International Journal of the Korean Society of Precision Engineering, Vol. 1, No. 1, June 2000.

Friction Analysis of Spindle Bearings

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ABSTRACT

Friction in bearing exerts an important effect upon power dissipation and heat generation of spindle system. This paper presents frictional moments derived from rotational axis coordinate system of spindle and frictional characteristics to spindle speed. A frictional moment of spindle bearings is derived by work-energy method. Differential sliding moments in outer raceway has a major effect upon frictional resistance; spin sliding moments in inner raceway has a secondary effect. As spindle speed increases, also the frictional moments increase. In high-speed region, ceramic ball bearing has smaller frictional moment than steel ball bearing.

Key Words: Friction, frictional moment, spindle bearing

Nomenclature

a = semi-major axis of projected contact ellipse
b = semi-minor axis of projected contact ellipse
D = ball diameter
E = kinetic energy
F = force
J = mass moment of inertia
M = moment
M_g = gyroscopic moment
M_R = rolling friction moment
M_S = spin friction moment
Q = normal load between ball and raceway
R = ring radius to neutral axis, radius of locus of raceway groove curvature centers
T = friction resistance
T_R = rolling friction resistance
W = work by moment
X, Y, Z = orthogonal right-handed coordinate system
α = contact angle
β = ball speed vector pitch angle
θ = angle defines a point on a contact ellipse
μ = coefficient of friction
ϕ = angle defines a normal direction of friction force at a point on a contact ellipse
σ_n = normal stress on differential area
Ω = absolute rotational speed of spindle
ω = rotational speed of spindle

subscripts
i refers to inner raceway
j refers to angular position of ball
o refers to outer raceway
R refers to rolling element
W refers to inner or outer raceway

1. Introduction

Spindle system determines the main performance of machine tools. Most of the bearings in spindle system are rolling bearings because of well-established standard, easy maintenance and low price. Current trend in spindle development is to increase the machining speed to reduce the machining time and cost, and higher stiffness of the spindle. These raise accuracy and damage problems of bearing system. For speed and stiffness cannot be achieved together and excessive preload causes too much generation of heat, preload amount has to be chosen properly.

In this paper, friction in angular-contact ball bearings used widely for machine tool spindle, is analyzed considering preload regulation.

2. Friction Characteristics of Spindle Bearing

2.1 Frictional Moment of Spindle Bearing

Frictional moment in rolling bearings depends on many factors like deformations of ball and raceway, characteristics of lubricant, conditions of oil film in contact area and rolling speed.

Frictional moment is generated at contact point of ball and bearing elements. Frictional force by contact stress between ball and raceway is along the sliding direction.

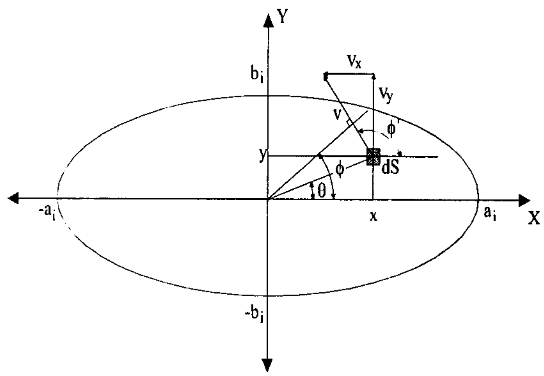

Fig. 1 Frictional force and sliding velocities acting on elliptical contact surface

The normal stress acting on the elliptic shape contact area in Fig.1 is given by

$$\sigma_n = \frac{3Q}{2\pi ab}\left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]^{1/2} \tag{1}$$

Frictional force acting on the contact area can be expressed by the product of normal force and coefficient of friction. Coefficient of friction that has frictional characteristics of spindle system should be determined by the dynamic condition of contact area. And it is proper to investigate experimentally in system level.

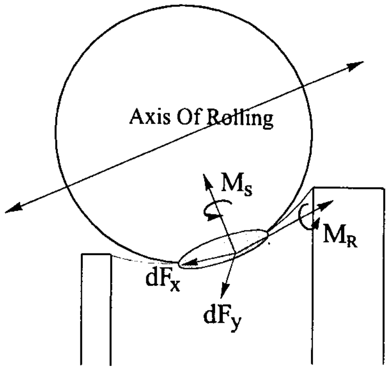

Fig. 2 Frictional forces and moments acting on a contact ellipse

Resist force, dF at micro-area dS in contact ellipse can be calculated by

$$dF = \frac{3Q}{2\pi ab}\left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]^{1/2} dS \tag{2}$$

Y-direction force at contact area by sliding motion is expressed as follows by area integration of equation 2

$$F_y = \oint_S (\sigma_n \cos\phi)\, dS$$
$$= \frac{3Q}{2\pi\, ab}\int_{-a}^{+a}\int_{-b\left(1-(x/a)^2\right)^{1/2}}^{+b\left(1-(x/a)^2\right)^{1/2}}\left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]^{1/2}\cos\varphi\, dydx \tag{3}$$

Also, x-directional force is by

$$F_y = \oint_S (\sigma_n \cos\phi)\, dS$$
$$= \frac{3Q}{2\pi\, ab}\int_{-a}^{+a}\int_{-b\left(1-(x/a)^2\right)^{1/2}}^{+b\left(1-(x/a)^2\right)^{1/2}}\left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]^{1/2}\cos\varphi\, dydx \tag{4}$$

Here, ϕ and θ are expressed as

$$\phi = \tan^{-1}\frac{\rho\omega_\delta \sin\theta - \upsilon_x}{\rho\omega_\delta \cos\theta + \upsilon_y} = \tan^{-1}\frac{y\omega_\delta - \upsilon_x}{x\omega_\delta + \upsilon_y} \tag{5}$$

$$\theta = \tan^{-1}\frac{y}{x} \tag{6}$$

In contact ellipse, relative motion has two component, difference of linear motion and spin motion.

$$dM_s = \rho\cos(\phi-\theta)\,dF \tag{7}$$

$$M_s = \oint_S (\rho\,\sigma_n \cos(\phi-\theta))\,dS$$
$$= \frac{3Q}{2\pi ab}\int_{-a}^{+a}\int_{-b\left[1-(x/a)^2\right]^{1/2}}^{+b\left[1-(x/a)^2\right]^{1/2}} \left(x^2+y^2\right)^{1/2} \tag{8}$$
$$\times\left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]^{1/2}\cos(\phi-\theta)\,dydx$$

Frictional moment in contact area for axis that pierces ball center and is perpendicular to line that determines contact angle is

$$dM_R = r'_w \cos\phi\, dF \tag{9}$$

$$M_R = \oint_S (r'_w\,\sigma_n \cos\phi)\,dS$$
$$= \frac{3Q}{2\pi ab}\int_{-a}^{+a}\int_{-b\left(1-(x/a)^2\right)^{1/2}}^{+b\left(1-(x/a)^2\right)^{1/2}} \left[1-\left(\frac{x}{a}\right)^2-\left(\frac{y}{b}\right)^2\right]^{1/2}$$
$$\times\left\{\left(R^2-x^2\right)^{1/2}-\left(R^2-a^2\right)^{1/2}+\left[\left(\frac{D}{2}\right)^2-a^2\right]^{1/2}\right\}$$
$$\cos\phi\, dydx \tag{10}$$

Frictional moment can be calculated from differential moment and spin moment with coefficient of friction. Using coefficient of friction, frictional moments for each raceway are

$$T_{Rw} = \mu M_{Rw}$$
$$T_{sw} = \mu M_{sw} \tag{11}$$

Each frictional moment shown in Fig. 3 is in equilibrium at static equilibrium condition and can be expressed by

$$-\mu M_{Ro}\cos\alpha_o - \mu M_{so}\sin\alpha_o$$
$$+\mu M_{Ri}\cos\alpha_i + \mu M_{si}\sin\alpha_i = 0 \tag{12}$$

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$$+\mu M_{Ri}\sin\alpha_i - \mu M_{si}\cos\alpha_i = 0 \tag{13}$$

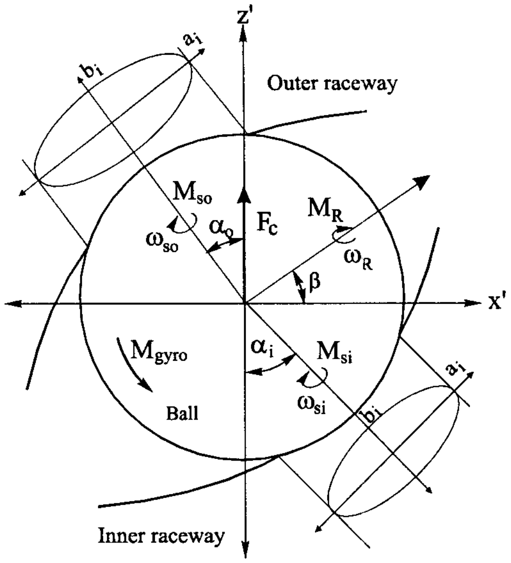

Fig. 3 Frictional and gyroscopic moments acting on a ball

These Equations don't include sliding by gyroscopic moment. Here, frictional moments of rotational direction in bearing raceway are in static equilibrium and gyroscopic moment is supported by frictional moment in this direction.

$$\frac{1}{2}\mu D\left(F_{\chi i}+F_{\chi o}\right)-M_{gy'}=0 \tag{14}$$

But, in case there is skid motion by high speed rotation of spindle, static equilibrium condition doesn't satisfied. Then the motion is described by

$$\frac{1}{2}\mu D\left(F_{\chi i}+F_{\chi o}\right) < M_{gy'} \tag{15}$$

$$\Sigma M_{y'} = M_{gy'} - \frac{1}{2}\mu D\left(F_{\chi i}+F_{\chi o}\right) = M_{y'} = J\frac{d\omega_y}{dt} \tag{16}$$

This non-equilibrium is non-damping and constant accelerating condition. Further torque transfer to the spindle system can destruct itself by increased motion in y'-direction.

In Equation 11, moment component is determined by load and velocity for the moment equation is derived by normal stress at contact area and direction of sliding velocity.

By the relation between ball and raceway, frictional moment of bearing can be expressed by the arithmetic sum of differential and spin sliding components.

$$T_B = T_{si} + T_{Ri} + T_{so} + T_{Ro} \quad (17)$$

But, because each moment component is expressed at different rotating axis, it is not plausible that this sum is a total frictional moment of bearing. By introducing the concept of work by moment, we now express work by product of moment and rotating angle at axis it resides.

$$W = M\,\theta \quad (18)$$

Works by moment component are

$$W_{sw} = M_{sw}\,\theta_{sw} \quad (19)$$
$$W_{Rw} = M_{Rw}\,\theta_{Rw} \quad (20)$$

Each rotating angle can be obtained by the ratio of spin speed to spindle rotating speed and rolling speed to spindle rotating speed.

$$\theta_{Rw} = \left(\frac{\theta_{Rw}}{\theta}\right)\theta = \left(\frac{\omega_{Rw}}{\Omega}\right)\theta \quad (21)$$

For spin rotating angle,

$$\theta_{sw} = \left(\frac{\theta_{sw}}{\theta}\right)\theta = \left(\frac{\omega_{sw}}{\Omega}\right)\theta \quad (22)$$

Moment-work relation is

$$W_{sw} = M_{sw}\,\theta_{sw} = M_{sw}\left(\frac{\theta_{sw}}{\theta}\right)\theta = M_{sw}\left(\frac{\omega_{sw}}{\Omega}\right)\theta \quad (23)$$
$$W_{Rw} = M_{Rw}\,\theta_{Rw} = M_{Rw}\left(\frac{\theta_{Rw}}{\theta}\right)\theta = M_{Rw}\left(\frac{\omega_{Rw}}{\Omega}\right)\theta \quad (24)$$

Ratio of rolling speed to spindle rotating speed by analysis of bearing velocity is

$$\frac{\omega_R}{\Omega} = \frac{-1}{\sum_{w=i,o}\left[\frac{r'_w(\cos\beta\cos\beta'\cos\alpha_w + \sin\beta\sin\alpha_w)}{(d_m/2) + c_w r'_w\cos\alpha_w}\right]} \quad (25)$$

Ratio of spin speed to spindle rotating speed is

$$\frac{\omega_{so}}{\Omega} = \left(\frac{\omega_R}{\Omega}\cos\beta\cos\beta'\sin\alpha_o - \frac{\omega_R}{\Omega}\sin\beta\cos\alpha_o - \sin\alpha_o\right)$$
$$\frac{\omega_{si}}{\Omega} = \left(-\frac{\omega_R}{\Omega}\cos\beta\cos\beta'\sin\alpha_i + \frac{\omega_R}{\Omega}\sin\beta\cos\alpha_i + \sin\alpha_i\right) \quad (26)$$

Frictional moment by spin motion at spindle rotation axis is expressed with coefficient of friction.

$$T_{sw} = \mu M_{sw}\left(\frac{\omega_{sw}}{\Omega}\right) \quad (27)$$

Likewise, differential sliding moment by rolling motion is

$$T_{Rw} = \mu M_{Rw}\left(\frac{\omega_{Rw}}{\Omega}\right) \quad (28)$$

Until now friction coefficient is still undetermined variable through the derivation of equation. By the work and energy relation, friction coefficient will be derived in forms expressed by determinable variables in following part.

In constant speed, kinetic energy of the spindle-bearing system is

$$E_1 = \frac{1}{2}\sum_{k=1,N}\left[\sum_{j=1,Z}\left(J_B\omega_R^2\right)_j\right]_k + \frac{1}{2}J_S\Omega^2 \quad (29)$$

Here, N is number of bearings, Z is number of bearing balls, J_B, J_S are mass moment inertia of ball and spindle respectively. If there is no external power input till spindle stops, kinetic energy of spindle-bearing system consumed as work by frictional moment. This relation is as follows

$$E_1 = \int_0^{\theta_1} \mu\, M_f(\Omega, Q)\, d\theta \quad (30)$$

Moment M_f is determined by geometric condition of bearing, rotating speed and applied load but viscosity of lubricant and coefficient of friction are not included. So the moment relation without coefficient of friction is

$$M_f = \sum_{k=1,N} \left[\sum_{j=1,Z} \left[\sum_{w=i,o} \left[M_{Rw} \left(\frac{\omega_{Rw}}{\Omega} \right) + M_{sw} \left(\frac{\omega_{sw}}{\Omega} \right) \right] \right]_j \right]_k \tag{31}$$

Coefficient of friction changes with the ratio of sliding speed to rolling speed and applied pressure. If we assume that the average frictional moment is constant during velocity change, the relation between kinetic energy, frictional moment, coefficient of friction and spindle speed can be simplified as follow.

$$E_1 = \mu \overline{M}_f \theta \tag{32}$$

In this equation, moment and kinetic energy terms are determined according to the running condition. So if the rotation count is known while the speed is changing, coefficient of friction can be obtained by this relation.

3. Simulation Result

For the analysis, 7911 angular-contact ball bearing is used. Bearing properties are shown below.

Table 1 Specifications of bearing used for analysis

Description	7911
Outer diameter of bearing	0.080 [m]
Inner diameter of bearing	0.055 [m]
Radius of outer raceway groove curvature	0.042 [m]
Radius of inner raceway groove curvature	0.042 [m]
Ball Diameter	0.00794 [m]
Number of Ball	23
Unloaded Contact Angle	15°

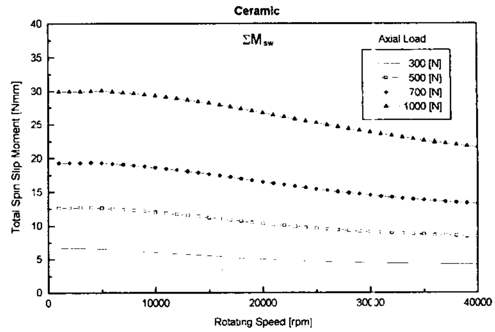

Fig. 4(a) Sum of inner and outer raceway components of spin moment vs. rotating speed for steel ball

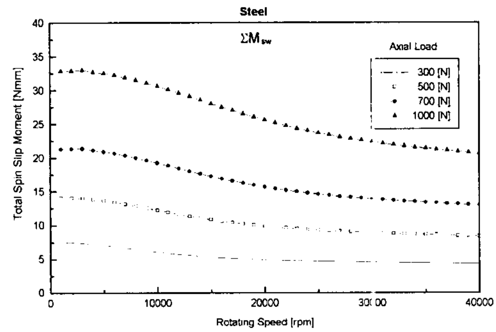

Fig. 4(b) Sum of inner and outer raceway components of spin moment vs. rotating speed for ceramic ball

The smaller applied load, the lower reduction rate of spin moment. Also, the amount of spin moment for ceramic ball bearing is smaller than that of steel ball bearing.

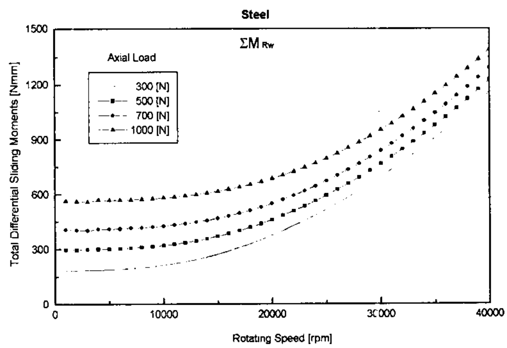

Fig. 5(a) Sum of inner and outer raceway components of differential sliding moment vs. rotating speed for steel ball

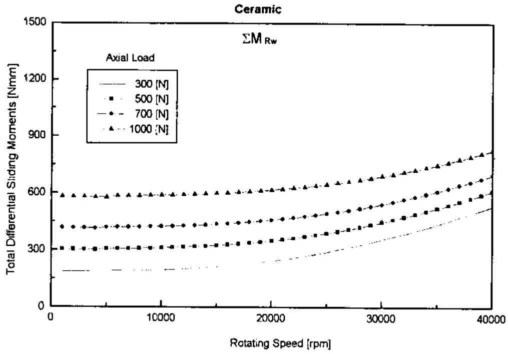

Fig. 5(b) Sum of inner and outer raceway components of differential sliding moment vs. rotating speed for ceramic ball

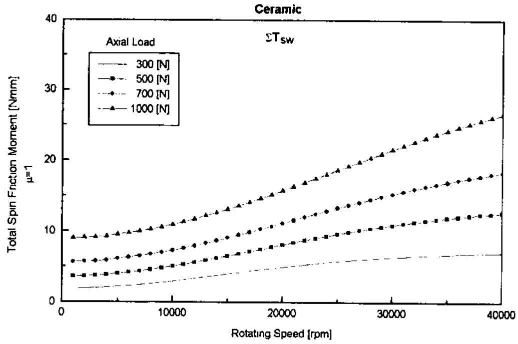

Fig. 6(b) Sum of inner and outer raceway components of spin friction moment vs. rotating speed for ceramic ball

With speed increase, differential sliding moment of steel ball bearing increases more rapidly than that of ceramic ball bearing. This is because that differential sliding moment is affected by increase of internal load such as centrifugal force and gyroscopic moment by increase of bearing speed.

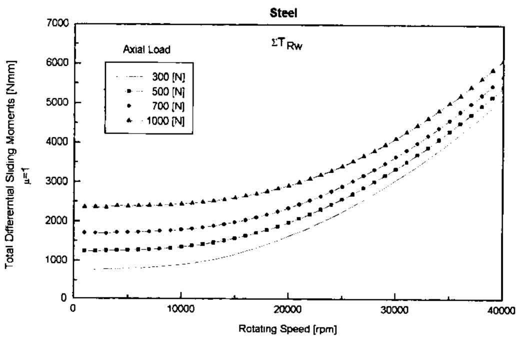

Fig. 7(a) Sum of inner and outer raceway components of differential friction moment vs. rotating speed for steel ball

Table 2 Properties of bearing ball materials

Property	Steel	NBD-200 (SI_3N_4)
Density of ball	7820 [Kg/m^3]	3160 [Kg/m^3]
Elastic modulus	200 [Gpa]	320 [Gpa]
Poisson's ratio	0.26	0.28

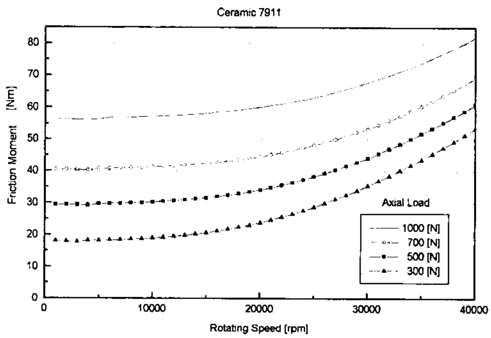

Fig. 7(b) Sum of inner and outer raceway components of differential friction moment vs. rotating speed for ceramic ball

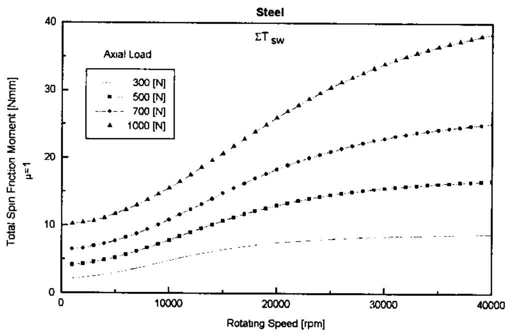

Fig. 6(a) Sum of inner and outer raceway components of spin friction moment vs. rotating speed for steel ball

Above result shows moment increase rate of steel ball bearing is larger than that of ceramic ball bearing.

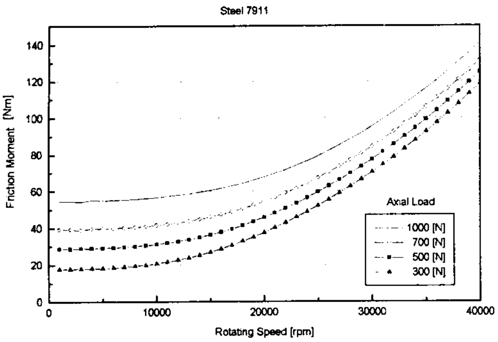

Fig. 8(a) Sum of inner and outer raceway components of all friction moments not including friction coefficient vs. rotating speed for steel ball

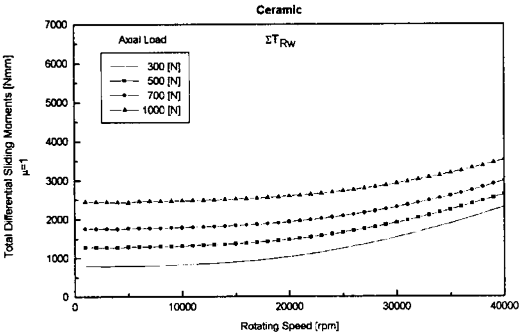

Fig. 8(b) Sum of inner and outer raceway components of all friction moments not including friction coefficient vs. rotating speed for ceramic ball

In Fig. 8, coefficient of friction is set as 1 so as to see the tendency of total frictional moment of the system.

4. Conclusions

Frictional moment is derived for coordinate of spindle rotation. Characteristic of friction is analyzed with spindle speed and preload. Major component of spindle bearing friction is differential sliding moment by rotational characteristics of bearing. Another component is spin sliding moment by ball and raceway's pitch angle.

With preload increase, spin friction moment increases in high-speed region. Amount of preload has an effect on frictional moment of bearings. Excessive preload causes increase of frictional moment.

Differential sliding moment of outer raceway is the first factor influencing on frictional moment. Second factor is spin sliding moment of inner raceway. Coefficient of friction has tendency to have linearity with increase of spindle speed in certain region.

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