OPTIMAL DESIGN FOR CAPACITY EXPANSION OF EXISTING WATER SUPPLY SYSTEM

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Abstract: This paper presents a two-phase search scheme for optimal pipe expansion of existing water distribution systems. In pipe network problems, link flows affect the total cost of the system because the link flows are not uniquely determined for various pipe diameters. The two-phase search scheme based on stochastic optimization scheme is suggested to determine the optimal link flows which make the optimal design of existing pipe network. A sample pipe network is employed to test the proposed method. Once the best tree network is obtained, the link flows are perturbed to find a near global optimum over the whole feasible region. It should be noted that in the perturbation stage the loop flows obtained from the sample existing network are employed as the initial loop flows of the proposed method. It has been also found that the relationship of cost-hydraulic gradient for pipe expansion of existing network affects the total cost of the sample network. The results show that the proposed method can yield a lower cost design than the conventional design method and that the proposed method can be efficiently used to design the pipe expansion of existing water distribution systems.

Key Words: two-phase search scheme, cost-hydraulic gradient curve, convex and nonconvex function, pipe network optimization, adjacent property

1. INTRODUCTION

The classical steady state approach to designing water distribution systems involves the following subproblems: (i) planning, (ii) design, and (iii) analysis. The planning aspect involves the determination of the water requirements of the system. The design problem involves the selection of an optimal topological layout as well as the sizes of the

distribution system components. The analysis step involves the evaluation of the flows and pressures in the system, based on the given layout and sizes of the system components.

Pipe flows affect the total cost of the pipe network. The global optimum can be obtained through the linear programming in tree pipe network with a single source, because the supply rate and pipe flows are uniquely determined. However, since supply rates or pipe flows are not uniquely determined in tree pipe network with multiple sources or in looped pipe networks, it is not easy to get the global optimum for the networks.

Alperovits and Shamir (1977) presented a two level algorithm that was called the linear programming gradient (LPG) method. The LPG method begins with known flow rates for the model and the optimal dual variables obtained from the linear programming are then used to determine the steepest descent direction with regard to a perturbation vector of flows. If the cost of the system decreases, a new LP problem is set up with the updated flow rates and the iterative procedure is repeated until no improvement in cost can be obtained. Quindry et al. (1981) presented the algorithms that would yield an optimal pipe flow distribution. Kessler and Shamir (1989) introduced the projection of the gradient of objective function onto constraints surface to improve the search procedure and obtained a local optimal solution.

Fujiwara and Khang (1990) and Kessler and Shamir (1991) proposed a two-phase procedure. These methods attempt to find a near global optimum. For the chosen flows based continuity. cost minimization on problem in the first phase has a nonlinear convex objective in terms of head loss with a linear constraint region. Then in the spirit of the LPG procedure, an improving direction is generated with the aid of the multipliers Lagrange of the Phase constraints called NLPG direction. The conventional optimization methods are relatively complicated, converge slowly, and find only a local optimum. Loganathan et al. (1995) used two global-search schemes. Multistart and Annealing, to permit a local-optimum-seeking method to migrate among various local minima. Ahn et al. (1995) and Ahn and Loganathan (1998) proposed stochastic optimization to find a near global optimum for two or three looped pipe networks.

In this study a two-phase search scheme is suggested to find a near global optimum over the whole feasible region and existing pipe network is considered to test the proposed approach. The optimal link flows and the optimal pipe diameters for capacity expansion are determined in the sample existing network. It is observed that either convex or nonconvex relationship of cost-hydraulic gradient for links yields either adjacent pipe diameters or nonadjacent pipe diameters respectively, and that the property of cost-hydraulic gradient for pipe expansion existing network affects. the total expansion cost of the sample network.

2. MODEL FORMULATION

The following mathematical programming formulation model 1 (M1) is adopted for looped pipe networks.

(M1): Minimize
$$\sum_{(i,j)} \sum_{m=1}^{M} C_{(i,j)m} x_{(i,j)m}$$
 (1)

Subject to

$$\sum_{j} Q_{(i,j)} - \sum_{j} Q_{(j,j)} = q, \text{ for } i \in \{N - S\}$$
 (2)

$$H_{s} - H_{s}^{m} - \sum_{(i,j) \in [r,k]} [+ \sum_{m} (J_{1(i,j)m} x_{(i,j)m}] \ge 0$$
for $s \in S$, and $k \in [N-S]$

$$\sum_{(i,j)\in b} \equiv \sum_{m} J_{(i,j)} x_{(i,j)m} = b_{p} \quad \text{for } p \in P \quad (4)$$

$$\sum_{m} x_{(i,j)m} - L_{(i,j)} = 0 \quad \text{for } (i,j) \in B$$

$$x_{(i,j)m} \ge 0$$
(5)

where $C_{(i,j)m}$ is unit cost for the m th diameter segment in a link (i, j); $x_{+i,j,m}$ is length of the m th diameter segment in a link (i, j), H_i is the fixed head; B is the set of links: L(i, i) is the length of link $(i, j) \in B$; N is the set of nodes; S is the set of fixed head nodes; $\{N-S\}$ is the set of junction nodes; $Q_{(r,t)}$ is flow rate through $(i, j) \in B$; q, is demand at node i: r(k) is a path through the network connecting a source node and demand node $k \in \{N-S\}$; H_k^{min} is minimum pressure head required at node k, b is path connecting fixed head nodes; P is set of paths connecting source nodes; b_b is zero in the loop and is head difference between the source heads.

The hydraulic gradient due to the frictional head loss is calculated from the Hazen-Williams equation with the SI system of unit: $J_{1(i,j)} = k_{1(i,j)} Q_{(i,j)}^{1.852} D_{(i,j)}^{-4.87}$, for each pipe in which $k_{1(i,j)} = 10.7/C^{1.852}$, Q is the Hazen-Williams coefficient, $Q_{(i,j)}$ is the given flow in link (i,j), and $D_{(i,j)}$ is the diameter of m th segment in link (i,j).

The objective function of Model 1 is the summation of the pipe network cost. Constraint (2) represents the flow continuity equations, constraint (3) represents the hydraulic head requirement at each node, constraint (4) represents the sum of head losses in a path, and constraint (5) represents the length constraints.

3. A TWO-PHASE SEARCH SCHEME

Since flow rate of each link affects the total cost of the system, the following equation is given: $Cost = f(Q_{(i,j)}), (i,j) \in B$ in which Cost is the total cost of the

system, $Q_{(i,j)}$ are the link flows, and B is the set of links. Let the feasible region be $X = \{ Q_{(t,t)} \mid g(Q_{(t,t)}) \ge 0 \}.$ A solution $Q_{(t,t)}^{-1}$ is said to be a local optimum if there exists a neighborhood E around $Q_{(i,j)}^{\perp}$ such that all $f(Q_{(i,j)}^1) \le f(Q_{(i,j)})$ for $Q_{(i,j)} \in E(Q_{(i,j)}^{1})$. A solution $Q_{(i,j)}^{*}$ is said be a global optimum $f(Q_{(i,j)}^*) \le f(Q_{(i,j)})$ for all $x \in X$. Since model 1 is a nonlinear, nonconvex programming problem with several minima, assuring the global optimal solution is an extremely difficult task because it requires that there should be no better point than $Q_{(i,j)}^*$ in every neighborhood of $Q_{(i,j)}^*$.

The following Model 2 is suggested for pipe networks to determine the optimal flow rates and diameters of pipes.

in which $Q_{(i,j)}$ are the perturbed flows of an underlying near optimal spanning tree of the looped layout satisfying constraints(2), N is the feasible region made up of constraints (3) \sim (5), and f(x) is the objective function of the Model 1 for link flows.

It is observed that the inner Model 3 of Model 2 is given by

Model 3:
$$\min_{x \in X} f(x)$$
 for fixed flows (7)

The model 3 is a nonlinear, discrete, nonconvex programming problem, which may have several local minima. For known flows the model 3 is a linear program which can be efficiently solved by using commercially available codes. The model 3 is equivalent to the model 1 if the link flows are known.

The two-phase search scheme consists of

global search phase and local search phase. The stochastic probing algorithm in the spirit of Laud et al. (1992) has been used in the global search phase. The stochastic probing algorithm is to find the location of a near global loop flow vector q^* . The method begins with the construction of a probing probability distribution, with the density function $P \sim N(q, \sigma)$, where q is location parameter and σ is scaling parameter. The costs of f(q) are evaluated at a few loop flows q sampled from the density function. The updating location of loop flow q and scale σ are based on Gibbs-like distribution and the entropy of the current distribution, respectively. In the local search phase, the Multistart algorithm used by Loganathan et al. (1995) has been adjusted to efficiently improve a local optimum obtained through the global search phase.

Because flows in the looped network are generated by perturbing the optimal loop flows, a probability density function for loop change flows with mean zero is a good initial choice for two loop pipe network (Ahn et al, 1995). However, other initial parameters of the density function must be adjusted by experimentation for the problem under consideration.

Global Search Phase

Step 0. (Initialization) Select an initial location of loop flow vector q^o and a scale vector σ^o of a probing distribution.

Step I. (Generation step) At stage $n(n \ge 0)$, generate k independent, identically distributed loop flows parameters q_{n1}, \dots, q_{nk} from $p(\cdot \mid q^o, \sigma^o)$. Let $q_{no} = q_n$ and $q^n = (q_{no}, \dots, q_{nk})$

Step 2. (Update location) The update

location q_{n+1} is chosen from a point that is the highest probability in the following Gibbs-like distribution:

$$P_r(q_{n+1} = q_{ni}) = \frac{1}{Z} e^{-B(\emptyset, i, q^n)}, \quad i = 0, 1, ..., k$$

where, $Z = \sum_{i=0}^{k} e^{-B(\emptyset, i, q^n)}$ and

$$B(\emptyset, i, q^n) = \frac{LP(q_m) - LP(q_m)}{\min(LP(q_m), LP(q_m)) - LP_{\min}}$$
 in which $LP(\cdot)$ is the value of objective function of the linear program given a set of loop flows.

Step 3. (Update scale) The rule for scale reduction is of the form

i) $q_{n+1} = q_n$ and $\sigma_{n+1} = \sigma_n$ if $f(q_{n+1}) \ge f(q_n)$ Go to step 1.

ii)
$$\sigma_{n+1} = w_n \sigma_n$$
 if $f(q_{n+1}) \langle f(q_n), \text{ where}$

$$w_n = \frac{Ent(n)}{\log(k+1)}$$

The scale reduction factor w_n is based on the entropy of the current distribution:

$$Ent(n) = -\sum_{i=0}^{k} P_r(q_{n+1} = q_m) \log P_r(q_{n+1} = q_m)$$
where $P_r(\cdot)$ is given in the current distribution.

Step 4. (Stopping rule)

- i) $\sigma_{n+1} \langle \sigma_n$, increase n by one and go to step 1.
- ii) $\sigma_{n+1} = \sigma_n$ and no improvement has been obtained in the last few iterations, save q_n^* and $f(q_n^*)$ and stop; otherwise go to step 1.

Local Search Phase

Step 0. (Initialization) Select the probability density function (pdf) with the known parameters from the best solution, $f(q^*_n)$, given by the *Global Search Phase* and assign the number of seed points to be generated, NMAX; determine an estimate of near global optimal value f_g ; set n=1.

Step 1. (Iterative Local Minimization) Until

n=NMAX do: Generate a seed point using the pdf. Obtain the set of loop flows by perturbing the tree link flows. Apply model 3 from the perturbed point. Update the best solution given by the *Global Search Phase*, Best.

Step 2. (Termination) If $|f_g - Best| <$ tolerance, Report Best and the optimal solution and stop. Otherwise, set n=1; go to step 1.

4. PIPE SELECTION FOR EXPANSION OF EXISTING NETWORKS

Pipe diameter selected from the linear program is an important aspect in the expansion of existing networks. Fujiwara and Dey (1987) pointed out that either one pipe diameter or two adjacent pipe diameters should be selected at each link if the cost versus hydraulic gradient curve for a link is a discrete, strictly convex function and that nonadjacent pipe diameters could be selected if the cost versus gradient curve for a link is a discrete, strictly concave function.

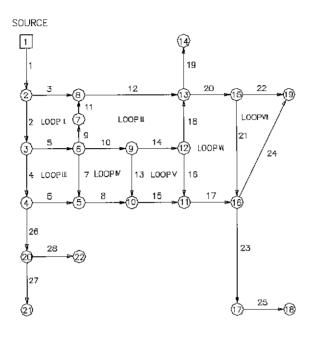
To detect links to be required to have parallel links in the sample network, the model I is solved with the restriction that existing pipes should not be undersized. Thus in the objective function of the model 1, the candidate diameters smaller than the existing diameter in a link are treated as the existing diameter. Putting the cost of the candidate diameter in the existing link affects the configuration of the cost - hydraulic gradient relationship. Two kinds of cost - hydraulic gradient function may be considered in the pipe expansion problem: convex and nonconvex function. the If costs and diameters of candidate diameters smaller than the existing diameter in a link are considered to be the cost and the diameter of the

existing diameter, the cost - gradient function becomes a convex function. However, if the candidate diameters are only considered to be the existing diameter, the cost - gradient curve becomes a nonconvex function.

5. ANALYSIS OF EXAMPLE NETWORK

The sample network, the Songtan water distribution system, as shown in Fig. 1 is the expanded water distribution network Pyongtack (Pyongtack city, 1998), Korea, which is solved using the proposed procedure. The network has one source, twenty one junction nodes, twenty eight links, and seven loops. The link lengths, the design heads, the demands, and the existing pipe diameters are given in Table 1. The Hazen-Wiliiams friction coefficient is 100 for all links and exponents for discharge and diameter are 1.85 and -4.87 respectively. Iron pipes are adopted in the study area. To obtain the annual cost of the system, the initial costs of pipe component are converted into annual capital recovery cost introducing the annual capital recovery factor. The annual capital recovery factor is given by: $R = i(1+i)^n/[(1+i)^n-1] = 0.1(1+0.1)^{30}$ /[$(1+0.1)^{30}-1$]=0.106, in which i is an interest rate per year, and n is the life span of a system. The annual payment equivalent to a present sum of each component of a system for life span at an interest rate i is computed: $c = C \cdot R$ where c is the annual capital recovery cost of a component, and C is the capital cost of the component. Table 2 shows commercially available pipe sizes, their capital cost, and annual costs.

The decision variables of the Songtan optimization model based on Model 1 are the unknown segment lengths of known twelve different candidate diameters while



Legend

demand node

link

Fig. 1. Songtan Water Distribution System

requirements. satisfying the design The algorithm TREESEARCH (Loganathan et al., 1990) is first applied to obtain the optimal tree layout and the link flows. The global tree network is obtained by deleting links 3, 6, 8, 12, 15, 20, and 21. The procedure for two-phase search scheme is then implemented to search the feasible region beginning with the perturbed optimal tree link flows as initial flows. The Model 3 is solved by the linear programming subroutine DLPRS from the International Mathematical and Statistical Libraries (IMSL).

A probing distribution for loop change flows with mean zero was a good initial choice for two loop pipe network(Ahn et al., 1995). However, the stochastic probing for the sample network yields a number of infeasible solutions with respect to mean zero of loop flows, which may be due to the expansion of existing network having seven loops. Thus, the initial loop flows are chosen from the results of a hydraulic simulator for the existing pipe network. It has been found that the stochastic probing method with the initial loop flows obtained from the existing pipe network migrates successfully to various local optima.

Fig. 2 and Fig. 3 show convex and nonconvex cost - hydraulic gradient curve for link 3, respectively. If the optimal hydraulic gradient for link 3 is 0.002918, the least cost should be produced by the combination of the diameters 0.6m and 0.7m in the convex

Table 1. Node and Link Data of Songtan Water Distribution Network

Node Demand		Minimum	Link	Langth	Existing Pipe	
number	(m^3/s)	Head, El. m	number	Length, m	Diameter, cm	
1	-0.57754	80.0			70.0	
2	0.01842	65 0	2	483.0	70.0	
3	0.02500	48.1	3	683.0	60.0	
4	0.00789	50.5	-1	475.0	15.0	
5	0.01191	39.9	5	620.0	60.0	
6	0.02391	55.0	6	535.0	30.0	
7	0.00254	55.0	7	520.0	15.0	
8	0.04159	56.5	8	510.0	31.8	
9	0.02657	48.6	9	286.0	10.0	
10	0.00887	38.0	10	624.0	60.5	
I 1	0.01302	39 5	11	226.0	10.0	
12	0.00639	39.0	12	1010.0	60.0	
13	0.07931	45.0	13	476.0	20.0	
14	0.07931	62.4	14	376.0	50.0	
15	0.05090	43,0	15	496.0	20.0	
16	0.06231	42.0	16	217.0	20.0	
17	0.00949	31.0	17	585.0	20.0	
18	0.08172	48.5	18	286.0	30.0	
19	0.01941	46.0	19	1060.0	30.0	
20	0.00017	51.0	20	700.0	45.0	
21	0.00323	77.0	21	487.0	60.0	
22	0.00575	68.3	22	691.0	40.0	
	i		23	300,0	0.0	
			24	834.0	20.0	
			25	800.0	0.0	
			26	1274.0	0.0	
			27	450,0	0.0	
	·		28	150.0	0.0	

Table 2. Diameter and Cost Data for Songtan Project

Diameter, cm	Annual cost Won/m/year	Diameter, cm	Annual cost Won/m/year	Diameter, cm	Annual cost Won/m/year
8.0	9116	25.0	14257	50.0	25222
10.0	9096	30.0	16069	60.0	28707
15.0	10720	35.0	17594	70.0	33800
20.0	12718	40.0	21075	80.0	39588

function. For the nonconvex function, the combination of the diameter 0.6m and 0.8m produces the least cost. If the relationship of the cost versus hydraulic gradient for the expanded links is said to be a nonconvex

function, the total cost in the Korean currency for capacity expansion of the Songtan water distribution system is 48,674,990 Won/year; whereas the total cost is 48,741,387 Won/year in case of a strictly

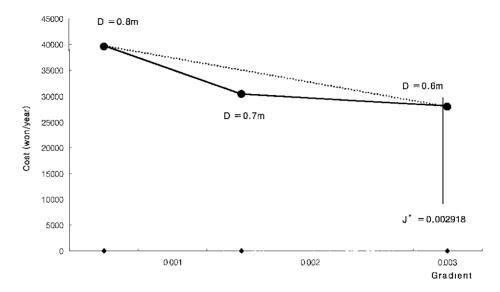


Fig. 2. Convex Cost-Hydraulic Gradient Curve

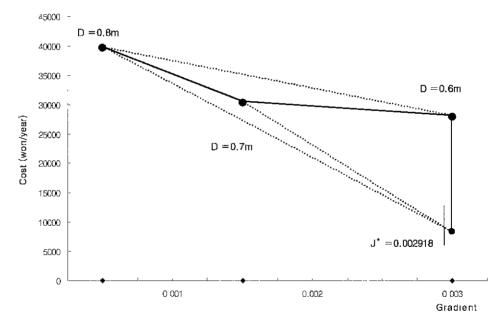


Fig. 3. Nonconvex Cost-Hydraulic Gradient Curve

convex. Table 3 shows the optimal solution obtained for either convex or nonconvex function. The optimal head losses for a link are same for both two functions. The two-phase search scheme has been applied to

search a near optimal loop flows over the whole feasible region. The stochastic probing method improves loop flows at each stage and the optimal loop flows are finally refined by the local search phase. The

Table 3. Optimal Solution to the Best Loop Flows for M1

nonconvex cost - gradient function				convex cost - gradient function			
link	diameter, cm	length, m	headloss, m	link	diameter, cm	length, m	headloss, m
1	70	260.0	1.138	1	70	260.0	1.138
	70	483.0	0.449	2	70	483.0	0.449
3	60	682.03	1.992	3	60	681.61	1.991
3	80	0.97	0.001	3	70	1.39	0.002
4	30	257.10	0.561	4	30	257.10	0.561
4	35	217.90	0.224	4	35	217.90	0.224
5	60	620.0	0.681	5	60	620.0	0.681
6	30	535.0	0.459	6	30	535.0	0.459
7	15	481.57	0.553	7	15	481.57	0.553
7	20	38.43	0.011	7	20	38.43	0.011
8	31.8	510.0	0.182	8	31.8	510.0	0.182
9	10	279.80	0.851	9	01	279.80	0.851
9	15	6.20	0.003	9	15	6.20	0.003
10	60.5	624.0	0.463	10	60.5	624,0	0.463
11	10	226.0	0.010	11	10	226 0	0.010
12	60	1010.0	2.263	12	60	1010.0	2.263
13	20	476.0	0.283	13	20	476.0	0.283
14	50	376.0	0.441	14	50	376.0	0.441
15	20	496.0	1.433	15	20	496.0	1.433
16	20	199.05	1.237	16	20	199.05	1.237
16	25	17.95	0 038	16	25	17.95	0.038
17	20	574.38	4.670	17	20	574.38	4.670
17	25	10.62	0.029	17	25	10.62	0.029
18	30	286.0	2.223	18	30	286.0	2.223
19	30	1060.0	7.303	19	30	1060.0	7.303
20	45	700.0	3.503	20	45	700.0	3.503
21	60	487.0	0.248	21	60	<u>4</u> 87.0	0.248
22	40	691.0	0.118	22	40	684.98	0.117
23	25	300.0	6.505	22	50	6.02	0.001
24	20	827.41	0.129	23	25	300.0	6.505
24	25	6.59	0.000	24	20	834.0	0.130
25	20	48.68	2.554	25	20	48.68	2.554
25	25	751.32	13.296	25	25	751.32	13.296
26	20	9.06	0.008	26	20	9.06	0.008
26	25	1264.94	0.376	26	25	1264.94	0.376
27	15	450.0	0.243	27	15	450,0	0.243
28	10	150.0	1.696	28	10	150.0	1.696

optimal loop flows in m^3/s (∇Q_1 , ∇Q_2 , ∇Q_3 , ∇Q_4 , ∇Q_5 , ∇Q_6 , ∇Q_7)=(0.309274, 0.267971, 0.25724, 0.018668, 0.017050, 0.194002, 0.022930) are obtained with the

procedure. In the local search phase, the direct search polytope algorithm is also adopted to improve the optimal solution. No improvement, however, is made by the direct

link number	length m	present study				design by Peongtack city('98)		
		dia., cm	length, m	cost Won/year	remark	dia., cm	length, m	cost Won/year
	260.0					<u> </u>		
2	483 0							
3	683.0	80.0	0.971	38,439	replacement	35.0	683.0	8,357,150
4	475.0	30.0	473.91	7,615,259	parallei			-
		35.0	1.09	19,177	pipe			
5	620.0							
6	535.0						_	
7	520.0	20.0	38.427	488,714	replacement			
8	510.0				_			
9	286.0	15.0	6.198	66,442	replacement	15.0	286.0	3,065,920
10	624.0		<u> </u>			15.0	624.0	6,689,280
11	226.0							
12	1010.0		-					
13	476.0							
14	376.0							
15	496.0					_		
16	217.0	25.0	17.95	255,913	replacement			
17	585.0	25.0	10.62	151,409	replacement			
18	286.0							
19	1060.0	_						
20	700.0			_				
21	487.0							_
22	691.0							
23	300.0	25.0	300.0	4,27 <u>7,1</u> 00	new pipe	60.0	300.0	8,612,100

93,982

619,112

115,225

10,711,569

18,034,249

4,824,000

1,364,400

48,674,990

replacement

new pipe

new pipe

new pipe

new pipe

Table 4. Optimal Solution for Songtan System

search polytope algorithm (IMSL subroutme UMPOL).

25.0

20.0

25.0

20.0

25.0

15.0

10.0

6 592

48.68

751.32

1264,94

450.0

150.0

9.06

834.0

0.008

1274.0

450.0

150.0

total cost

24

25

26

27

Table 4 shows the optimal solution for the Songtan water distribution system and the solution by Pyongtack city. The design of the sample pipe network involves mainly two decisions: the selection of diameters for expanded links in addition to the existing pipe(see Table 1) and the selection of new

pipe diameters of new links 23, 25, 26, 27, and 28. The link 4 needs parallel expansion and link 7, 9, 15, 17, 18, and 24 need to replace a segment of each link.

800.0

1274.0

450.0

150.0

12,855,200

20,471,906

7,231,050

2,410,350

65,426,396

300

30.0

30.0

30.0

30.0

6. SUMMARY AND CONCLUSIONS

A two phase search scheme for expanding a looped pipe network has been described. A sample pipe network is employed to test the

proposed procedure. Since model 1 for pipe expansion has several local minima, each perturbation of link flows by the procedure moves towards a better local optimum. The various locally optimal designs obtained by the proposed method, greatly aid in understanding the feasible region in terms of the objective function surface. As a practical matter, the methodology helps the designer in understanding how close the various designs are in terms of cost. The analysis of the sample pipe network involves mainly three decisions: the selection of the tree layout from the given network, the selection of link to be expanded, and the selection of diameters of links. Ιn this study TREESEARCH algorithm is first used to determine the optimal tree lavout. two-phase search scheme then perturbs link flows to obtain loop flows for the optimal design of the pipe network. The two phase search scheme iteratively improves objective function by finding successive better points and by being escaped out of a local optimum.

The optimal solution obtained from both convex and nonconvex cost-gradient functions are all hydraulically feasible. optimal head losses for existing links are same for the two functions. The solution by the nonconvex function is better than the one by the convex function. It has been also found that in the perturbation stage, selecting the initial loop flows as the loop flows obtained from the existing network moves efficiently towards a better local optimum. The results show that the proposed method can yield a lower cost design than the conventional design method. In conclusion, the proposed method can be efficiently used to design the pipe expansion of existing

networks.

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