			315
5.0-5.5						18	60	69	50	23	13	10	5	4	4	1			257
5.5-6.0					1	8	32	40	31	17	10	13	3	6	4	4			169
6.0-6.5							6	28	21	22	6	10	2	4	2	2	2	1	106
6.5-7.0							2	20	18	21	14	2	4						81
7.0-7.5								3	9	15	13	3	1	1	1				46
7.5-8.0									8	12	4	3	3						30
8.0-8.5								3	5	11	4	5	3						31
8.5-9.0									3	3	4	4	1						15
9.0-9.5										1	4	2	3		1		1		12
9.5-10.0										3	1								4
10.0-10.5										1									1
10.5-11.0								1						1		1			3
Sum	23	147	463	796	1115	1408	1201	936	743	583	348	216	113	58	38	14	5	5	8212

density function of H_S and T_P can be approximated by the log normal distribution as shown below

$$f_{T_p|H_s}(t \mid h) = \frac{1}{\sqrt{2\pi}\phi t} \exp\left\{-\frac{(\ln t-\mu)^2}{2\phi^2}\right\} \quad (3)$$

where μ is the mean of $\ln T_P$ and ϕ^2 is the variance of $\ln T_P$. μ and ϕ^2 are estimated from the sample for each case of significant wave height. The smoothed estimates are given below

$$\mu = 1.59 + 0.42\ln(h+2) \quad (4)$$

$$\phi^2 = 0.005 + 0.085\exp(-0.13h^{1.34}) \quad (5)$$

3. Equation of Motion

If the influence of all other degrees of freedom can be neglected, the equation of motion of a ship or an offshore structure in random waves can be written in the following form

$$x + D(x) + R(x) = N(t) \quad (6)$$

where x is the displacement of response, D represents the nonlinear damping function, R represents a restoring function, and $N(t)$ is a Gaussian random process with zero mean with spectrum $S_N(\omega)$. $N(t)$ is a non-white noise. The damping and restoring can be represented as

$$R(x) = r_1 x + r_3 x^3 \quad (7)$$

$$D(\dot{x}) = d_1 + d_2 \mid \dot{x} \mid \dot{x} \quad (8)$$

In this study the excitation term $N(t)$ is obtained by introducing the transfer function as

$$S_N(\omega) = \mid H_{\eta N}(\omega) \mid^2 S_\eta(\omega) \quad (9)$$

where $\eta(t)$ represents the wave height, $S_\eta(\omega)$ is the wave spectrum, $H_{\eta N}(\omega)$ is the transfer function from $\eta(t)$ to excitation $N(t)$. It can be written

$$H_{\eta N}(\omega) = \frac{1}{(c_3 - c_1\omega^2) + i c_2\omega} \tag{10}$$

where c_1, c_2 and c_3 are arbitrary constants, and ω is the angular frequency. The Pierson-Moskowitz spectrum is used for $S_\eta(\omega)$ in the following form

$$S_\eta(\omega) = 5\sigma^2 \frac{\omega^{-5}}{\omega_0^{-4}} \exp\left[-1.25\left(\frac{\omega}{\omega_0}\right)^{-4}\right] \tag{11}$$

where ω_0 is a peak frequency, and σ^2 represents the 0th moment of a wave spectrum. It can be written as

$$\sigma^2 = \int_{-\infty}^{\infty} S_\eta(\omega)\, d\omega \tag{12}$$

4. Long-term Prediction

For long term prediction various conditions of ocean is essentially required. In processing the long-term prediction, the short term distributions are first computed. The response amplitude can be expressed as follows

$$F_{Y_s}(y) = 1 - \exp\left\{-\left(\frac{y}{\sigma_x^2}\right)^2\right\} \tag{13}$$

where y is the random variable and represents the response amplitude. $F_{Y_s}(y)$ is the cumulative distribution function of response amplitude calculated from simulation. σ_x^2 can be calculated from the time history of response. The mean zero upcrossing frequency can be obtained from the following equation

$$\nu_{x,0}^+ = \left\{\frac{m_X^{(2)}}{m_X^{(0)}}\right\}^{\frac{1}{2}} \tag{14}$$

where $m_X^{(n)}$ is the nth moment of response spectrum. The specific form of the 0th and 2nd moment can be written as

$$m_X^{(0)} = \sigma_x^2 = E[x^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x^2 f_X(x, \dot{x})\, dx d\dot{x} \tag{15}$$

$$m_X^{(2)} = \sigma_{\dot{x}}^2 = E[\dot{x}^2] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \dot{x}^2 f_X(x, \dot{x})\, dx d\dot{x} \tag{16}$$

The long-term distribution of response amplitude can be written as

$$F_{Y_L}(y) = \frac{1}{\overline{\nu_{x,0}^+}} \int_h\int_t \nu_{x,0}^+(h, t) \cdot F_{Y_S|H_s,T_p}(y \mid h, t) f_{H,T_p}(h, t)\, dh dt \tag{17}$$

where $\overline{\nu_{x,0}^+}$ is the long-term mean frequency of zero-upcrossing, i.e.

$$\overline{\nu_{x,0}^+} = \int_h\int_t \nu_{x,0}^+(h, t) f_{H,T_p}(h, t)\, dh dt \tag{18}$$

The long-term cumulative distribution function can be obtained by substituting eq(13) and eq(14) into eq(17). Since the response process are assumed to be Gaussian, the long-term response can be obtained from the following equation

$$1 - F_{Y_l}(\overline{y_n}) = \frac{1}{n_N}$$

where n_N represents the expected number of observations for the N years.

5. Simulation

The conventional way of performing simulation is to sum up cosine series in which the phase is randomly chosen. That is

$$N_S(t) = \sum_{i=1}^{n} A_i \cos(\omega_i t - \theta_i) \tag{20}$$

where θ_i is the random number which is distributed uniformly from 0 to 2π(Shinizuka, 1991, Yang, 1986). The mean and variance of $N_S(t)$ can be represented as

$$E[N_S(t)] = \sum_{i=1}^{n} A_i E[\cos(\omega_i t - \theta_i)] = 0 \tag{21}$$

$$E[N_S(t)^2] = E\left[\sum_{i=1}^{n} A_i \cos(\omega_i t - \theta_i) \sum_{j=1}^{n} A_j \cos(\omega_j t - \theta_j)\right] = \sum_{i=1}^{n} \frac{1}{2} A_i^2 \tag{22}$$

The variance of excitation $N(t)$ can be calculated from the 0th moment of the spectrum as follows

$$\sigma_N^2 = \int_0^{\infty} 2S(\omega) d\omega = \sum_{i=1}^{n} 2S(\omega_i)\Delta\omega \tag{23}$$

From eq(22) and eq(23) the relation between A_i and spectrum $S_N(\omega)$ can be obtained

$$A_i = \sqrt{4S_N(\omega_i)\Delta\omega} \tag{24}$$

The expression for the simulation $N_S(t)$ can be obtained by substituting eq(24) into eq(20)

$$N_S(t) = \sum_{i=1}^{n} \sqrt{4S_N(\omega)\Delta\omega} \cdot \cos(\omega_i t - \theta_i) \tag{25}$$

6. Numerical Results and Discussion

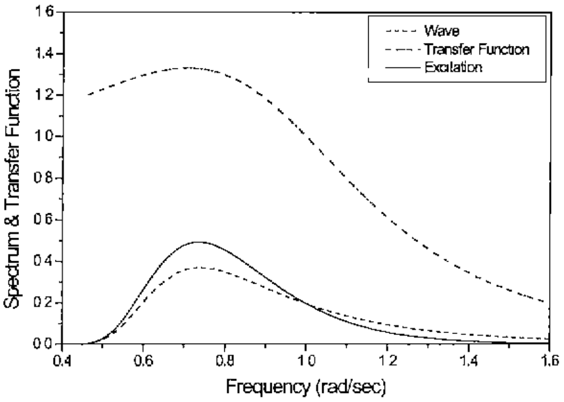

Fig. 1 Comparison of Spectrums

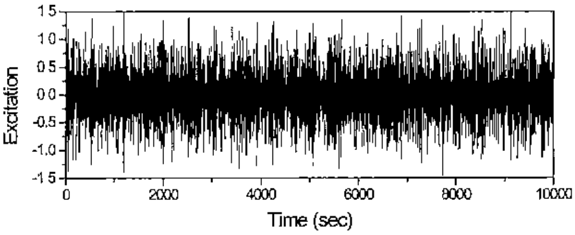

Fig. 2 Time History of Exciting Displacement

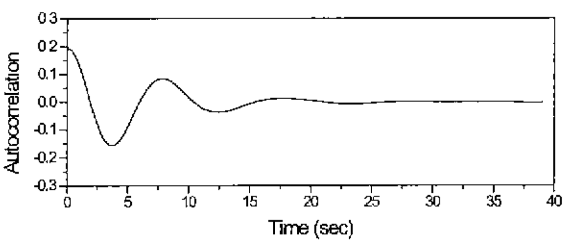

Fig. 3 Auto-correlation of Exciting Displacement

The wave spectrum, transfer function, and exciting spectrum are shown in Fig. 1. The dashed line is the typical wave spectrum when the significant wave height corresponds to 1.75m and peak period is 8.5 seconds. Dashed line and dotted line is the transfer function as $c_1 = c_2 = c_3 = 1$, and solid line is the exciting spectrum.

The time history of numerical simulation of the excitation is shown in Fig.2. Fig.3 shows the auto-correlation function of this time history. The shape of the auto-correlation function proves that there is no periodicity in the simulation. The target spectrum and simulated spectrum are plotted in Fig.4 to check the accuracy of the simulation. They are in good agreement with each other. The probability density function of the significant wave height is shown in Fig.5. The solid circle is calculated from the scatter diagram and solid line represents the calculated results from eq.(2). The conditional probability density function of T_P for given values of H_S is shown in Fig.6. The rectangle, circle and triangle represent probability density function calculated from the Table 1, and solid, dashed, and dotted line show the shapes calculated from eq.(3). The typical significant wave heights 0.75m, 1.75m, and 2.75m are used.

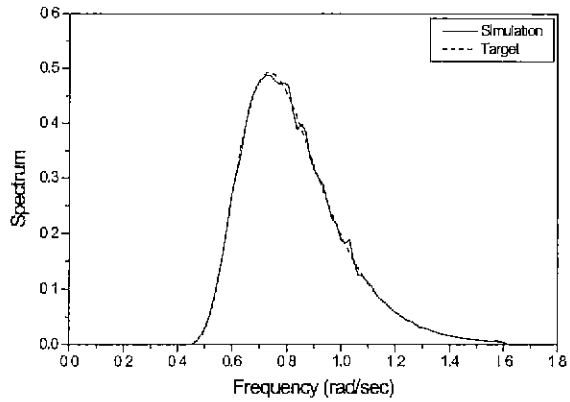

Fig. 4 Comparison of Exciting Spectrums between the Given Simulated Spectrum

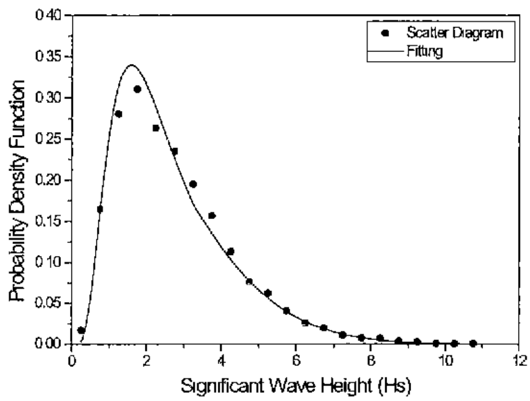

Fig. 5 Probability Density Function of Significant Wave Height

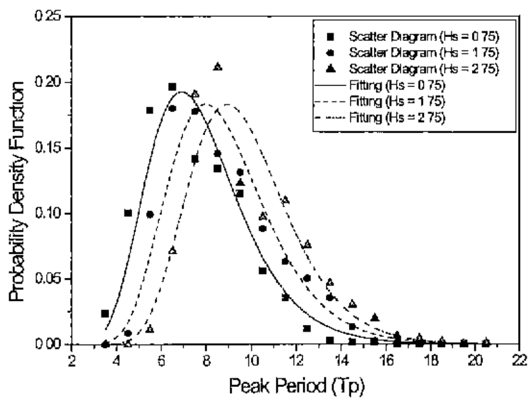

Fig. 6 Probability Density Functions of Peak Period

In this study, we consider a linear system whose analytic solution is known to check the accuracy of the simulation. The linear system we consider is

$$\ddot{x} + d\dot{x} + rx = N(t) \tag{26}$$

where d and r represent damping and restoring coefficients, respectively. The analytic expression of the variance of the response for the given linear system can be written as

$$\sigma_x^2 = \int_{-\infty}^{\infty} |H_L(\omega)|^2 S_N(\omega)\, d\omega \tag{27}$$

where $H_L(\omega)$ is the frequency response function of eq.(26). The functional form of the $H_L(\omega)$ is

$$H_L(\omega) = \frac{1}{(r-\omega^2) + id\omega}$$

The unit values are used for the linear coefficients of the equation of motion. The computational results for 100 year return period of simulation and analytic results are shown in Table 2. The comparison shows that simulation is very accurate.

Table 2 Comparison of analytic solution and simulation for 100 year return period

	analytic solution	simulation
extream value	10.3348	10.3671

The variances of simulated response are shown in Fig.7 and Fig.8. These variances are calculated from 20000 samples. The total cpu time for the calculation took approximately 20 hours on pentiumIII 450 MHz computer.

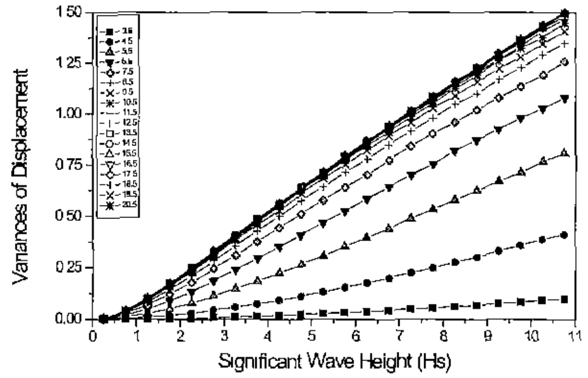

Fig. 7 Variances of Response Displacement

The values of the coefficients used are $d_1 = 1$, $d_2 = 0.5$, $r_1 = 1$ and $r_3 = 0.5$. Fig.9 shows the cumulative distribution function of the response by the long-term prediction. The long-term pediction of the structure is shown in Table3.

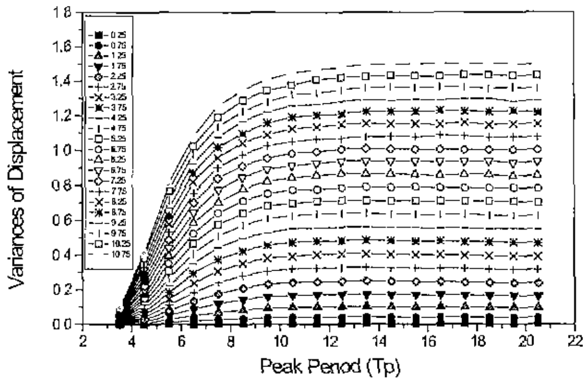

Fig. 8 Variances of Response Displacement

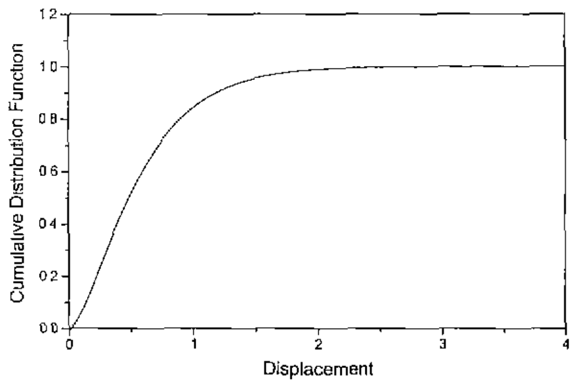

Fig. 9 Cumulative Distribution Function of Response Displacement

Table 3 Long-term prediction of responses fo various return periads

return period	extreme value
10 year	3.6830
20 year	3.8874
50 year	4.1782
100 year	4.3645

The computational results for other systems of equations are listed in Table 4. These values are the long-term responses for 100 year return period.

Table 4 Long-term responses for 100 year return period

d_2	r_3	extreme value
0.1	0.1	6.367
0.3	0.3	4.923
0.5	0.5	4.365

7. Conclusions

The long-term prediction scheme was proposed in this paper. The short-term statistics for all the combinations of significant wave heights and spectral peak periods were generated by simulation. The computational power of computer made this huge amount of calculation possible. These are extended to the whole life time of the structure. The accuracy of the simulation was tested on the linear system whose analytic solution could be known in advance. The results show good agreement with each other. Then the scheme was tested on nonlinear systems. This paper demonstrated that the proposed scheme can be an efficient tool for estimating responses of structures in the ocean wave.

Acknowledgement

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