Parameter Calibration of the Nonlinear Muskingum Model using Harmony Search

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ABSTRACT: A newly developed heuristic algorithm, Harmony Search, is applied to the parameter calibration problem of the nonlinear Muskingum model. The Harmony Search could, mimicking the improvisation of music players, find better parameter values for in the nonlinear Muskingum model than five other methods including another heuristic method, genetic algorithm, in the aspect of SSQ (the sum of the square of the deviations between the observed and routed outflows) as well as in the aspects of SAD (the sum of the absolute value of the deviations), DPO (deviations of peak of routed and actual flows) and DPOT (deviations of peak time of routed and actual outflow). Harmony Search also has the advantage that it does not require the process of assuming the initial values of design parameters. The sensitivity analysis of Harmony Memory Considering Rate showed that relatively large values of Harmony Memory Considering Rate makes the Harmony Search converge to a better solution.

(KEY TERMS: Harmony Search; Nonlinear Muskingum Method; Parameter Calibration; Genetic Algorithm.)

1 INTRODUCTION

There are two types of basic approaches to route floods: the hydrologic routing approach and the hydraulic routing approach. The former routes using the storage-continuity equation and the latter routes using the Saint-Venant equation (Fread, 1976).

One of the hydrologic routing techniques, the Muskingum method (McCarthy, 1938) has been frequently used to route floods in natural channels and rivers. In the Muskingum model, the following continuity and storage equations are used:

$$\frac{dS_t}{dt} = I_t - O_t \tag{1}$$

$$S_t = K[xI_t + (1-x)O_t] \tag{2}$$

in which S_t = the channel storage at time t; I_t and O_t = the rates of inflow and outflow at time t, respectively; K = the storage-time constant for the river reach, which has a value close to the flow travel time through the river reach; and x = a weighting factor varying between 0 and 0.3 for stream channels.

Commonly, the parameters K and x in the Muskingum model are graphically estimated by a trial and error procedure. After x is assumed, the values of $[xI_t + (1 - x)O_t]$ are computed using recorded data and plotted against S_t. The value of x which makes the width of the plotted loop minimum can be chosen as the correct value of x, and the line slope for the correct value of x can be chosen as K. However, the above procedure is subjective and inefficient, and the relationship between $[xI_t + (1 - x)O_t]$ and S_t is not always and basically linear. Gill (1978) proposed a nonlinear Muskingum model as

$$S_t = K[xI_t + (1-x)O_t]^m \tag{3}$$

This nonlinear Muskingum model has an additional parameter m used as an exponent, which presumably makes the model fit closer to the nonlinear relation between accumulated storage and weighted flow. However, the calibration procedure for finding the correct values of three parameters K, x, m becomes more complicated. These values cannot be determined graphically from historical inflow and outflow hydrographs. Alternative parameter estimation methods are required. Several endeavors have been executed by Gill (1978), Tung (1985), Yoon and Padmanabhan (1993), and Mohan (1997). Gill (1978) suggested a technique for finding the values of three parameters in the nonlinear Muskingum model using the least squares method. However, his technique is arbitrary in the process of selecting three points for solving simultaneous nonlinear equations (Tung, 1985). Tung (1985) proposed three techniques for parameter calibration using various curve fitting techniques. He employed the Hooke-Jeeves (HJ) pattern search (Hooke and Jeeves, 1961), whose philosophy is that any set of moves that have been improving objective function values in early trials will deserve repeating, in conjunction with simple linear regression (LR), the conjugate gradient (CG), and the Davidon-Fletcher-Powell (DFP) techniques. He compared these three techniques with Gill's and concluded that the techniques (HJ + CG) and (HJ + DFP) yield better results. Yoon and Padmanabhan (1993) proposed one technique for linearity determination of hydrologic data. With this determination, the model parameters are calibrated using six different techniques: three for a linear model; two for a piece-wise linear model; and one for the nonlinear model (NONLR) in the Muskingum routing equations. Mohan (1997) pointed out that all of the above techniques require careful choosing of initial values and if initial values are far from the global optimum, they are apt to fall into local optima. He also suggested a calibration technique for the nonlinear Muskingum model using a genetic algorithm (GA), one of heuristic search methods. The results showed that the calibration technique using a genetic algorithm is better than the above techniques and does not require the process of assuming initial values close to the optimum.

At present, another heuristic algorithm named Harmony Search has been developed and applied to various problems including the travelling salesman problem, a specific continuous function, the layout of a pipe network, and the design of a pipe network (Geem et al., submitted). Harmony Search algorithm has outperformed other heuristic and mathematical algorithms such as simulated annealing, genetic algorithm, evolutionary programming and the generalized reduced gradient (GRG) method. Consequently, the Harmony Search algorithm gives a possibility of success in the parameter calibration for the nonlinear Muskingum model.

2 HARMONY SEARCH

Following the idea that existing heuristic algorithms are found in the paradigm of natural processes such as th metallic annealing process in simulated annealing (Kirkpatrick, 1983) and the evolutionary process of Darwin' natural selection theory in genetic algorithm (Holland, 1975) and in evolutionary programming (Fogel et al. 1966), new heuristic algorithm can be found from a natural process in a musical performance (for example, a jazz trio involving searching for better harmony. Musical performances seek a best state (fantastic harmony) determined b aesthetic estimation, as the optimization algorithms seek a best state (global optimum: minimum cost or maximur benefit or maximum efficiency) determined by objective function evaluation. Aesthetic estimation is done by the se of the sounds played by joined instruments, as objective function evaluation is done by the set of the values produce by composed variables; the sounds for better aesthetic estimation can be improved practice after practice, as th values for better objective function evaluation can be improved iteration by iteration. The new algorithm is name Harmony Search. The steps in the procedure of Harmony Search are as follows:

Step 1. Initialize the harmony memory (HM).
Step 2. Improvise a new harmony from HM.
Step 3. If the new harmony is better than the minimum harmony in HM,
include the new harmony in HM
and exclude the minimum harmony from HM.
Step 4. If stopping criteria are not satisfied, go to Step 2.

The structure of Harmony Memory is shown in Fig. 1.

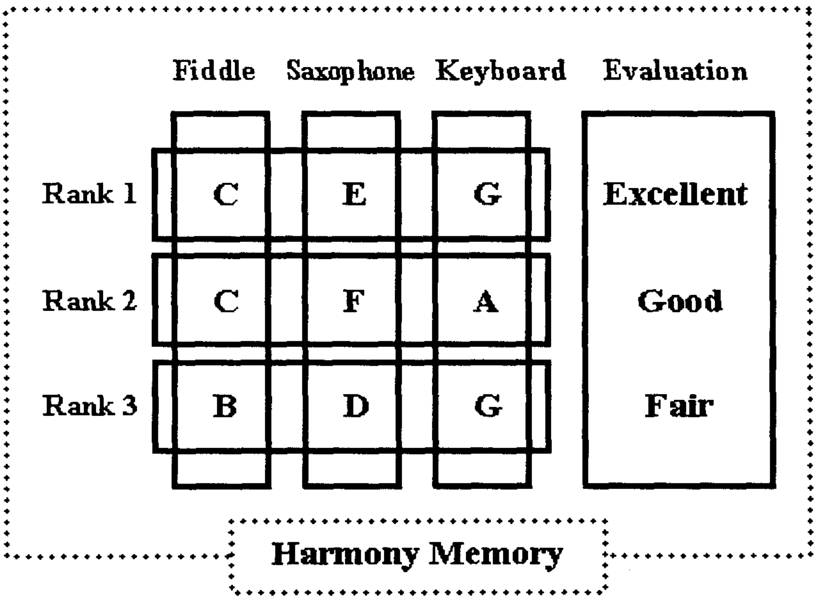

Fig. 1. Structure of Harmony Memory (HM).

Consider a jazz trio composed of fiddle, saxophone and keyboard. Initially, the memory is stuffed with random harmonies: (C, E, G), (C, F, A), and (B, D, G), which are sorted by aesthetic estimation. In the improvising procedure, three instruments produce new harmony, for example, (C, D, A): fiddle sounds {C} out of {C, C, B}; saxophone sounds {D} out of {E, F, D}; and keyboard sounds {A} out of {G, A, G}. If the newly made harmony (C, D, A) is better than any of the existing harmonies in the harmony memory (HM), the new harmony is included in HM and the worst harmony (in this example (B, D, G)) is excluded from the HM. This process is repeated until the best (or near best) harmony is obtained.

3 ROUTING PROCEDURE AND HARMONY SEARCH

By rearranging Equation (3), the rate of outflow O_t can be expressed as

$$O_t = \left(\frac{1}{1-x}\right)\left(\frac{S_t}{K}\right)^{1/m} - \left(\frac{x}{1-x}\right)I_t \tag{4}$$

Combining Equation (4) and the continuity equation, Equation (1), the state equation can be obtained as

$$\frac{dS_t}{dt} = -\left(\frac{1}{1-x}\right)\left(\frac{S_t}{K}\right)^{1/m} + \left(\frac{1}{1-x}\right)I_t \tag{5}$$

The parameter calibration procedure using Harmony Search involves the following seven steps:

Step 1. – Generate three parameters, K, x, and m, in the nonlinear Muskingum model from Harmony Memory.
Step 2. – Calculate initial storage using Equation (3), where initial outflow is the same as initial inflow.
Step 3. – Calculate the time rate of change of storage volume in the channel reach using Equation (5).
Step 4. – Estimate the next accumulated storage as

$$S_{t+1} = S_t + dS_t \tag{6}$$

Step 5. – Calculate the next outflow using Equation (4).
Step 6. – Compare the observed outflow and computed outflow, and if the difference is smaller than existing value, Include the parameters K, x, and m in the Harmony Memory and exclude worst parameters from Harmony Memory.
Step 7. - Go to Step 1 until the stopping criteria are satisfied

4 APPLICATIONS

The calibration technique for the nonlinear Muskingum model using Harmony Search is applied to an example from Wilson (1974), which has been tackled by Gill (1978), Tung (1984), Yoon and Padmanabhan (1993) and Mohan (1997). This example is demonstrated to show a nonlinear relationship between weighted-flow and storage volume

(Mohan, 1997). The objective function for assessment of optimal K, x, and m values in the nonlinear Muskingun model is to minimize the residual sum of squares (SSQ) between actual and routed outflows:

$$min\ SSQ = \sum_{t=1}^{n} \left(O_t - \hat{O}_t\right)^2 \tag{7}$$

where $\hat{O}_t$ is routed outflow at time t.

The routing procedure is that mentioned above using the algorithm parameters (the size of Harmony Memory (the number of ranks) = 100; Harmony Memory Consulting Rate = 0.95; and the maximum iterations = 5,000) and the ranges of three parameters in the nonlinear Muskingum model (K = 0.01 – 0.20; x = 0.2 – 0.3; and m = 1.5 – 2.5). After the computation, Harmony Search appeared to have found a solution that is better than those found by other methods, as shown in Table 1 and 2.

Table 1. Comparison of the Routing Results in Various Methods

Time	Inflow	Observed Outflow	Computed Outflow						
			Gill	Tung[a]	Tung[b]	NONLR	GA	HS[a]	HS[b]
0	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0	22.0
6	23.0	21.0	22.0	22.0	22.0	22.6	22.0	22.0	22.0
12	35.0	21.0	22.8	22.4	22.4	23.0	22.4	22.4	22.4
18	71.0	26.0	29.6	26.8	26.7	24.2	26.3	26.6	26.5
24	103.0	34.0	39.1	34.9	34.8	33.2	34.2	34.4	34.5
30	111.0	44.0	47.6	44.5	44.7	47.1	44.2	44.1	44.3
36	109.0	55.0	58.0	56.7	56.9	56.8	56.9	56.8	56.9
42	100.0	66.0	67.1	67.3	67.7	66.2	68.2	68.1	68.0
48	86.0	75.0	74.8	75.9	76.3	75.0	77.1	77.1	76.8
54	71.0	82.0	80.4	81.9	82.2	80.7	83.2	83.3	83.0
60	59.0	85.0	83.2	84.5	84.7	83.5	85.7	85.9	85.5
66	47.0	84.0	82.8	83.4	83.5	84.3	84.2	84.5	84.2
72	39.0	80.0	80.1	79.9	79.8	79.9	80.2	80.6	80.3
78	32.0	73.0	74.5	73.6	73.3	74.3	73.3	73.7	73.5
84	28.0	64.0	67.2	65.8	65.5	65.3	65.0	65.4	65.4
90	24.0	54.0	58.1	56.9	56.5	55.9	55.8	56.0	56.1
96	22.0	44.0	48.1	47.8	47.5	45.1	46.7	46.7	46.9
102	21.0	36.0	37.6	38.9	38.7	35.4	38.0	37.8	38.1
108	20.0	30.0	28.2	31.5	31.4	28.7	30.9	30.5	30.9
114	19.0	25.0	21.9	25.8	25.9	24.3	25.7	25.3	25.6
120	19.0	22.0	19.1	22.0	22.1	20.9	22.1	21.8	22.0
126	18.0	19.0	19.0	20.1	20.2	20.4	20.2	20.0	20.1

Table 2. Comparison of the Solutions in Various Methods.

Method	K	x	m	SSQ	SAD	DPO	DPOT
Gill	0.0100	0.2500	2.3470	143.60	46.4	1.80	0
Tung[a]	0.0669	0.2685	1.9291	49.64	25.2	0.50	0
Tung[b]	0.0764	0.2677	1.8978	45.54	24.8	0.30	0
NONLR	0.0600	0.2700	2.3600	43.26	25.2	0.70	1
GA	0.1033	0.2813	1.8282	38.23	23.0	0.70	0
HS[a]	0.0883	0.2873	1.8630	36.78	23.41	0.92	0
HS[b]	0.0883	0.2803	1.8640	37.96	22.96	0.50	0

Table 2 shows the calibrated parameters and the performances of four measurements (SSQ, SAD, DPO, and DPOT) from comparison between observed and computed outflows (Table 1) in various methods studied by Gill, Tung[a] (HJ+CG), Tung[b] (HJ+DFP), Yoon and Padmanabhan (NONLR), Mohan (GA) and Harmony Search. It shows that among the results, the SSQ in Harmony Search (HS[a]) is the smallest (SSQ = 36.78). The values of the other measurements are somewhat larger considering other measurements: the sum of the absolute value of the deviations between the computed and observed outflows (SAD); deviations of peak of routed and observed outflows (DPO); and deviations of peak time of routed and observed outflows (DPOT). For satisfying all four measurements, another values of parameters (HS[b]) can be chosen from Harmony Memory. These values guarantee smaller SAD, DPO, DPOT as well as smaller SSQ (37.96), except that DPO (0.5) of HS[b] is larger than that (0.3) of Tung[b] (HJ+DFP). Fig. 2 shows the computed outflow hydrographs in a genetic algorithm and in Harmony Search and also the observed inflow and outflow hydrographs.

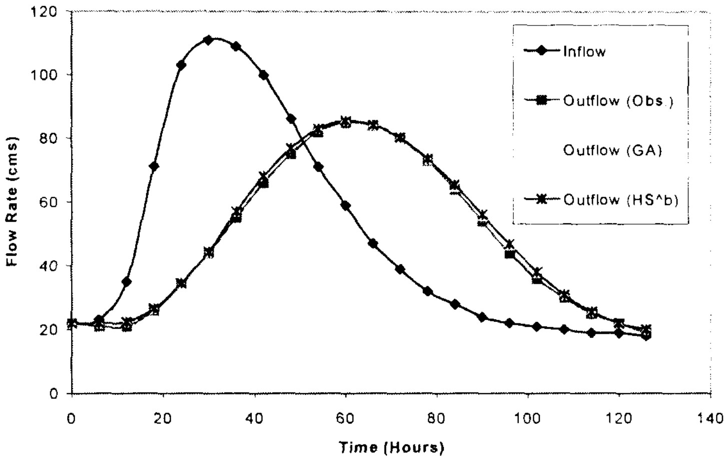

Fig. 2. Inflow and Outflow Hydrographs.

For the sake of avoiding local optima, Harmony Search initiates Harmony Memory Considering Rate that ranges from 0 to 1. If a uniformly generated value between 0 to 1 occurs outside of this parameter, Harmony Search finds values (of parameters, K, x, and m) randomly within the possible range without considering Harmony Memory. The results of a sensitivity analysis of Harmony Memory Considering Rate shows that relatively large value of HM considering rate (near 0.95) causes the lower solution (Fig. 3).

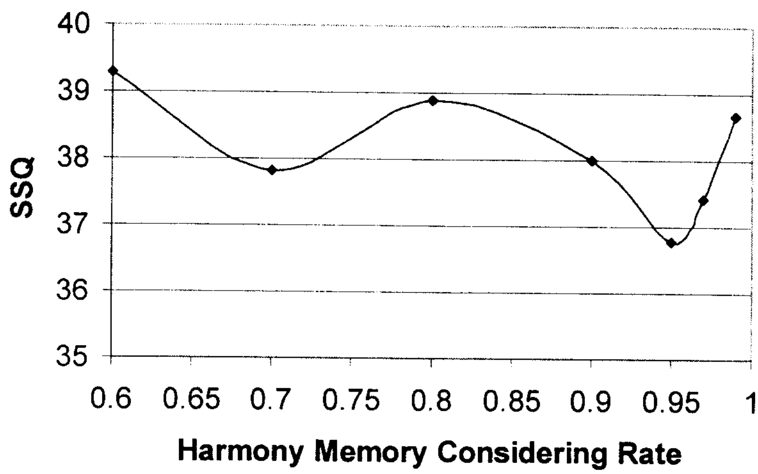

Fig. 3. Results of Sensitivity Analysis of Harmony Memory Considering Rate.

5 CONCLUSIONS

The relationship between weighted-flow and storage volume is not always linear in the Muskingum model. Sometimes nonlinear Muskingum model is more suitable for routing in a certain area. However, the calibration procedure for finding the correct values of the parameters in nonlinear Muskingum model is more complicated than in the linear Muskingum model. Up to now, various efforts have been made by several researchers for finding the values of three parameters in the nonlinear Muskingum model.

The newly developed heuristic algorithm, Harmony Search is also applied to the parameter calibration problem of the nonlinear Muskingum model. Harmony Search produced better results than other methods proposed by several researchers. not only in the aspect of the objective function, SSQ (the sum of the square of the deviations between the observed and routed outflows) but also in the aspects of SAD (the sum of the absolute value of the deviations), DPO (deviations of peak of routed and actual flows) and DPOT (deviations of peak time of routed and actual outflow).

Some techniques require careful choosing of initial values, and if initial values are far from the global optimum, they are apt to fall into local optima. However, Harmony Search does not require the process of assuming the initial values of design parameters (K, x, and m) as in a genetic algorithm model.

The sensitivity analysis of Harmony Memory Considering Rate has been performed. The results of the sensitivity analysis showed that relatively large value of Harmony Memory Considering Rate (near 0.95) makes the Harmony Search converge to a better solution.

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