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Application of the Improved Green Integral Equation to the Radiation-Diffraction Problem for a Floating Ocean Structure in Waves and Current

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ABSTRACT: *The improved Green integral equation for the calculation of time-harmonic potentials in the radiation -diffraction problem about a freely floating body in the presence of moderate or weak current is presented. The forward-speed Green function presented by Brard is used. The correct free surface boundary conditions on the physical free surface are employed as well as an appropriate boundary conditions on the non-physical inner free surface. The default in the existing Green integral equation as well as in the source integral equation is discussed in detail.*

1. Introduction

The wave loads on a large-volume offshore structure without the presence of the current, can be calculated by making use of the solution of the potential boundary value problem in the frequency domain(John 1950). The radiation-diffraction potential on the hull of a floating structure can be obtained from the solution of the improved Green integral equation using Kelvin-type Green function(Hong 1987).

When the current is present, the wave field is modified and it is difficult to formulate the potential problem. In this study, it is assumed that the boundary conditions for the waves are linear and that the waves are transferred by the current without deformation. Under the assumptions, the boundary value problem for the radiation-diffraction potential in the presence of the uniform horizontal current is equivalent to the so called, forward speed three dimensional radiation-diffraction problem in the field of ship hydrodynamics. The time harmonic forward speed Kelvin type three dimensional Green function was presented by Brard(1948) and various numerical methods based on a source integral equation using this Green function have been presented to calculate wave loads and motion of a surface ship advancing in waves(Chang 1977, Bougis 1980, Inglice and Price 1981, Chan 1990). Unfortunately, it seems that all these three-dimensional methods in the frequency domain are not conclusive yet.

More recently, in the field of moored body hydrodynamics, it has been found that the wave drift damping or the increase of the time mean drift force due to the relative current is similar to the increase of the resistance of a ship advancing in waves(Wichers and Sluijs 1979, Huijsmans 1986, Newman 1993). According to the comparative numerical studies on the wave drift damping, very large discrepancy between numerical methods has been found(Grue 1992, Park and Choi 1996, Huijsmans 1996, Lee et al 2000). The cause of the discrepancy is quite ambiguous. The effect of the so-called Neumann-Kelvin potential or the steady potential has been taken into account by some authors while not accounted by others; besides, the source integral equation has been employed by some authors which has been known to be not appropriate for the so-called surface-piercing body.

In this paper, the boundary value problem of the radiation-diffraction potential for a freely floating body in the presence of the uniform horizontal current is solved by making use of the improved Green integral equation under the assumption that the current speed is low for full-shaped bodies and relatively high for fine-shaped bodies positioned parallel with the direction of the current(Hong 2000). In other words, it is assumed that the magnitude of the steady potential is as small as that of the unsteady potential so that the effect due to their coupling may be neglected. The water depth where the offshore structures are installed is not deep but in this study it is assumed to be deep in order to simplify the comparative numerical studies for the unsteady potential which will be followed soon. In this way, the cause of the discrepancy mentioned above may be explained in the near future. An approximate method for weak current is also presented

2. Linearized Boundary-Value Problem in the Frequency Domain

A body is freely floating in the free surface of deep water under gravity and in the presence of plane progressive sinusoidal incident wave of small amplitude a_0 transferred by a horizontal current with uniform speed U. The current speed U is assumed to be of $O(1)$. Let $oxyz$ be a Cartesian co-ordinate system attached to the mean position of the body, with z vertically upward, x in the negative direction of the current velocity and o in the mean waterplane W which is also denoted by the inner free surface F_i. The body performs simple harmonic oscillations of small amplitude about its mean position with circular frequency ω which is equal to the apparent frequency of incident wave. It is assumed that the disturbance of the free surface is of $O(\varepsilon)$ where ε, being as small as the wave slope, is the measure of smallness in the present study.

With the usual assumptions of the incompressible, inviscid fluid and irrotational flow without capillarity, the fluid velocity $\vec{v}$ can be given by the gradient of a velocity potential Φ which satisfies the Laplace equation,

$$\nabla^2\Phi = 0 \tag{1}$$

in the fluid region.

Under the assumptions given above, Ψ at P in the fluid region can be decomposed as follows:

$$\Phi(P,\ t) = \Phi_S(P) - Ux + Re\{\Psi(P)\,e^{-i\omega t}\} \tag{2}$$

where Φ_S denotes a steady potential due to the presence of the body in the current, Ψ a complexed-valued unsteady potential and ω the apparent frequency of the incident wave. The velocity potential of incident wave is as follows:

$$\Phi_0 = Re\{\Psi_0\,e^{-i\omega t}\} \tag{3}$$

where

$$\Psi_0 = -\frac{a_0 g}{\omega_0}\,e^{k_0[z+i(x\cos\beta+y\sin\beta)]} \tag{4}$$

for

$$\omega = (\omega_o - Uk_o\cos\beta) > 0 \tag{5}$$

and

$$\Psi_0 = -\frac{a_0 g}{\omega_0}\,e^{k_o[z-i(x\cos\beta+y\sin\beta)]} \tag{6}$$

for

$$\omega = (Uk_o\cos\beta - \omega_o) > 0 \tag{7}$$

where g is the gravitational acceleration, β the angle between the phase velocity of the incident wave and the current velocity, ω_0 the circular frequency of incident wave and $k_0 = \frac{\omega_0^2}{g}$ the wavenumber expressed in a relative co-ordinate system $\overline{o}\,\overline{x}\,\overline{y}\,\overline{z}$ attached to the uniform horizontal current as follows:

$$\overline{x} = x + Ut,\quad \overline{y} = y,\quad \overline{z} = z \tag{8}$$

Here, it should be noted that the magnitude of both Φ_S and Ψ is of $O(\varepsilon)$.

The equation of the mean free surface is

$$z = 0 \tag{9}$$

The free surface boundary condition on $z=0$ is as follows:

$$\left(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla\right)p = 0 \quad on \quad z = 0 \tag{10}$$

where p denotes the pressure :

$$p = -\rho\left(\frac{\partial\Phi}{\partial t} + \frac{v^2}{2}\right) \tag{11}$$

Substituting (2) and (11) into (10) and neglecting second order quantities, the following free surface boundary conditions for Φ_S and Ψ can be found respectively:

$$\left[U^2\frac{\partial^2}{\partial x^2} + g\frac{\partial}{\partial z}\right]\Phi_S = 0 \quad \text{on} \quad z = 0 \tag{12}$$

$$\left[\left(-i\omega - U\frac{\partial}{\partial x}\right)^2 + g\frac{\partial}{\partial z}\right]\Psi = 0 \quad on \quad z = 0 \tag{13}$$

Under the assumption of small amplitude oscillation, the displacement vector $\vec{A}(M)$ of a point M on the wetted surface S of the body at its mean position is of $O(\varepsilon)$. The expression of $\vec{A}(M)$ is as follows.

$$\vec{A}(M) = Re\{\vec{a}(M)\,e^{-i\omega t}\},\qquad M\in S \tag{14}$$

$$\vec{a}(M) = \sum_{k=1}^{3} a_k\vec{e}_k + \vec{\theta}\times\overrightarrow{OM},\qquad M\in S \tag{14a}$$

$$\vec{\theta} = \sum_{k=4}^{6} a_k\vec{e}_{k-3} \tag{14b}$$

where $a_k(k=1,2,,,6)$ denotes complex valued amplitude of surge, sway, heave, roll, pitch, yaw respectively and O the center of rotation of the body.

It should be noted that the time-harmonic quantities correspond to the real part of terms involving $e^{-i\omega t}$ and it will not be shown hereafter unless its presence is necessary.

Applying impermeability condition on S, the following body boundary condition can be found:

$$\begin{aligned}(\vec{n} + \vec{\theta}\times\vec{n})\cdot\nabla(\Phi_S - Ux + \Psi)\\ = (\vec{n} + \vec{\theta}\times\vec{n})\cdot(-i\omega\vec{a})\end{aligned} \tag{15}$$

where $\vec{n}$ denotes a unit normal to S directed into the fluid region, at its mean position and $(\vec{n}+\vec{\theta}\times\vec{n})$ the Taylor expansion of the normal at its instantaneous position.

Neglecting second-order quantities, the following linearized body boundary condition for Φ_S and Ψ can be found respectively:

$$\frac{\partial \Phi_S}{\partial n} = U n_1 \quad on \quad S \tag{16}$$

$$\frac{\partial \Psi}{\partial n} = -i\omega \vec{a}\cdot\vec{n} + U(a_5 n_3 - a_6 n_2) \quad on \quad S \tag{17}$$

With these linearized boundary conditions on S and on $z=0$, the unsteady potential and the steady potential problems can be solved independently and the latter will be dropped from the present study.

The unsteady potential Ψ can further be decomposed as follows:

$$\Psi = \Psi_0 + \Psi_7 + \Psi_R \tag{18}$$

where the sum of Ψ_0 and Ψ_7 is known as the diffraction potential and Ψ_R the radiation potential which can be decomposed as follows:

$$\Psi_R = -i\omega \sum_{k=1}^{6} a_k \Psi_k - U(a_6 \Psi_2 - a_5 \Psi_3) \tag{19}$$

Then the body boundary conditions for $\Psi_k(k=1,2,,,7)$ are

$$\frac{\partial \Psi_k}{\partial n} = n_k \quad on \quad S, \qquad k=1,2,3 \tag{20a}$$

$$\frac{\partial \Psi_k}{\partial n_0} = (\overrightarrow{e_{k-3}}\times\overrightarrow{OM})\cdot\vec{n} \quad on \quad S, \qquad \text{for} \quad k=4,5,6 \tag{20b}$$

$$\frac{\partial \Psi_7}{\partial n} = -\frac{\partial \Psi_0}{\partial n} \quad on \quad S \tag{21}$$

The potentials $\Psi_k(k=1,2,,,7)$ also satisfy the free surface boundary condition given by the equation (13):

$$\left[\left(-i\omega - U\frac{\partial}{\partial x}\right)^2 + g\frac{\partial}{\partial z}\right]\Psi_k = 0 \quad on \quad F, \qquad \text{for} \quad k=1,2,,,7 \tag{22}$$

It is also assumed that they vanish at infinity as $\frac{1}{r^{\infty}}$ where r^{∞} denotes the distance from the body. They must also satisfy the radiation condition presented by Brard(*see Appendix I*).

3. Review on the Source Integral Equation

It has just been shown that the present boundary value problem is equivalent to the so called, forward speed three dimensional radiation-diffraction problem in the field of ship hydrodynamics. Therefore it can be solved by making use of the improved Green integral equation(*see Appendix II*)..

Here, we will present brief review on the source integral equation. When a body is present in the free surface, the fluid region D_e is bounded by the mean wetted surface of the body S, the outer free surface $F_e = F - F_i$ and some arbitrary surface S_∞ at infinity. Let C and C_∞ denote the closed intersection contours of F with S and S_∞ respectively. Applying Green's theorem to the potential Ψ and the Green function G over the fluid region D_e, the following Green integral equation can be obtained :

$$\begin{aligned}&\frac{1}{2}\Psi(P) + \int\int_S \Psi(M)\frac{\partial G(P,M)}{\partial n_M}ds \\ &+ 2i\gamma\int_C \Psi(M)G(P,M)dy_M \\ &- \frac{U^2}{g}\int_C\left[\Psi(M)\frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M}G(P,M)\right]dy_M \\ &= \int\int_S \frac{\partial \Psi(M)}{\partial n_M}G(P,M)ds, \qquad P\in S\end{aligned} \tag{23}$$

Let Ψ^i the interior potential defined inside D_i bounded by S and F_i. Applying Green's theorem to Ψ^i and the Green function G over D_i, the following Green integral equation for Ψ^i analogous to (23) can be obtained:

$$\begin{aligned}&\frac{1}{2}\Psi^i(P) - \int\int_S \Psi^i(M)\frac{\partial G(P,M)}{\partial n_M}ds \\ &- 2i\gamma\int_C \Psi^i(M)G(P,M)dy_M \\ &+ \frac{U^2}{g}\int_C\left[\Psi^i(M)\frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi^i(M)}{\partial x_M}G(P,M)\right]dy_M \\ &= -\int\int_S \frac{\partial \Psi^i(M)}{\partial n_M}G(P,M)ds, \qquad P\in S\end{aligned} \tag{24}$$

Adding (23) and (24), we have

$$\begin{aligned}&\frac{\Psi(P)+\Psi^i(P)}{2} + \int\int_S[\Psi(M)-\Psi^i(M)] \\ &\frac{\partial G(P,M)}{\partial n_M}ds + 2i\gamma\int_C[\Psi(M)-\Psi^i(M)] \\ &G(P,M)dy_M - \frac{U^2}{g}\int_C\{[\Psi(M)-\Psi^i(M)] \\ &\frac{\partial G(P,M)}{\partial x_M} - \left[\frac{\partial \Psi(M)}{\partial x_M} - \frac{\partial \Psi^i(M)}{\partial x_M}\right] \\ &G(P,M)\}dy_M = \int\int_S\left[\frac{\partial \Psi(M)}{\partial n_M} - \frac{\partial \Psi^i(M)}{\partial n_M}\right] \\ &G(P,M)ds, \qquad P\in S\end{aligned} \tag{25}$$

Imposing

$$\Psi(M) - \Psi'(M) = 0 \quad on \quad S \tag{26}$$

and denoting

$$\frac{\partial \Psi(M)}{\partial n_M} - \frac{\partial \Psi'(M)}{\partial n_M} = \sigma(M) \quad on \quad S \tag{27}$$

we have

$$\Psi(P) + \frac{U^2}{g}\int_C [\frac{\partial \Psi(M)}{\partial x_M} - \frac{\partial \Psi'(M)}{\partial x_M}] G(P,M)\, dy_M = \int\int_S \sigma(M)\, G(P,M)\, ds, \quad P \in S \tag{28}$$

According to the potential theory, the condition (26) entails the following conditions .

$$\frac{\partial \Psi(M)}{\partial l_M} - \frac{\partial \Psi'(M)}{\partial l_M} = 0 \quad on \quad S \tag{29a}$$

$$\frac{\partial \Psi(M)}{\partial \tau_M} - \frac{\partial \Psi'(M)}{\partial \tau_M} = 0 \quad on \quad S \tag{29b}$$

where $\vec{l}$ is a unit vector tangent to C whose direction along which one, traveling in D_e, would proceed in keeping W to his left, is defined positive and τ a unit vector tangent to S forming a right-hand vector triad $\vec{\tau} = \vec{l} \times \vec{n}$.

The derivative of Ψ or Ψ' with respect to x_M can be decomposed as follows:

$$\frac{\partial \Psi(M)}{\partial x_M} = \vec{e_1} \cdot [\frac{\partial \Psi(M)}{\partial n_M}\vec{n_M} + \frac{\partial \Psi(M)}{\partial l_M}\vec{l_M} + \frac{\partial \Psi(M)}{\partial \tau_M}\vec{\tau_M}], \quad M \in S \tag{30}$$

Substituting (30) into (28) and taking account of (27) (29a) and (29b), we have

$$\Psi(P) + \frac{U^2}{g}\int_C \sigma(M)\, G(P,M)(\vec{e_1} \cdot \vec{n_M})\, dy_M = \int\int_S \sigma(M)\, G(P,M)\, ds, \quad P \in S \tag{31}$$

Taking the normal derivative of (31) on S with respect to the field point P, the so-called source integral equation is obtained:

$$\frac{\sigma(P)}{2} + \int\int_S \sigma(M)\frac{\partial G(P,M)}{\partial n_P}\, ds - \frac{U^2}{g}\int_C \sigma(M)\frac{\partial G(P,M)}{\partial n_P}(\vec{e_1} \cdot \vec{n_M})\, dy_M = \frac{\partial \Psi(P)}{\partial n_P}, \quad P \in S \tag{32}$$

This integral equation, considered in the strict sense of the integral equation, is not complete since the condition (26) is imposed only on the open boundary S. In fact, it was necessary to impose the condition (26) over the closed surface $S \cup F_e \cup S_\infty$. Thus, in order to ensure the uniqueness of the solution, it is necessary to impose the following condition.

$$\Psi(M) - \Psi'(M) = 0 \quad on \quad F_e \tag{33}$$

The condition on S_∞ can be omitted since Ψ on S_∞ vanishes in the limit.

We may think that the condition (33) can be replaced by an appropriate condition on the inner free surface F_i. But, since the unknown in the equation (32) is the source density σ over S, it will be extremely difficult to combine the appropriate condition into the source integral equation. Moreover, it is hard to find an appropriate condition on the inner free surface F_i.

4. Improved Green Integral Equation for Weak Current

The speed U of the current was assumed to be of $O(1)$. From now on, we assume that U^2 is of $O(\varepsilon)$. Then Brard's Green function can be simplified as follows:

$$G^s(P,M,t) = Re\{G^s(P,M)e^{-i\omega t}\} \tag{34}$$

where

$$G^s(P,M) = G_0(P,M) - G_1(P,M) + G_f^s(P,M) \tag{35}$$

$$G_j(P,M) = -\frac{1}{4\pi}\frac{1}{r_j}, \quad j = 0,1 \tag{36}$$

$$r_j = \{(x_P - x_M)^2 + (y_P - y_M)^2 + (z_P - (-1)^j z_M)^2\}^{\frac{1}{2}}, \quad j = 0,1 \tag{37}$$

$$G_f^s(P,M) = \frac{1}{4\pi^2}(H_1^s + H_2^s) \tag{38}$$

$$H_l^s = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta \int_0^\infty \frac{1}{D_l^s} e^{\zeta} gk\, dk, \quad l = 1,2 \tag{39}$$

$$D_l^s = \omega^2 - 2(-1)^{(l+1)}\omega Uk\cos\theta - gk + i\nu[\omega - (-1)^{(l+1)}Uk\cos\theta], \quad l = 1,2 \tag{40}$$

$$\zeta = k\{z_P + z_M + i[(x_P - x_M)\cos\theta + (y_P - y_M)\sin\theta]\} \tag{41}$$

where ν is an artificial damping parameter infinitely small, positive, which will determine the path of integration in the complex plane K associated with the variable k shown in the expressions of H_1 and H_2.

The above Green function $G^s(P,M)$ satisfies the following equations:

$$\nabla_P^2 G^s(P,M) = 0 \quad \text{for} \quad P \neq M \tag{42}$$

$$\begin{aligned}(-\omega^2 + 2i\omega U\frac{\partial}{\partial x_P} + g\frac{\partial}{\partial z_P})\, G^s(P,M) \\ = 0 \quad \text{for} \quad z_M < 0 \quad \text{and} \quad z_P \leq 0\end{aligned} \tag{43}$$

The properties of this Green function has been presented by Grekas(1981). It should be noted that the integrand of H_l^s has simple pole and therefore the integration with respect to k can be done more easily compared to that in the Brard's Green function having double pole.

The adjoint free surface condition for $G^s(P,M)$ is as follows :

$$\begin{aligned}(-\omega^2 - 2i\omega U\frac{\partial}{\partial x_P} + g\frac{\partial}{\partial z_M})\, G^s(P,M) \\ = 0 \quad \text{for} \quad z_P < 0 \quad \text{and} \quad z_M \leq 0\end{aligned} \tag{44}$$

Applying Green's theorem to the potential Ψ and the Green function G^s over the fluid region D_e, the following improved Green integral equation for weak current can be obtained :

$$\begin{aligned}&\frac{1}{2}\ \Psi(P)\,\delta[1-\delta(z_P-0)] \\ &+ \int\int_{S^-} \Psi(M)\frac{\partial G(P,M)}{\partial n_M}\, ds \\ &+ \delta[1-\delta(z_P-0)] \\ &2i\gamma\int_C \Psi(M)\, G(P,M)\, dy_M \\ &= \int\int_{S^+} \frac{\partial \Psi(M)}{\partial n_M}\, G(P,M)\, ds \\ &\quad \text{for} \quad P \in S^+ \cup F_i^-\end{aligned} \tag{45}$$

where S^+ denotes the positive side of S and F_i^- the negative side of F_i(see Appendix II).

5. Discussions and Conclusion

The improved Green Integral equation which contains the correct boundary conditions on the free surface as well as the supplementary condition on the inner free surface, for the potential boundary value problem about the radiation-diffraction wave by a freely floating body in the presence of current, has been presented.

The improved Green Integral equation for a weak current has been presented too. The latter can represent the weak contribution of the current in a consistent manner while the existing source integral equation cannot.

Numerical tests which is necessary to validate the present improved Green integral equations will be followed soon. Since the kernels of the present integral equations are not square, a least-square approach should be employed for their numerical solutions.

It will also be necessary to employ the B-spline higher-order panel method rather than the low order panel methods since the treatment of the line integral demands computation of very high accuracy.

We expect that numerical tests of the present improved Green integral equations for moderate and weak currents would be done by many researchers in this field.

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Appendix I. Brard's Green Function

The Green function derived by Brard(Brard 1948) characterizes the potential induced at P by a pulsating source of unit strength at M advancing under the free surface with uniform velocity $U\vec{e_1}$. The point M is the so-called source point and P the field point. It has been obtained as follows:

$$G(P,M,t) = Re\{G(P,M)e^{-i\omega t}\} \tag{A1}$$

where

$$G(P,M) = G_0(P,M) - G_1(P,M) + G_f(P,M) \tag{A2}$$

$$G_j(P,M) = -\frac{1}{4\pi}\frac{1}{r_j}, \quad j=0,1 \tag{A3}$$

$$r_j = \{(x_P-x_M)^2 + (y_P-y_M)^2+(z_P-(-1)^j z_M)^2\}^{\frac{1}{2}}, \quad j=0,1 \tag{A4}$$

$$G_f(P,M) = \frac{1}{4\pi^2}(H_1+H_2) \tag{A5}$$

$$H_l = \int_{-\frac{\pi}{2}}^{+\frac{\pi}{2}} d\theta \int_0^{\infty} \frac{1}{D_l} e^{\zeta} gk\, dk, \quad l=1,2 \tag{A6}$$

$$D_l = [\omega-(-1)^{(l+1)}Uk\cos\theta]^2 - gk + i\nu[\omega-(-1)^{(l+1)}Uk\cos\theta], \quad l=1,2 \tag{A7}$$

$$\zeta = k\{z_P+z_M+i[(x_P-x_M)\cos\theta + (y_P-y_M)\sin\theta]\} \tag{A8}$$

where ν is an artificial damping parameter infinitely small, positive, which will determine the path of integration in the complex plane K associated with the variable k shown in the expressions of H_1 and H_2.

The function G_0 is the Rankine-type Green function which is singular when $P=M$ and regular otherwise. The function G_1 and G_f are regular for $z_P \le 0$.

Brard's Green function satisfies the following equations:

$$\nabla_P^2 G(P,M) = 0 \quad \text{for} \quad P \neq M \tag{A9}$$

$$[(-i\omega - U\frac{\partial}{\partial x_P})^2 + g\frac{\partial}{\partial z_P}]G(P,M) = 0 \quad \text{for} \quad z_M < 0 \quad \text{and} \quad z_P \le 0 \tag{A10}$$

It has been shown that the radiation condition for $G(P,M)$ is satisfied when the artificial damping parameter is present in the denominators D_1 and D_2(Hong 1996). Since the Green function $G(P,M)$ is of $O(\frac{1}{r})$, it tends to zero as $r \to \infty$.

The Green function $G(P,M)$ also satisfies the so-called adjoint free surface condition

$$[(-i\omega + U\frac{\partial}{\partial x_M})^2 + g\frac{\partial}{\partial z_M}]G(P,M) = 0 \quad \text{for} \quad z_P < 0 \quad \text{and} \quad z_M \le 0 \tag{A11}$$

which can be derived from the free surface condition (A10) according to the reciprocal property of the forward-speed Green function(Timman and Newman 1962, Brard 1972).

The integrations with respect to k in H_1 and H_2 can be done analytically by making use of the complex exponential integral $E_1(\zeta)$ as shown by Hong(Hong 1978) in his report on the radiation problem of a cylinder advancing under the free surface. This method of integration was generalized by Guevel(Guevel et al. 1979) and was applied to the three-dimensional radiation-diffraction problem with forward speed by Bougis(Bougis 1980).

Appendix II. Improved Green Integral Equation

When a body is present in the free surface, the fluid region D_e is bounded by the mean wetted surface of the body S, the outer free surface $F_e = F - F_i$ and some arbitrary surface S_∞ at infinity. Let C and C_∞ denote the closed intersection contours of F with S and S_∞ respectively. Applying Green's theorem to the potential Ψ and the Green function G over the fluid region D_e, the following integral identities can be obtained:

$$\alpha \Psi(P) = -\int\int_{S \cup F^e \cup S^\infty} [\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)]ds, \quad \text{for} \quad z_P < 0 \tag{A12}$$

where $\vec{n}$ denotes a unit normal to the boundary surface directed into the fluid region D_e.

The number α in the left-hand side of (A12) takes the value of 1, $\frac{1}{2}$ or 0 according as the field point P lies inside, on and outside the closed surface $S \cup F_e \cup S_\infty$. The $\frac{\partial \Psi}{\partial n}$ and Ψ on the boundary surface denote the densities of sources and normal doublets known as the fundamental hydrodynamic singularities, distributed over there.

Since Ψ and G tend to zero as $\frac{1}{r^\infty}$, the integral over S_∞ vanishes in the limit and, in D_e, we have

$$\Psi(P) = -\int\int_S [\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)]ds - I_F \quad \text{for} \quad z_P < 0 \tag{A13}$$

where

$$I_F = \int\int_{F_e} [\Psi(M)\frac{\partial G(P,M)}{\partial n_M} - \frac{\partial \Psi(M)}{\partial n_M} G(P,M)]ds, \quad z_P < 0 \tag{A14}$$

Substitution of the free surface condition (A10) and the adjoint free surface condition (A11) into the normal derivatives of Ψ and G in (A13) respectively, yields

$$I_F = I_\gamma + I_U \tag{A15}$$

where

$$\begin{aligned} I_\gamma &= -2i\gamma \int\int_{F^e} [\Psi(M)\frac{\partial G(P,M)}{\partial x_M} + \frac{\partial \Psi(M)}{\partial x_M} G(P,M)]ds \\ &= -2i\gamma \int\int_{F^e} \frac{\partial}{\partial x_M}[\Psi(M)G(P,M)]ds \\ &\quad \text{for} \quad z_P < 0 \end{aligned} \tag{A16}$$

and

$$I_U = \frac{U^2}{g}\int\int_{F_e} \frac{\partial}{\partial x_M}[\Psi(M)\frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P,M)]ds, \quad z_P < 0 \tag{A17}$$

The γ in (A16) is a non-dimensional parameter known as the Brard number.

$$\gamma = \frac{U\omega}{g} \tag{A18}$$

Application of Stokes's theorem to (A16) yields

$$\begin{aligned} I_\gamma &= -2i\gamma \int_{C_\infty} \Psi(M)G(P,M)dy_M \\ &\quad + 2i\gamma \int_C \Psi(M)G(P,M)dy_M, \quad z_P < 0 \end{aligned} \tag{A19}$$

where the positive directions around both C and C_∞ are defined counterclockwise when one would see them from above the free surface.

The line integral of the product Ψ and G along C_∞ vanishes in the limit since both Ψ and G tend to zero as $\frac{1}{r^\infty}$ and we have

$$I_\gamma = 2i\gamma \int_C \Psi(M)G(P,M)dy_M, \quad z_P < 0 \tag{A20}$$

Similarly, application of Stokes's theorem to (A17) yields

$$I_U = -\frac{U^2}{g}\int_C [\Psi(M)\frac{\partial G(P,M)}{\partial x_M} - \frac{\partial \Psi(M)}{\partial x_M} G(P,M)]dy_M, \quad z_P < 0 \tag{A21}$$

Substitution of (A20) and (A21) into (A15) yields

$$\begin{aligned} I_F = &\ 2i\gamma \int_C \Psi(M)G(P,M)dy_M \\ &- \frac{U^2}{g}\int_C [\Psi(M)\frac{\partial G(P,M)}{\partial x_M} \\ &- \frac{\partial \Psi(M)}{\partial x_M} G(P,M)]dy_M, \quad z_P < 0 \end{aligned} \tag{A22}$$

Substituting the final expression of I_F into the integral relation

(A13) and taking account of the potential jump across S, we can obtain the following Green integral equation for Ψ:

$$\begin{aligned}\frac{1}{2}\Psi(P) &+ \int\int_S \Psi(M)\frac{\partial G(P,M)}{\partial n_M}ds \\ &+ 2i\gamma\int_C \Psi(M)G(P,M)dy_M \\ &- \frac{U^2}{g}\int_C[\Psi(M)\frac{\partial G(P,M)}{\partial x_M} \\ &- \frac{\partial\Psi(M)}{\partial x_M}G(P,M)]dy_M \\ &= \int\int_S \frac{\partial\Psi(M)}{\partial n_M}G(P,M)ds, \qquad P\in S\end{aligned} \tag{A23}$$

The derivative of Ψ with respect to x_M can be decomposed as follows:

$$\begin{aligned}\frac{\partial\Psi(M)}{\partial x_M} &= \vec{e_1}\cdot[\frac{\partial\Psi(M)}{\partial n_M}\vec{n_M} + \frac{\partial\Psi(M)}{\partial l_M}\vec{l_M} \\ &+ \frac{\partial\Psi(M)}{\partial\tau_M}\vec{\tau_M}], \quad M\in S\end{aligned} \tag{A24}$$

where $\vec{l}$ is a unit vector tangent to C whose direction along which one, traveling in D_e, would proceed in keeping W to his left, is defined positive and τ a unit vector tangent to S forming a right-hand vector triad $\vec{\tau}=\vec{l}\times\vec{n}$.

Substitution of (A24) into (A23) yields

$$\begin{aligned}\frac{1}{2}\Psi(P) &+ \int\int_S \Psi(M)\frac{\partial G(P,M)}{\partial n_M}ds \\ &+ 2i\gamma\int_C \Psi(M)G(P,M)dy_M \\ &- \frac{U^2}{g}\int_C\Big\{\Psi(M)\frac{\partial G(P,M)}{\partial x_M} \\ &- \vec{e_1}\cdot[\frac{\partial\Psi(M)}{\partial l_M}\vec{l_M} \\ &+ \frac{\partial\Psi(M)}{\partial\tau_M}\vec{\tau_M}]G(P,M)]\Big\}dy_M \\ = \int\int_S &\frac{\partial\Psi(M)}{\partial n_M}G(P,M)ds \\ - \frac{U^2}{g}&\int_C\frac{\partial\Psi(M)}{\partial n_M}G(P,M)\vec{e_1}\cdot\vec{n_M}dy_M,\ P\in S\end{aligned} \tag{A25}$$

It should be noted that the expression of the Green function in the line integral can be reduced as follows

$$G(P,M) = G_f(P,M) \quad on\ C \tag{A26}$$

since the first and second terms in the right-hand side of (A2) cancel out when $z_M=0$.

The equation (A25) is the Green integral equation which contains the correct free surface boundary conditions(Hong 2000). However, some boundary conditions are missing in equation (A25). According to the theory of integral equation, an integral equation must contain all the boundary conditions of the boundary value problem in question. Let the surface in contact with the fluid be the positive side of the boundary surface and the other side of the same surface outside D_e the negative side of the surface. The wetted surface will be denoted by S^+ hereafter. According to the potential theory, the potential jump across S which has been incorporated in (51) implies that the condition $\Psi=0$ is imposed on S^-, the negative side of S. In fact, it was necessary to impose $\Psi(P)=0$ when P lies on the negative side of the closed surface $S\cup F_e\cup S_\infty$ as it is done in the Green integral equation with the Rankine-type Green Function. Thus, in order to ensure the uniqueness of the solution, it is necessary to impose the following condition.

$$\Psi=0 \quad on \quad F_e^- \tag{A27}$$

The condition on S_∞ can be omitted since Ψ on S_∞ vanishes in the limit.

The plane F_e^- denotes the negative side of F_e where $z=+\varepsilon$, ε being an infinitesimal positive number.

But, since the integral over the boundary surface F_e was already replaced by the line integral along the waterline C, it is not desirable to reintroduce F_e into the present Green integral equation. Instead we are going to find a supplementary condition for Ψ which can compensate for the condition (A27). To do this, let us introduce the so-called adjoint interior boundary value problem for the interior potential Ψ^i defined inside D_i bounded by S^- and F_i^-. The plane F_i^- where $z=-\varepsilon$, denotes the negative side of F_i which was previously denoted by the waterplane W. Applying Green's theorem to Ψ^i and the Green function G over D_i, the following integral identities analogous to (A12) can be obtained:

$$\begin{aligned}\Psi^i(P) = \int\int_{S^-\cup F_i^-}&[\Psi^i(M)\frac{\partial G(P,M)}{\partial n_M} \\ &- \frac{\partial\Psi^i(M)}{\partial n_M}G(P,M)]ds, \qquad z_P<0\end{aligned} \tag{A28}$$

where $\vec{n}$ denotes a unit normal to the boundary surface directed into the outside of the region D_i.

It is not necessary to solve the boundary value problem for Ψ^i. However we will assume that Ψ^i satisfies the following conditions.

$$\Psi^i=0 \quad on \quad S^- \tag{A29}$$

$$\frac{\partial\Psi^i}{\partial n}=0 \quad on \quad S^- \tag{A30}$$

$$[(-i\omega - U\frac{\partial}{\partial x})^2 + g\frac{\partial}{\partial z}]\Psi^i = 0 \quad on \quad F_i^- \tag{A31}$$

If the condition (A29) is imposed on the closed surface $S^- \cup F_i^-$, Ψ^i would vanish identically in D_i. In that case, the conditions (A30) and (A31) would be satisfied only in the trivial way. But, here, the nature of the problem demands that these conditions are satisfied in both the trivial and the non-trivial ways. Therefore the conditions (A29), (A30) and (A31) are necessary *simultaneously*.

Substituting (A29), (A30) and (A31) for Ψ^i as well as the adjoint free surface condition (A11) for the Green function G, into (A28) and applying Stokes's theorem to the integral over F_i^-, we have

$$\begin{aligned}\Psi^i(P) &= 2i\gamma\int_C \Psi^i(M)G(P,M)dy_M \\ &- \frac{U^2}{g}\int_C [\Psi^i(M)\frac{\partial G(P,M)}{\partial x_M} \\ &- \frac{\partial \Psi^i(M)}{\partial x_M}G(P,M)]dy_M, \quad z_P < 0\end{aligned} \tag{A32}$$

where the positive directions around C is defined counterclockwise when one would see them from above the free surface as defined previously for the line integral involving Ψ.

Let Ψ^e denote the potential on F_i^- induced by the hydrodynamic singularities distributed over $S^+ \cup C$.

$$\begin{aligned}\Psi^e(P) &= -\int\int_{S^+}[\Psi(M)\frac{\partial G(P,M)}{\partial n_M} \\ &- \frac{\partial \Psi(M)}{\partial n_M}G(P,M)]ds \\ &- 2i\gamma\int_C \Psi(M)G(P,M)dy_M \\ &+ \frac{U^2}{g}\int_C [\Psi(M)\frac{\partial G(P,M)}{\partial x_M} \\ &- \frac{\partial \Psi(M)}{\partial x_M}G(P,M)]dy_M, \ P \in F_i^-\end{aligned} \tag{A33}$$

Now we impose the following condition on F_i^-:

$$\Psi^e + \Psi^i = 0 \quad on \quad F_i^- \tag{A34}$$

We expect that the condition (A34) would be the compensation for the condition (A27).

Substituting (A32) and (A33) into (A34), we have

$$\begin{aligned}\Psi^e(P) &= -\int\int_{S^+}[\Psi(M)\frac{\partial G(P,M)}{\partial n_M} \\ &- \frac{\partial \Psi(M)}{\partial n_M}G(P,M)]ds \\ &- 2i\gamma\int_C \Psi(M)G(P,M)dy_M \\ &+ \frac{U^2}{g}\int_C [\Psi(M)\frac{\partial G(P,M)}{\partial x_M} \\ &- \frac{\partial \Psi(M)}{\partial x_M}G(P,M)]dy_M, \quad P \in F_i^-\end{aligned} \tag{A35}$$

Now we impose

$$\Psi(M) - \Psi^i(M) = 0, \qquad M \in C \tag{A36}$$

$$\frac{\partial[\Psi(M) - \Psi^i(M)]}{\partial x_M} = 0, \qquad M \in C \tag{A37}$$

Then the equation (A35) becomes as follows:

$$\begin{aligned}\int\int_{S^+}&[\Psi(M)\frac{\partial G(P,M)}{\partial n_M} \\ &- \frac{\partial \Psi(M)}{\partial n_M}G(P,M)]ds = 0, \ P \in F_i^-\end{aligned} \tag{A38}$$

Combining the equation (A38) with (A25), we have the improved Green integral equation :

$$\begin{aligned}&\frac{1}{2}\Psi(P)\delta[1-\delta(z_P-0)] \\ &+ \int\int_{S^+}\Psi(M)\frac{\partial G(P,M)}{\partial n_M}ds \\ &+ \delta[1-\delta(z_P-0)]\Big[2i\gamma\int_C \Psi(M)G(P,M)dy_M \\ &- \frac{U^2}{g}\int_C\Big\{\Psi(M)\frac{\partial G(P,M)}{\partial x_M} - \\ &\vec{e_1}\cdot[\frac{\partial \Psi(M)}{\partial l_M}\vec{l_M} + \frac{\partial \Psi(M)}{\partial \tau_M}\vec{\tau_M}]G(P,M)]\Big\}dy_M\Big] \\ &= \int\int_{S^-}\frac{\partial \Psi(M)}{\partial n_M}G(P,M)ds \\ &- \delta[1-\delta(z_P-0)] \\ &\frac{U^2}{g}\int_C[\frac{\partial \Psi(M)}{\partial n_M}G(P,M) \\ &(\vec{e_1}\cdot\vec{n_M})]dy_M\end{aligned} \tag{A39}$$

for $P \in S^+ \cup F_i^-$