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Robust Control of a Glass Fiber Composite Beam using μ-Synthesis Algorithm

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ABSTRACT

A study on the robust control of a composite beam with a distributed PVDF sensor and piezo-ceramic actuator is presented in this paper. 1^{st} and 2^{nd} natural frequencies are considered in the modeling, because robust control theory which has robustness to structured uncertainty is adopted to suppress the vibration. If the controllers designed by H_∞ theory do not satisfy control performance, it is improved by μ-synthesis method with *D-K* iteration so that the μ-controller based on the structured singular value satisfies the nominal performance and robust performance. Simulation and experiment were carried out with the designed controller and the verification of the robust control properties was presented by results.

Keywords : Robust control, PVDF sensor, structured singular value, *D-K* iteration, weighting function, μ-synthesis, piezo-ceramic actuator.

Nomenclature

b : width of a PVDF
C^E_{pq} : stiffness matrix
C^s_p : capacitance of a sensor
E_k : electric field
D_i : electric displacement
d_{kp}, e_{iq} : piezo coefficients
E : effective young's modulus
i, j, k :subscript of piezo elements(=1,⋯,3)
I : inertia moment
K_s : constant of a PVDF(= $Y_p d_{31} b y_c / C^s_p$)
K_a : constant of a PZT (= $(1/2) b d_{31} Y_p (t_a + t_b)$)
p, q : subscript of piezo elements(=1,⋯,6)
r : radius of curvature
s^E_{pq} : compliance matrix
S : sensitivity function(= $(I + G_{nom}K)^{-1}$)
S_p : strain matrix
T_q : stress matrix
T : complementary sensitivity (= $KG_{nom}(I + KG_{nom})^{-1}$)
y_c : distance from the sensor to neutral axis
Y_p : young's modulus of piezoelectric materials
$y'(x)$: modal function of a normalized beam
$v_p(t)$: output voltage of a PVDF sensor
ρ : density of a beam
ε^T_{ik} : permeability coefficient
$\eta_i(t)$: generalized modal coordinate
$\phi_i(t)$: modal function of a composite beam
β_i : satisfaction coefficient($\beta_i^4 = \rho A \omega_i^2 / EI$)
ω_i : natural frequencies of a cantilever beam

1. Introduction

Glass-Fiber-Reinforced(GFR) polymeric composite materials provides the desirable properties of high stiffness and strength as well as low specific weight. Therefore, this material is now used in a variety of components for automotive, aerospace, marine, and architectural structure[1-2]. The advantages of GFR thermoplastic composites are very short cycle time and good surface finish since solid-phase forming techniques can be used. Solid-phase forming is a process in which the part is formed at temperatures between the glass transition temperature and melting point of the polymer matrix[3-5]. But the structures using these materials have low flexural rigidity and are lightly damped due to the small material damping.

This may lead to the destructive large-amplitude vibration and long vibration decay times and thus result in fatigue, instability and poor operation of the structures. In order to solve these problems, the composite beams bonded or embedded with sensors and actuators called smart structure have been recently studied.

Smart structure incorporating piezoelectric sensors and actuators have found a wide range of applications in the fields of active vibration control[6], shape control[7], self-sensing control[8] and noise control in the past decade. Piezoelectric elements have been used successfully in the closed loop control of a variety of active structures with shape memory alloy[9], electro-rheological fluids[10], and optic fiber [11].

Through a number of studies have been reported on the vibration control using these materials, but there are few studies concerning the active robust vibration control of a random directional glass fiber reinforced thermoplastic composite.

In this work, using a PVDF sensor and piezo-ceramic actuator, a study on the robust control of a cantilever composite beam is presented. The state equation of a composite beam is obtained by using a modal approach and modal coordinates. A robust controller is designed to suppress the vibration of a reinforced beam using μ-synthesis theory. Experiment was carried out and the results were compared with the simulation results.

2. Theoretical Analysis

The PVDF sensor and piezo-ceramic actuator are embedded to a random directional glass fiber reinforced thermoplastic composite. The properties of materials used in this study are shown in Table 1. Assuming that piezo-material is isotropic, the general piezo-material equations can be expressed in the following form.

$$S_p = s_{pq}^E T_q + d_{kp} E_k \quad (1)$$

$$D_i = d_{iq} T_q + \varepsilon_{ik}^T E_k \quad (2)$$

$$T_p = c_{pq}^E S_q + e_{kp} E_k \quad (3)$$

$$D_i = e_{iq} S_q + e_{ik}^s E_k \quad (4)$$

Supposed that composite material is a Bernoulli-Euler beam, the Eq. (1) and Eq. (2) can be written by :

$$S_1 = \frac{1}{Y_p} T_1 + d_{31} E_3 \quad (5)$$

$$D_3 = d_{31} T_1 + \varepsilon_{33} E_3 \quad (6)$$

Equations (5) and (6) can be used to verify that the stress and strain can be determined from the electromechanical equations when reduced to the scalar case. The sensor equation of PVDF can be calculated using piezoelectric effect. In the case of a Bernoulli-Euler beam, the stresses have the following Eq. (7).

$$T_1(x, y) = -\frac{y_c Y_p}{r} \quad (7)$$

$$\frac{1}{r} = \frac{\partial^2 y(x,t)}{\partial x^2} \quad (8)$$

Using Eq. (6) and Eq. (7), the expression of Eq. (6) can be written by ;

$$D_3(x, t) = d_{31} \frac{y_c Y_p}{r} \quad (9)$$

where, $1/Y_p = S_{31}^E$. The charge $q(t)$ from a PVDF sensor is calculated by integrating Eq. (9), and the result is shown by :

$$q(t) = \int_A D_i n_i \, dA = \int_{x_1}^{x_2} Y_p y_c d_{31} b \frac{\partial^2 y(x,t)}{\partial x^2} dx$$
$$= Y_p y_c d_{31} b [\tilde{y}'(x_2) - \tilde{y}'(x_1)] \quad (10)$$

Then, the voltage can be expressed by Eq. (11).

$$v_p(t) = \frac{q(t)}{C_p^s} = K_s [y'(t, x_2) - y'(t, x_1)] \quad (11)$$

If the actuator is pasted in the part of a composite beam, The voltage distribution is given by Eq. (12)

$$V(x,t) = [H(x-x_1) - H(x-x_2)]V_a(t) \tag{12}$$

where, $H(\cdot)$ is Heaviside function which is used to locate the distributed moment between x_1 and x_2. x_1 and x_2 is the coordinated position of a piezo-ceramic actuator. The moment induced by voltage is written by :

$$M(x,t) = \int_{t_b/2}^{t_b/2+t_a/2} \sigma_1 bzdz \tag{13}$$

where, t_a is the thickness of the piezoelectric actuator, t_b is the thickness of a beam. The distributed moment that the piezoelectric actuator applies to the beam is proportional to the applied voltage by following Eq. (14).

$$M(t,x) = K_a v_a(t)[y'(x_2) - y'(x_1)] \tag{14}$$

The equation of a flexible composite beam with a PVDF sensor and piezo-ceramic actuator is given by :

$$EI\frac{\partial^4 w}{\partial x^4} + \rho A\frac{\partial^2 w}{\partial t^2} = M \tag{15}$$

The boundary conditions are given by :

$$w(0,t) = 0 \qquad \frac{\partial w(0,t)}{\partial x} = 0$$

$$\frac{\partial^3 w(l,t)}{\partial x^3} = 0 \qquad \frac{\partial^2 w(l,t)}{\partial x^2} = 0 \tag{16}$$

The vibration displacement can be divided into time function $\eta_i(t)$ and space function $\phi_i(x)$ using the classical modal analysis, and which has the form as:

$$W(x,t) = \sum_{i=1}^{n} \phi_i(x)\eta_i(t) \tag{17}$$

where, $\eta_i(t)$ is generalized modal coordinate and $\phi_i(x)$ is modal function. Modal function can be written in the following form with boundary conditions.

$$\phi_i(x) = \cosh\beta_i x - \cos\beta_i x - \frac{\sinh\beta_i L - \sin\beta_i L}{\cosh\beta_i L + \cos\beta_i L} \times (\sinh\beta_i x - \sin\beta_i x) \tag{18}$$

Substituting Eq. (17) into Eq. (15), a set of ordinary differential equations in modal coordinates can be given by:

$$\ddot{q}_i + 2\zeta_i\omega_i\dot{q}_i + \omega^2 q_i = K_a v_a(t)[y'(x_2) - y'(x_1)] \tag{19}$$

Neglecting the high-order modes, ordinary differential equation can be presented by the state space equation.

$$\begin{aligned} x(t) &= Ax(t) + Bv(t) \\ y(t) &= Cx(t) \end{aligned} \tag{20}$$

where, $x(t) = [\eta_1\ \dot{\eta}_1,\ \ldots,\ \eta_n\ \dot{\eta}_n]$,

$$A = \begin{bmatrix} 0 & 1 & \cdots & & \\ -\omega_1^2 & -2\zeta_1\omega_1 & & & \\ & & \ddots & & \\ & & & 0 & 1 \\ & & \cdots & -\omega_n^2 & -2\zeta_n\omega_n \end{bmatrix},$$

$B = [0,\ b_1,\ \ldots,\ 0,\ b_n]^T$,

$C = [c_1,\ 0,\ \ldots,\ c_n,\ 0]$.

3. μ-Synthesis Theory

There are many methods available for active control synthesis such as PID, LQG, H_∞ and sliding mode control. In this paper, a newly developed design method for robust vibration control using μ-synthesis is proposed which enables to obtain the robust stability as well as the robust performance against the perturbations.

Table 1 Properties of materials

Property	PVDF	PZT	Beam
d_{31}(pC/N)	23	260	-
g_{31}(10-3)Vm/N	21.6	-	-
E_1(GPa)	2	63	5
E_2(GPa)	2	63	5
ν_{12}	-	0.34	0.3

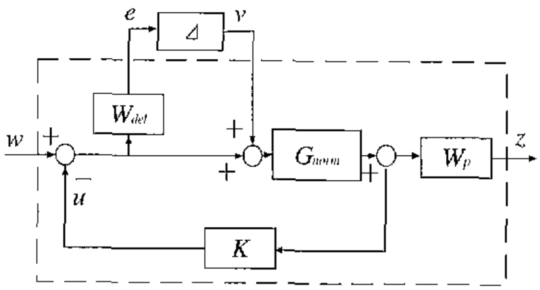


Fig. 1 Block diagram of closed-loop system

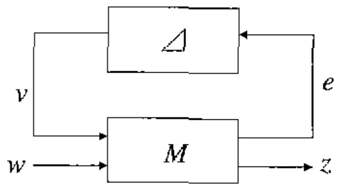


Fig. 2 General plant for μ-synthesis

A robust controller with the structured singular value was designed for the system by using the μ -synthesis theory. The block diagram in Fig. 1 is composed of weight function Wdel for uncertainty of the model, weight function Wp for performance of the controller, a nominal model Gnom, model error Δ , and controller K. The nominal model Gnom means the theoretical model written by the state equation and Δ is the error between the real structural system and nominal model. It is assumed that the model error Δ is not known, except that it satisfies condition $\|\Delta\|_\infty < 1$.

The uncertainty of the model is occurred because of the approximation of the continuous structural systems with a discrete system of finite degrees of freedom, neglect of high-order modes, and other approximations in the modeling process. The μ -synthesis control system shown in Fig. 1 can be expressed to a general plant shown in Fig. 2 in which M is the transfer function of the sub-system in Fig. 1. The designed controller K satisfies the robust control performance defined by weight functions Wdel and Wp. Therefore, μ -synthesis controller has the form in which the H∞ norm of M is less than 1. A robust controller which based on the structured singular value has the nominal performance and robust performance.

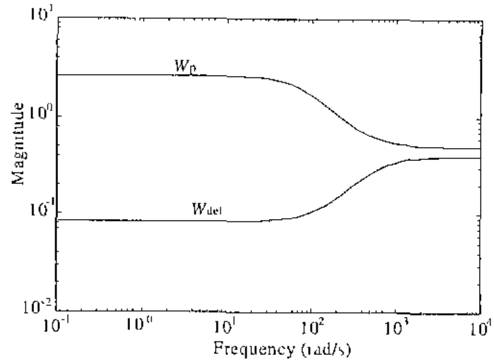


Fig. 3 Frequency characteristics of weighting functions

First, we determined weight functions Wdel and Wp in order to design the controller K. As we know, high suppression ratio to the disturbance is required only in the low-frequency range since only the low-order modes are strongly excited. On the other hand, the error of the nominal system model is small in the low-frequency range, but it can be large in the high-order modes since the high-order modes are neglected. For this reason, the following weight functions are chosen:

$$W_{del} = \frac{0.4(s+125)}{0.5(s+600)} \tag{21}$$

$$W_p = \frac{0.5(s+424)}{s+80} \tag{22}$$

The weight function Wdel means that the modeling error is 8% in the low-frequency range and 40% in the high-frequency range. The function Wp equals 2.5 in the low-frequency range and 0.5 in the high-frequency range, which means that the disturbance in the output path of the closed-loop system should be reduced by 60% in the low-frequency range and could be amplified by as much as 100% in the high-frequency range.

The frequency characteristics of weight function W_{del}, W_p are plotted in Fig. 3. The transfer function of expanded plant M can be expressed in the following form:

$$M = \begin{bmatrix} W_{del}T & W_{del}KS \\ W_pSG_{nom} & W_pS \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \tag{23}$$

where, in the transfer function M, $W_{del}T$ is the robust stability condition and W_pS is nominal performance condition. The μ-synthesis has to satisfy the robust performance given by Eq. (24).

$$Sup_{\bar{\sigma}(\Delta)\le 1}|F_u(M, \Delta)|_\infty \le 1 \tag{24}$$

where F_u is linear fractional transformation(LFT) which is defined by

$$F_u(M, \Delta) = M_{22} + M_{21}\Delta(I - M_{11}\Delta)^{-1}M_{12} \tag{25}$$

The Eq. (24) can be expressed by the following equivalent equation

$$\mu_\Delta(M) \le 1 \tag{26}$$

which μ is structured singular value defined by Eq. (27).

$$\mu_\Delta(M) = \frac{1}{\min[\bar{\sigma}(\Delta) \cdot det(I - M\Delta) = 0]} \tag{27}$$

Using LFT, the transfer function $F_u(M, \Delta)$ is found from w to z by the Eq. (28)

$$z = F_u(M, \Delta)w \tag{28}$$

Using H_∞ control theory, controller is obtained with $\gamma = ||M||_\infty = 1.3$. It should be noticed that the robust performance of this system is not satisfied since γ is larger than 1. If the designed robust controller is not satisfied, a reasonable approach can be done by iterative method for D and K as shown in Eq. (29). This is so-called D-K iteration.

$$\mu_\nabla(F_l(P_i, K_i)) = \min_{D,K}||D_{i+1}F_l(P_i, K_i)D_{i+1}^{-1}||_\infty \tag{29}$$

After three steps of $D-K$ iteration of the μ-synthesis method, γ is reduced to 0.98 in Fig. 4, this means that the desired robust performance is satisfied. On the other hand, the order of controller rises to 12 after one-step of $D-K$ iteration.

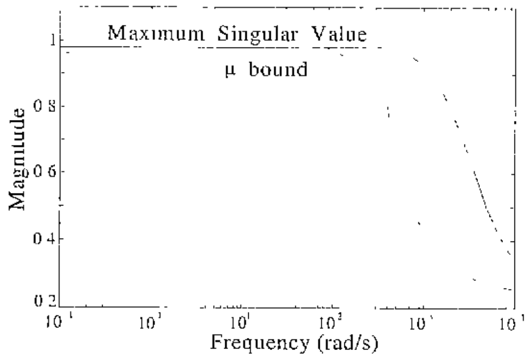


Fig. 4 Robust performance

In order to reduce the amount of computation and increase the sampling frequency, the order of controller is reduced from 8 to 6 using an optimal norm approximation. Also it is difficult to satisfy the desired performance only if H_∞ is used. However, the robust performance could be satisfied with the peak value less than 1 when the μ-synthesis was used. Therefore, the designed μ-controller is expressed by the following form:

$$K(s) = \frac{4655400(s+5.2)(s+4.2+407.2i)}{(s+132180)(s+8.4+4.1i)(s+8.4-4.1i)} \times \frac{(s+4.2-407.2i)(s+3.8+109.9i)(s+3.8-109.9i)}{(s+49.5+184.8i)(s+49.5-184.8i)(s+4.3)} \tag{23}$$

4. Experimental Setup

The experimental set-up of a flexible beam is shown in Fig. 5. The material used for the tests is a random glass fiber reinforced polypropylene composite. The average glass fiber length and diameter were obtained from the manufacturer to be 12 mm and 11 μm respectively. The composite sheet with glass fiber weight fractions of 40% was used for the tests. The thickness of the sheet was 2.54 mm for 40% glass sheet The PVDF was LDT1-028K(AMP, 28μm thickness). The piezoceramic used is a multi-layer bender PZT actuator which is PL-128.255 Lead Zirconate Titanate (PZT) of PI(Physik

Instrument company). The dimension is 31× 9.6× 0.65mm.

The PZT and PVDF were bonded to the beam with adhesive, and the electrical leads were soldered to the electrode of piezo-elements. The overall experimental configuration of a flexible beam is shown in the following Fig. 6. Tensile test using UTM(United Testing Machine) was done in order to measure Young's modulus according to ASTM D3039 method. Natural frequencies and damping ratios of a flexible beam are calculated by modal analysis which is done by A&D 3254 FFT analyser system. Natural frequencies found by modal analysis are also compared by ANSYS FEM analysis, and Rayleigh-Ritz method in Table 2. The experiment was carried out using DSP board (TMS320C40, dSPACE company) and amplifier(PI company). The voltage-follower and north-filter is used by LM324 IC. The signal from A/D converter with sampling ratio 1ms is sent to the DSP system and the calculated control input is sent to a PZT actuator for control through D/A converter and amplifier.

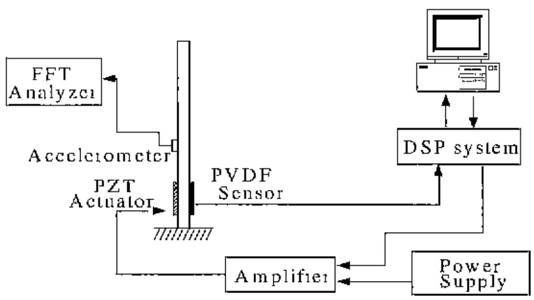


Fig. 5 Overall experimental set-up

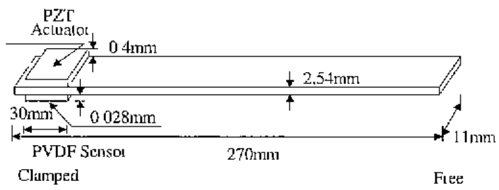


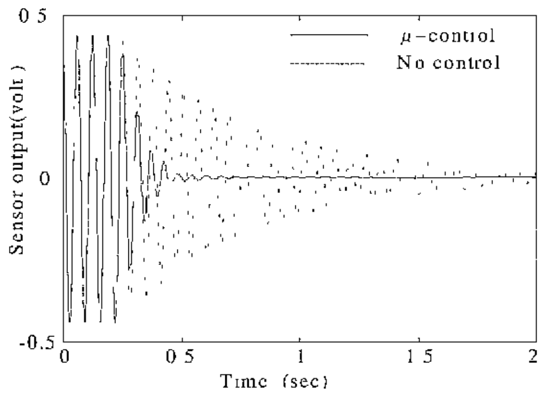
Fig. 6 Experimental configuration of a composite beam

Table 2 Comparison of modal frequencies for a composite beam(unit: Hz)

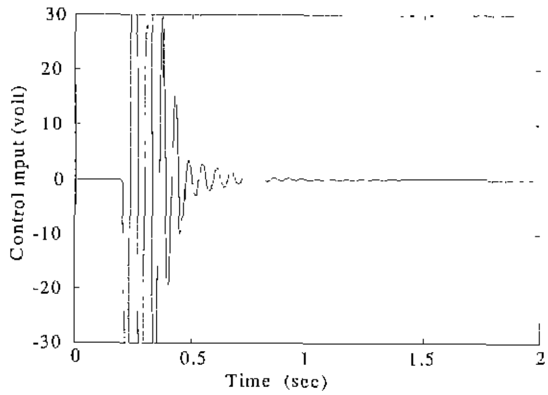
	f_1	f_2	f_3
Rayleigh-Ritz method	15.56	63.62	185.95
FEM	16.59	65.51	188.83
Modal analysis	15.61	65.24	190.74

5. Simulation and Experimental Results

Simulations and experiments were performed at the frequency of the first mode. Fig. 7 shows the simulation results by μ-synthesis control. The exciting frequency is the first natural frequency with the magnitude of 20V for 0.2s to the actuator and control is started after 0.2s later. Fig. 7(a) is the tip displacement from the sensor(Dotted line is controlled output and dashed line is uncontrolled output) and Fig. 7(b) is the control input to the actuator. The controller was discretized at every sampling interval of 0.001s. It is shown that the vibration of a composite beam was reduced to less then 1/10 in about 0.02s after control was started. Hence, the transient vibration of a composite beam was effectively suppressed by using the output feedback of the designed controller.

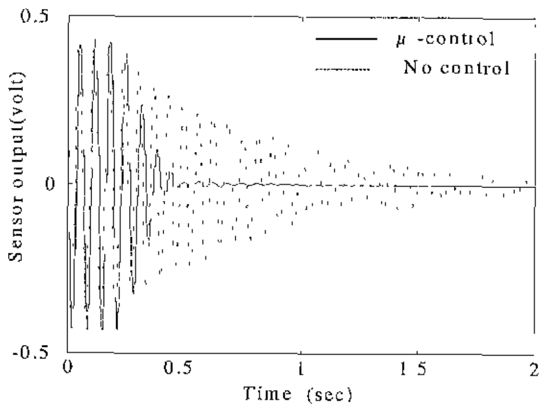


(a) Sensor output of a composite beam

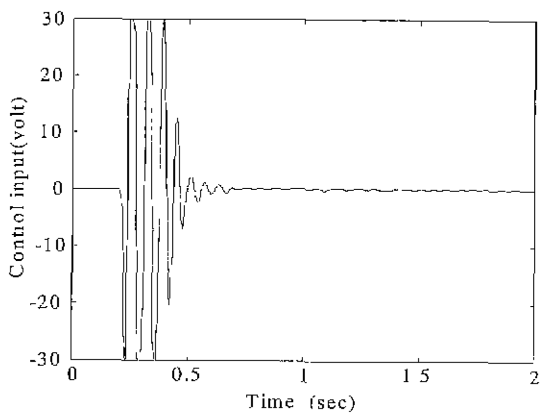


(b) Control input of a composite beam

Fig. 7 Simulation results for transient vibration control of a composite beam: 1st bending mode



(a) Sensor output of a composite beam



(b) Control input of a composite beam

Fig. 8 Experimental results for transient vibration control of a composite beam: 1st bending mode

Fig. 8(a) is the experimental results of robust control by using μ-synthesis. The sensor voltage of vibration was reduced well and it can also be found that the experimental results are in good agreement with the simulation results.

6. Conclusion

The robust control of a composite beam using μ -synthesis theory is carried out to suppress the vibration when there is the uncertainty of the model. From the simulation and experimental results, we can see the following results.

1) There was a possibility that we couldn't find a robustness using H_∞ theory, but using the μ -synthesis with structured singular value, the designed controller satisfied the robust performance and nominal performance.
2) The vibration of a glass fiber composite beam using a PVDF sensor and piezo-ceramic actuator was effectively suppressed using μ-synthesis control theory.

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