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A General Multivariate EWMA Control Chart

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ABSTRACT

This paper proposes a general approach of the multivariate exponentially weighted moving average (MEWMA) chart, in which the smoothing matrix has full elements instead of only diagonal elements The average run length (ARL) properties of this scheme are examined for a diverse set of quality control environments and the information to design the chart is provided. Performance of the scheme is measured by estimating ARL and compared to those of two group cumulative sum (CUSUM) charts. The comparison results show that the MEWMA chart can improve its ARL performance in detecting a small shift by using appropriate nondiagonal components in the smoothing matrix. When the process shifts out-of-control in the start-up stage, the general MEWMA chart of a full smoothing matrix appears to offer an exceptional protection against departures from control in the process mean.

1. INTRODUCTION

A quality control (QC) chart scheme alternative to the standard Shewhart χ^2 chart is the EWMA chart. Since Roberts [12] introduced the use of EWMA technique for constructing a control chart for an univariate normal process with independent and identical distribution, the EWMA control scheme has been exploited and its properties has been evaluated numerically and analytically (Robinson and Ho [13]; Crowder [2]; Lucas and Saccucci [10]). Assuming that the observations are independent over time, the QC procedure uses an EWMA technique for detecting when special causes of variation enter into a system. The univariate EW-

MA procedure was extended to a multivariate control chart scheme for controlling the mean of a multivariate normal process (Lowry, Woodall, Champ and Rigdon [9]). The MEWMA chart is a straightforward generalization of the corresponding univariate procedure, using a smoothing weight of diagonal matrix.

A general MEWMA chart is proposed in this paper. In this control chart scheme, the smoothing weight is defined with a general matrix including nondiagonal elements. The MEWMA chart is directionally invariant when using a diagonal smoothing matrix, that is, its ARL performance is determined by the mean vector μ and covariance matrix Σ only through the value of the noncentrality parameter

$$\eta_c = \sqrt{(\mu - \mu_0)' \Sigma^{-1} (\mu - \mu_0)}.$$

The MEWMA chart of nondiagonal smoothing matrix does not give this directional invariance property, however. This study evaluates the ARL performance of the general MEWMA chart for various covariance structures and mean shift directions of process on the basis of 10,000 Monte Carlo independent simulation runs. Design of general MEWMA chart is discussed in the next section and the following section which contains numerical results. This is followed by the performance comparison of general MEWMA charts for various QC characteristics with the directionally-variant group CUSUM control charts that operate multiple univariate CUSUM charts simultaneously. For the case of using a diagonal matrix, Lowry, Woodall, Champ and Rigdon [9] compared the MEWMA chart in ARL with the other multivariate control charts which are directionally invariant, and performance of the group CUSUM charts was extensively evaluated including comparison with the directionally-invariant multivariate control charts in (Choi and Lee [1]). In a subsequent section, some conclusions are presented.

2. GENERAL MULTIVARIATE EWMA CHARTS

Recently. there has been interested in using EWMA charts to detect shifts in the process mean level for QC. Crowder [2] and Lucas and Saccucci [10] thoroughly investigated properties of univariate EWMA charts and suggested design strategies, and Lowry, Woodall, Champ and Rigdon [14] proposed a multivariate EW-

MA chart using a smoothing matrix of diagonal form. Suppose that the successive measurements $\{\mathbf{x}_n,\ n=1,2,...\}$ are independent and identically distributed multivariate normal random vectors $\mathbf{x}_n \sim N(\mu, \Sigma)$. For simplicity, assume that the in-control mean vector $\mu_0 = (0,0,\cdots,0)' = \mathbf{0}$, and the covariance matrix Σ is known and normalized such that all diagonal elements are 1. Similar to the univariate EWMA chart, the MEWMA chart for a p-variate normal process was implemented by defining an MEWMA vector

$$\mathbf{y}_n = \mathbf{R}\mathbf{x}_n + (\mathbf{I} - \mathbf{R})\mathbf{y}_{n-1} \tag{1}$$

for $n = 1, 2, \cdots$ where $\mathbf{y}_0 = \mathbf{0}$ and the smoothing matrix $\mathbf{R}$ is a diagonal matrix whose diagonal elements are $\{0 < r_i \le 1,\ i = 1, 2, \cdots, p\}$. Unless there is any reason to differently weight the quality characteristic measurements related to the normalized covariance matrix Σ, all diagonal elements of the weight matrix can be set to an equal value, that is, $r_1 = r_2 = \cdots = r_p = r$. This MEWMA control scheme gives an out-of-control signal as soon as

$$T^2 = \mathrm{y}_n' \Sigma_{\mathrm{y}_n}^{-1} \mathrm{y}_n > \mathrm{h} \tag{2}$$

where

$$\Sigma_{\mathrm{y}_n} = \frac{r\,[1-(1-r)^{2n}]}{2-r}\Sigma \tag{3}$$

for a given threshold $h > 0$. In QC practice, it is likely that the process stays in control for a sufficiently long period but later shifts out of control. Based on this fact, the control chart can be alternatively designed with the asymptotic covariance matrix

$$\Sigma_{\mathrm{y}_n} = \frac{r}{2-r}\Sigma. \tag{4}$$

The EWMA chart using the exact covariance matrix has the fast initial response (FIR) feature and then detects more quickly for the initial out-of-control.

A natural extension is to use a smoothing matrix $\mathbf{R}$ having full elements in (1) if there exists interaction between the variables in multivariate process. This study examines the MEWMA chart using a smoothing matrix of general form, in which the smoothing weights associated with each variable are equivalent under the assumption that the scale of each variables is uniform. If r_{ij} is the (i, j) th element of the smoothing matrix $\mathbf{R}$, the matrix for a p-variate process is formed in this study such that $r_{ii} = r_{on}$ for $i = 1, 2, \cdots, p$ and $r_{ij} = r_{off}$ for $i, j = 1, 2, \cdots, p$ and $i \neq j$. To investigate the effects of smoothing manner with the same total weight for each variable of the MEWMA vector, the row sums of $\mathbf{R}$ are constantly fixed such that

$$r = \sum_{j=1}^{p} r_{ij} = r_{on} + (p-1)\, r_{off}, \ \forall i. \tag{5}$$

If the smoothing matrix has off-diagonal elements, the covariance matrix of the MEWMA vector $\mathbf{y}_n$ is more complicated and is recursively calculated in the control chart procedure:

$$\Sigma_{y_{n-1}} = \Sigma_{y_1} + \overline{\mathbf{R}} \Sigma_{y_n} \overline{\mathbf{R}} \tag{6}$$

for $n = 1, 2, \cdots$ where $\Sigma_{y_1} = \mathbf{R} \Sigma \mathbf{R}$ and $\overline{\mathbf{R}} = \mathbf{I} - \mathbf{R}$. It is easily shown that covariance matrix of (6) is converged as $n \to \infty$ for the smoothing matrix satisfying (5) and $0 < r < 1$ because the matrix $\overline{\mathbf{R}}$ is diagonally dominant (Cullen [3]). Since it is not appropriate to use the off-diagonal weights greater than the on-diagonal weight in the smoothing manner for the multivariate process, this study uses the matrix $\mathbf{R}$ such that $r_{off} = c r_{on}$ for $0 \leq c < 1$. Then, given r and c,

$$r_{on} = \frac{r}{1 + (p-1)\, c} \quad \text{and} \quad r_{off} = \frac{cr}{1 + (p-1)\, c}.$$

The MEWMA chart of Lowry, Woodall, Champ and Rigdon [9] corresponds to the scheme using $\mathbf{R}$ with $c = 0$. Table 1 contains an example of the full smoothing matrix $\mathbf{R}$ with $r = 0.1$ and $c = 0.75$. In the next section, the properties of the general MEWMA chart are examined for variation of the values of r and c.

Table 1 Covariance matrix Σy_n of MEWMA vector $\mathbf{y}_n$ with $r = 0.1$ and $c = 0.75$ for negative correlation type N−8 in time n.

R of $r = 0.1$ and $c = 0.75$				Σ of N-8			
0.031				1			
0.023	0.031			-0.8	1		
0.023	0.023	0.031		0.8	-0.8	1	
0.023	0.023	0.023	0.031	-0.8	0.8	-0.8	1
Σy_n							
(n = 101)				(n = 201)			
0.0055				0.0061			
0.0000	0.0055			-0.0005	0.0061		
0.0049	0.0000	0.0055		0.0054	-0.0005	0.0061	
0.0000	0.0049	0.0000	0.0055	-0.0005	0.0054	-0.0005	0.0061
(n = 301)				(n = 401)			
0.0063				0.0063			
-0.0006	0.0063			-0.0006	0.0063		
0.0055	-0.0006	0.0063		0.0055	-0.0006	0.0063	
-0.0006	0.0055	-0.0006	0.0063	-0.0006	0.0055	-0.0006	0.0063

3. ARL PERFORMANCE OF THE GENERAL MEWMA CHARTS

The ARL performance of general MEWMA control schemes are evaluated using multivariate normal processes that are simulated with various QC characteristics. The QC characteristics considered are the correlation structure and the direction of the shift in the mean vector. The correlation structure equivalent to the normalized covariance matrix represents the most important attribute of the measurements from a multivariate normal process with zero mean, and the out-of-control condition is usually characterized by the shift direction for the problem controlling the mean of a multivariate process. We choose the six correlation structures and three shift directions, which are used in (Choi and Lee [1]). The six correlation structures are categorized into two classes: the positive type, in which variables i and j for $i \neq j$ have a negative correlation if $i + j$ is odd and a positive correlation if $i + j$ is even. For simplicity, assume that absolute magnitudes of correlation between the variables are uniform. Then, the positive types are denoted by P-2, P-5, P-8 with the absolute magnitudes of 0.2, 0.5, 0.8 respectively and the negatives types by N-2, N-5, N-8. An example of the correlation type of N-8 is illustrated in Table 1. The out-of-control mean process is modeled with tree types of shift direction :

(1) Equal Shift, in which all components of μ are equal ;
(2) Symmetric Shift, which differs from Equal Shift in that the first half components of μ have different signs to the second half;
(3) Only Shift, in which only a single component of μ is nonzero.

For Only Shift, we randomly choose one of all the variables for the out-of-control situation in each simulation run. The ARL performance is evaluated for six amounts of shift distance from the desired mean level corresponding to the values of noncentrality parameter $\eta_c = 0.1, 0.2, 0.4, 0.8, 1.6, 3.2$ respectively. We also consider two states related to the time point for the out-of-control situation : the "steady" state, in which the process becomes out of control after initially staying in control for a substantial period, and the "initial" state, in which the process changes into an undesirable condition from the very first point. Table 1 shows the covariance matrices of (6) in the MEWMA procedure using $r = 0.1$ and $c = 0.75$ for several time steps. It is an example for the case in which the covariance matrix most slowly converges among our charts examined in this study. As illustrated in the table, the general MEWMA chart procedure achieves the steady parameter at least after the initial in-control period of 300 for the 4-variate process. We assume that out-of-control conditions are suddenly introduced on the process after running in control for 300 sampling stages. The ARL values were estimated from 10,000 simulation runs using independent random seed number for each case considered.

First, we investigate the properties of the ARL performance of the MEWMA charts for 4-variate normal processes using the control limits which result in ARL = 300 for in-control processes in steady state. Table 2 contains the estimated values of threshold h for six correlation types based on independent 10,000 simulation runs. The estimated values of h vary in the correlation types except $c = 0$, but h values differ by less than 0.5% of the values. The MEWMA chart uses a greater value of threshold h as increasing r and c for the same r. Table 3 displays the averages and standard deviations of the estimated ARLs for six correlation types for the MEWMA charts of $r = 0.1, 0.2, 0.5$ and $c = 0, 0.25, 0.5, 0.75$. Using the h values of in-control ARL = 300 in Table 2, these results were obtained from total 60,000 simulation runs by applying the six types to each 10,000 runs respectively. Table 3 shows that the scheme has the best ARL performance with different combinations of r and c according to the distances and directions of the shift in the mean vector. The chart appears to performs best with $r = 0.1$, and $c = 0.75$ for $\eta_c < 1.6$, $r = 0.2$ and $c = 0.75$ for $\eta_c = 1.6$ and $r = 0.5$ and $c = 0.0$ for the largest shift distance. In the other shift directions, the shortest ARL is given by the nondiagonal smoothing schemes with $r = 0.1$ for $\eta_c < 0.8$ and the charts of the diagonal smoothing matrices with $r = 0.1$, 0.2, 0.5 show the best performance for $\eta_c = 0.8$, 1.6, 3.2 respectively Generally, the MEWMA chart yields a shorter ARL with smaller r for a small shift and larger r for a large shift. For a small shift, the ARL performance is improved in Symmetric and Only Shifts greater than in Equal Shift by using a nonzero c. When shifting a large amount in the mean vector, ARL decrease in Equal Shift except for $r = 0.5$, but is longer in the other directions with a large value of c. Compared to the size of the corresponding length, the standard deviations of ARL for the six correlation types are not significant in the Equal and Symmetric Shift directions, but is not for Only Shift. The ARL results in Table 4 show that the nondiagonal smoothing scheme has a little variation in ARL for the different classes of correlation type in the manner that its ARL is shorter in the positive class for a small shift and the negative class for a large one. The performance of the MEWMA charts when $c \neq 0$ also appears to be sensitive on the different shift types of the process mean. For examples, the chart of $r = 0.1$ and $c = 0.5$ has the ARLs of 147.9, 118.7, 122.9 for $\eta_c = 0.2$ and 6.77, 11.53, 10.26 for $\eta_c = 1.6$ in the Equal, Symmetric and Only Shifts respectively.

Table 2. h values of in-control ARL = 300 for general MEWMAs for steady state processes of $p = 4$.

r	c	N-8	N-5	N-2	P-2	P-5	P-8
	0	13 83	13.83	13.83	13 83	13 83	13 83
0.1	0.25	12.61	12.62	12.63	12 61	12 62	12.63
	0.5	11 49	11.50	11.50	11.51	11.48	11.49
	0.75	10.11	10.12	10.11	10.09	10 09	10.13
	0	14.93	14.93	14 92	14.93	14.93	14.93
0.2	0.25	13.96	13.97	13 97	13.98	13.97	13.98
	0.5	13.05	13.04	13.02	13.00	13.00	12.98
	0.75	11.78	11.79	11.80	11.76	11.77	11 76
	0	15.62	15.62	15 62	15.62	15.62	15.62
0.5	0.25	15.22	15.21	15 20	15.19	15.21	15.21
	0.5	14.49	14.51	14.49	14.52	14.52	14.55
	0.75	13.46	13.45	13.45	13.48	13 47	13.49

Table 3. Average (standard deviations) of ARLs of general MEWMAs for six correlation types using h of in−control ARL = 300 for steady state processes of $p = 4$.

				Equal Shift			
r	c	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0.4$	$\eta_c = 0.8$	$\eta_c = 1.6$	$\eta_c = 3.2$
0.1	0	245.6(0.5)	158.6(0.9)	61.3(0.2)	18.6(0.0)	7.06(0.00)	3.34(0.00)
	0.25	242.2(0.9)	153.8(0.5)	59.0(0.2)	18.3(0.0)	6.97(0.01)	3.27(0.00)
	0.5	240.1(0.6)	147.9(1.1)	57.4(0.1)	17.9(0.0)	6.77(0.01)	3.17(0.00)
	0.75	*234.1(1.8)	*141.4(1.3)	*54.1(0.2)	*16.8(0.0)	6.40(0.01)	3.00(0.00)
0.2	0	267.6(0.9)	200.6(1.3)	88.3(0.5)	22.4(0.0)	6.36(0.01)	2.69(0.00)
	0.25	263.9(1.3)	192.9(1.1)	83.9(0.7)	22.1(0.0)	6.38(0.01)	2.68(0.00)
	0.5	263.7(2.9)	188.2(0.8)	80.9(0.6)	21.9(0.1)	6.31(0.01)	2.63(0.00)
	0.75	258.6(1.8)	180.4(0.9)	77.9(0.6)	21.1(0.1)	*6.05(0.01)	2.54(0.00)
0.5	0	283.2(1.4)	244.4(0.6)	152.6(0.8)	47.3(0.3)	8.10(0.02)	*2.17(0.01)
	0.25	284.3(1.7)	246.7(2.1)	155.3(1.7)	48.4(0.1)	8.39(0.03)	2.24(0.01)
	0.5	285.3(2.6)	243.2(1.0)	148.1(0.8)	46.8(0.5)	8.50(0.06)	2.24(0.01)
	0.75	281.7(1.6)	237.5(1.3)	139.8(1.4)	45.2(0.4)	8.40(0.05)	2.18(0.00)
				Symmetric Shift			
r	c	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0.4$	$\eta_c = 0.8$	$\eta_c = 1.6$	$\eta_c = 3.2$
0.1	0	246.9(1.3)	159.2(0.9)	62.0(0.2)	*18.7(0.0)	7.06(0.01)	3.33(0.00)
	0.25	225.8(1.3)	129.2(0.5)	*51.6(0.3)	19.9(0.1)	8.94(0.01)	4.44(0.00)
	0.5	213.1(0.9)	*118.7(0.6)	53.5(0.1)	24.1(0.0)	11.53(0.01)	5.84(0.00)
	0.75	*206.5(0.9)	124.3(0.3)	64.4(0.2)	32.3(0.0)	16.23(0.01)	8.32(0.01)
0.2	0	271.6(0.6)	200.8(0.7)	88.9(0.5)	22.7(0.1)	*6.36(0.01)	2.69(0.00)
	0.25	244.9(1.8)	155.8(1.1)	59.2(0.2)	18.7(0.0)	7.35(0.01)	3.51(0.00)
	0.5	227.3(1.3)	129.7(0.3)	52.1(0.2)	20.5(0.0)	9.26(0.00)	4.59(0.01)
	0.75	208.3(0.3)	119.6(0.5)	56.4(0.1)	26.6(0.0)	12.93(0.01)	6.56(0.00)
0.5	0	284.7(0.7)	246.8(0.8)	154.2(0.5)	48.1(0.3)	8.20(0.02)	*2.18(0.00)
	0.25	271.8(1.9)	204.1(1.9)	92.8(0.8)	23.5(0.1)	6.36(0.02)	2.63(0.00)
	0.5	250.3(2.6)	163.3(0.9)	63.0(0.2)	18.9(0.1)	7.13(0.02)	3.34(0.01)
	0.75	222.6(0.3)	126.8(1.1)	51.8(0.1)	20.9(0.0)	9.50(0.02)	4.71(0.01)
				Only Shift			
r	c	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0.4$	$\eta_c = 0.8$	$\eta_c = 1.6$	$\eta_c = 3.2$
0.1	0	248.3(1.0)	158.6(0.9)	61.8(0.3)	*18.6(0.1)	7.07(0.01)	3.35(0.01)
	0.25	228.1(1.1)	133.7(2.7)	*52.3(0.9)	19.7(0.2)	8.51(0.28)	4.15(0.19)
	0.5	216.6(2.8)	*122.9(2.8)	53.6(0.4)	22.8(0.8)	10.26(0.80)	4.97(0.51)
	0.75	*210.7(2.1)	126.1(1.5)	62.0(1.5)	28.3(2.5)	12.72(2.04)	6.00(1.23)
0.2	0	267.2(1.7)	199.4(0.9)	88.6(0.6)	22.5(0.1)	*6.35(0.02)	2.69(0.00)
	0.25	247.8(1.8)	160.0(3.6)	62.4(2.1)	19.2(0.2)	7.17(0.13)	3.33(0.12)
	0.5	230.3(4.2)	137.0(5.7)	54.7(2.2)	20.6(0.1)	8.62(0.41)	4.02(0.35)
	0.75	217.1(4.9)	126.1(4.7)	58.0(1.2)	25.5(0.7)	11.05(1.20)	5.00(0.88)
0.5	0	283.8(1.2)	244.0(1.1)	153.3(1.4)	47.3(0.1)	8.10(0.04)	*2.18(0.01)
	0.25	272.0(3.8)	209.8(6.8)	100.3(6.2)	25.8(1.7)	6.57(0.16)	2.56(0.05)
	0.5	255.5(3.2)	173.1(7.5)	70.4(5.2)	20.7(1.2)	7.26(0.07)	3.12(0.16)
	0.75	231.0(4.5)	139.1(7.4)	57.7(3.9)	22.3(0.9)	9.35(0.14)	4.05(0.43)

Table 4. ARLs of general MEWMAs using h of in−control ARL = 300 for steady state processes of p = 4 in Only Shift.

η_c	r	c	N-8	N-5	N-2	P-2	P-5	P-8
		0	158.1	155.8	156.5	156.3	154.6	156.3
	0.1	0.25	133.1	131.6	131.7	127.5	125.1	124.7
		0.5	119.3	119.1	118.8	115.6	113.3	112.5
		0.75	114.6	114.8	114.6	113.2	111.3	111.4
		0	197.0	196.1	195.8	193.6	194.7	196.9
0.2	0.2	0.25	163.4	161.7	159.2	155.9	152.5	151.8
		0.5	138.9	138.2	136.4	130.5	127.1	126.1
		0.75	123.8	122.5	122.7	116.3	113.1	112.7
		0	245.1	242.1	241.9	245.2	245.5	245.2
	0.5	0.25	218.4	214.9	213.1	202.9	201.8	201.7
		0.5	182.4	181.7	176.2	165.1	162.1	160.0
		0.75	144.6	143.4	139.8	130.8	126.5	125.2
		0	6.6	6.7	6.6	6.6	6.6	6.6
	0.1	0.25	7.6	7.6	7.6	8.0	8.1	8.2
		0.5	8.5	8.6	8.7	9.6	10.1	10.5
		0.75	9.3	9.4	9.6	11.4	12.6	13.5
		0	6.1	6.1	6.1	6.1	6.1	6.1
1.6	0.2	0.25	6.6	6.6	6.6	6.8	6.9	6.9
		0.5	7.5	7.6	7.7	8.1	8.4	8.6
		0.75	8.7	8.8	9.1	10.4	11.2	11.7
		0	8.0	8.0	8.0	7.9	8.0	8.0
	0.5	0.25	6.5	6.5	6.4	6.2	6.1	6.1
		0.5	6.9	6.9	6.9	6.8	6.8	6.8
		0.75	8.5	8.5	8.5	8.7	8.8	8.9

Since the processes in QC practice are often away from the in-control values due to start-up problems or the ineffective control action for the previous signal, a FIR feature of the charts is important. Next, the ARL properties of general MEWMA charts are examined for out-of-control processes in the initial state. This experiment is conducted in the same way with the previous one except using the control limit of in-control ARL = 100. Tables 5 an 6 contain the estimated values of threshold h and ARL. Like for the steady state process, a diagonal smoothing MEWMA chart can be designed equivalently for a same in-control ARL regardless of the correlation characteristics of the measurements and its ARL is only dependent on the value of noncentrality parameter. When $c > 0$, the control limits is not much different over the correlation types to generate the same in-control ARL, but the performance changes over the correlation types in Only Shift and varies in the shift directions of the mean vector. These variations are not substantial for relatively large shifts with $r = 0.1$. As illustrated in Table 6, the scheme appears to always have shorter ARLs for smaller rs and larger cs in all the direction of the shift, and the ARL performance is then best with $r = 0.1$ and $c = 0.75$ for all the situations considered. When shifting a small distance, the ARL is reduced by using the nondiagonal components in Symmetric and Only Shifts more than in Equal Shift. It is reverse for a large shift.

Table 5. hs of in-control ARL = 100 for general MEWMAs for initial state processes of $p = 4$.

r	c	N-8	N-5	N-2	P-2	P-5	P-8
	0	10.99	10.99	10.99	10.99	10.99	10.99
0.1	0.25	9.80	9.80	9.80	9.80	9.81	9.82
	0.5	9.01	8.99	8.97	8.98	8.99	9.01
	0.75	8.32	8.32	8.33	8.35	8.33	8.35
	0	12.13	12.13	12.13	12.13	12.13	12.13
0.2	0.25	11.12	11.13	11.14	11.14	11.13	11.13
	0.5	10.23	10.24	10.25	10.22	10.23	10.24
	0.75	9.40	9.40	9.40	9.41	9.40	9.38
	0	13.11	13.11	13.11	13.11	13.11	13.11
0.5	0.25	12.48	12.49	12.50	12.50	12.52	12.50
	0.5	11.73	11.71	11.74	11.73	11.75	11.74
	0.75	10.79	10.79	10.77	10.79	10.81	10.82

Although the EWMA charts can be designed give a quick signal for a small shift in mean, Yashchin [15] remarked the "inertia" problem of the EWMA chart schemes that are likely to reacting slowly to outliers. The general MEWMA chart can be subject to the same problem, that was discussed for the MEWMA chart by Lowry, Woodall, Champ and Rigdon [9]. We can provide a Shewhart rule in the MEWMA chart procedure to protect against outliers. As they mentioned, however, there will be a trade-off between protection against inertia and quick detection of small shifts when using the combined Shewhart EWMA control scheme.

From the results of the previous analysis, we can conclude that the best chart scheme is designed with the total smoothing weight $r = 0.1$ among our cases. Tables 7 and 8 display the ARL results of the general MEWMA charts using $r = 0.1$ with fixed values of h over different correlation structures for $p = 2$, 4, 10. These results provides useful information for designing a general MEWMA chart according to the quality characteristics with Tables 2 and 5. In the next section, the investigation of the ARL performance for general MEWMA charts will be extended by comparing with the other multivariate QC charts.

Table 6. Averages(standard deviation) of ARLs of general MEWMAs for six correlation types using h of in-control ARL = 100 for initial state process of $p = 4$

Equal Shift							
r	c	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0.4$	$\eta_c = 0.8$	$\eta_c = 1.6$	$\eta_c = 3.2$
	0	88.8(0.3)	66.0(0.2)	32.9(0.2)	12.1(0.0)	4 51(0.01)	1 97(0.00)
0 1	0 25	86.0(0.2)	61.9(0.4)	29.6(0 1)	10.9(0 0)	4 14(0.00)	1.85(0 00)
	0.5	84.2(0.6)	58.1(0.5)	26 6(0.1)	9.9(0.0)	3 87(0.01)	1 76(0.00)
	0 75	*81.4(0.8)	*53.4(0.2)	*24.0(0 1)	*9.1(0.0)	*3.65(0.01)	*1.68(0.00)
	0	92.2(0.2)	74.5(0.4)	41.0(0 1)	14.1(0.0)	4.73(0 00)	1.99(0.00)
0 2	0.25	91.4(0.2)	72.0(0.2)	38.7(0.2)	13 2(0.1)	4.46(0.01)	1.90(0.00)
	0.5	90 5(0.4)	69.1(0.3)	35.3(0.2)	12 0(0.0)	4.18(0 01)	1.81(0.00)
	0.75	87 7(0.6)	64 9(0.4)	31.2(0.1)	10 8(0 0)	3.90(0 01)	1.73(0.00)
	0	96.5(0 5)	86.9(0.2)	59.9(0.2)	23 6(0 0)	5.76(0.03)	1.84(0.00)
0.5	0.25	96.7(0 5)	86.6(0 2)	59.2(0.2)	23.1(0 ,1)	5.70(0.05)	1.83(0.00)
	0.5	95.7(0.6)	84.8(0.4)	56.4(0.3)	21.6(0 1)	5 38(0.04)	1 77(0.00)
	0.75	95.0(0.6)	82.3(0.5)	52.3(0.4)	19.2(0 0)	4.87(0.03)	1.68(0.00)
Symmetric Shift							
r	c	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0\ 4$	$\eta_c = 0.8$	$\eta_c = 1.6$	$\eta_c = 3.2$
	0	88.4(0.3)	66.5(0.4)	33.1(0.2)	12.1(0.0)	4 50(0.01)	1 98(0.00)
	0.25	83.7(0.1)	58.2(0.2)	28 2(0.1)	10.9(0.0)	4 24(0.01)	1 90(0.00)
0 1	0 5	80.6(0.4)	53.9(0.5)	26.0(0 1)	10.2(0.0)	4.01(0.00)	1.83(0 00)
	0.75	*77.3(0.6)	*50.5(0.2)	*24.3(0 1)	*9.6((0.0)	*3.81(0 00)	*1.76(0.00)
	0	92.6(0.3)	75.1(0.3)	41.1(0 3)	14 2(0.0)	4.71(0.01)	1.99(0 00)
0.2	0.25	87.9(0.4)	65.1(0.3)	32.3(0.2)	12.1(0.0)	4.55(0 01)	2.00(0 00)
	0 5	84.3(0.4)	58.7(0 4)	28.8(0.1)	11 3(0.0)	4.38(0 01)	1.95(0.00)
	0.75	80 8(0.5)	53 9(0.2)	26.6(0.1)	10 6(0 0)	4.15(0 01)	1.88(0.00)
	0	96 7(0 3)	86 9(0 2)	60.6(0.2)	23 8(0 1)	5.74(0.02)	1.85(0.01)
0.5	0.25	93.1(0.	76.1(0.6)	42.5(0.2)	14 5(0.0)	4 77(0.01)	2.00(0.00)
	0.5	89.0(0.2)	66.7(0.3)	33.6(0.1)	12.5(0.0)	4 66(0.01)	2 03(0.00)
	0.75	84.7(0.5)	58.7(0.3)	29.1(0.1)	11.6(0.0)	4 51(0.01)	1 99(0.00)
Only Shift							
r	c	$\eta_c = 0.1$	$\eta_c = 0.2$	$\eta_c = 0\ 4$	$\eta_c = 0.8$	$\eta_c = 1.6$	$\eta_c = 3.2$
	0	88.4(0..2)	66.0(0.2)	32 8(0.1)	12.0(0.0)	4 49(0.01)	1 97(0.00)
0.1	0.25	84.3(0.7)	58.7(0.8)	28 2(0.1)	10.9(0.1)	4 22(0.02)	1.89(0.01)
	0 5	81.3(0.6)	54.2(0.6)	26.0(0 1)	10.1(0.1)	3.99(0 03)	1.81(0 01)
	0 75	*77.7(0.2)	*50.2(0.2)	*24.1(0.1)	*9.5(0.1)	*3.78(0 03)	*1.74(0.01)
	0	92.6(0.3)	75.0(0.3)	40.9(0.1)	14.1(0.9)	4.71(0 01)	1.99(0.00)
0 2	0.25	88.6(0.6)	65 9(0.8)	33.1(0.6)	12 2(0.2)	4.53(0.02)	1.98(0.01)
	0.5	85.3(0 9)	60.0(1 4)	29.5(0 6)	11 4(0 1)	4.33(0 03)	1.92(0.02)
	0.75	81.7(0.9)	55 1(1.1)	27.1(0.4)	10.6(0.0)	4.10(0.04)	1.84(0.02)
	0	97.1(0.2)	86.7(0.3)	60.0(0.2)	23.7(0.1)	5.72(0.03)	1.84(0.00)
0.5	0.25	93.0(0.8)	77.5(1.3)	44.6(1 6)	15.5(0 7)	4 89(0 07)	1.96(0.02)
	0.5	90.5(1.0)	69.5(2.1)	36 2(1 9)	13.3(0.6)	4 74(0.05)	1 98(0.03)
	0.75	86 4(1.1)	61.7(2.1)	31 2(1 4)	12 3(0.5)	4 56(0.02)	1 92(0.04)

Table 7. In−control ARLs of the general MEWMAs of $r = 0.1$ for various h value in steady state.

	$p = 2$			$p = 4$			$p = 10$		
c	h	N-5	P-5	h	N-5	P-5	h	N-5	P-5
0	7 0	100.0	100 0	10.7	99.8	99.8	20.1	99 4	99.4
	10.8	507.9	507 9	15.2	500.2	500 2	25 8	501 1	501 1
0.25	6.6	101 2	102.2	9.3	100.9	101 2	15.4	100.6	99.6
	10.4	497 2	503.1	14 1	500.5	502.3	22.5	505.9	506.9
0.5	6 1	99.0	100.5	8.0	99.6	99.1	11.6	100.1	100.0
	10 0	496.1	500.8	13.0	496 0	493.8	19 8	500 8	500.0
0.75	5.6	99.9	101 3	6.7	101 3	101.6	8 4	100 7	100 5
	9.5	497.2	498 7	11.8	501.8	500 9	16.6	504.6	505.3

Table 8. In−control ARLs of the general MEWMAs of $r = 0.1$ for various h values in initial state.

	$p = 2$			$p = 4$			$p = 10$		
c	h	N-5	P-5	h	N-5	P-5	h	N-5	P-5
0	9.7	307 9	307.9	14.0	304.9	304 9	24.4	306 8	306 8
	10.8	494.1	494 1	15.3	507.5	507 5	25.9	498.7	498.7
0.25	9.3	301 8	301.1	12 9	302.8	302.6	20.9	261.0	262.3
	10 5	498 3	507.1	14 3	503.5	507.5	23.0	499.3	497.7
0.5	9.0	302.4	306.4	12.1	303.2	301.9	19.7	302 4	304.3
	10.1	490.0	495.0	13.4	500 2	498.3	21 3	508.3	509.0
0 75	8.6	300.8	307 3	11.2	300.4	300 0	18 4	306.3	304 0
	9.7	492.6	495.8	12.5	506.8	497.8	19.7	497.6	496 4

4. COMPARATIVE PERFORMANCE OF MEWMA CHARTS WITH GROUP CUSUM CHART

One approach for controlling multivariate normal processes utilizes a control chart based on a multivariate statistic that involves information on the dependence between separate measurements. The MEWMA chart of a diagonal smoothing matrix was made comparison of ARL performance with the multivariate control charts, the Shewhart χ^2 chart and two multivariate CUSUM chart, by Lowry, Woodall, champ and Rigdon [9]. These multivariate CUSUM charts proposed by Pignatiello and Runger [11] employs the Hotelling statistic [7] in an univariate CUSUM procedure. Another approach to CUSUM control for multivariate processes is the proposal of Woodall and Ncube [14]. It operates a group of individual univariate CUSUM charts of all the variables simultaneously and a signal is given if any of the chart in the group exceeds its control limits. The interpretation of this group chart is simpler than that of the other multivariate control charts, since the signal instantaneously identifies with the out-of-control variable. Woodall and Ncube [14] and Hawkins [6] also suggested to apply the group approach to a linear transformation of the original variables. The ARL performance comparison between the group CUSUM charts and the multivariate control charts are presented in (Pignatiello and Runger [11]; Choi and Lee [1]). This study investigates the relative ARL performance of the general MEWMA charts and the group CUSUM charts.

Woodall and Ncube [14] described how a p component multivariate normal process can be monitored with p two-sided univariate CUSUM charts. Assume the zero in-control mean vector and the normalized covariance matrix such as our case. If $x_{n,i}$ is the observation on the ith variable at time n, the ith univariate CUSUM is operated for a given reference value $k > 0$ by forming the cumulative sums

$$U_{n,i} = \max(0, U_{n-1,i} + x_{n,i} - k) \tag{7}$$

and

$$L_{n,i} = \min(0, L_{n-1,i} + x_{n,i} + k) \tag{8}$$

for n =1, 2, $\cdots$ and the group CUSUM chart then signal when

$$\max_{i} [\max(U_{n,i}, - L_{n,1})] > h \tag{9}$$

for a given threshold h. With the idea that departures from control in multivariate processes may be expected to affect only a minority of the variables, Hawkins [5, 6] proposed using the vector of scaled residuals from the regression of each variable on all the others. Realization rm bold $\mathbf{x}_n$ is transformed to the regression-adjusted vector

$$\mathbf{z}_n = [\text{diagonal}(\Sigma^{-1})]^{-1/2}\Sigma^{-1}\mathbf{x}_n. \tag{10}$$

The principal component analysis for successive multivariate measurement provides the possibility of separate control of the individual variables of a multivariate normal process (Woodall and Ncube [14]; Jackson [4]; Pignatiello and Runger [11]). The normalized principal component vector is obtained by the spectral decomposition of the covariance matrix:

$$\mathbf{w}_n = \Lambda^{-1/2}\,\psi\,\mathbf{x}_n \tag{11}$$

where ψ is the matrix of eigenvectors and Λ the diagonal matrix of eigenvalues associated with ψ. The group CUSUM charts, MCZ and MCW apply $\mathbf{z}_n$ and $\mathbf{w}_n$ respectively to the procedures of (7), (8) and (9).

In this section, the ARL performance of the general MEWMA charts with $r = 0.1$ is examined for $p = 2, 4, 10$ including the comparison to that of the group CUSUM charts. The comparison are made for the out-of-control condition in both the initial and steady states using two correlation types of different class by shifting a distance in the mean vector in two extreme directions of Equal and Only Shifts. The control schemes are designed to give an out-of-control signal when the test statistic is greater than the threshold h of in-control ARL = 300 in the steady state and in-control ARL = 100 in the initial state. Table 9 contains h values used for the group CUSUM charts, and the values for the MEWMA chart can be found in Tables 2 and 10. As in the previous section, all the threshold were estimated such that the chart results in having the specified in-control ARL for the in dependent 10,000 simulation runs. The reference values of the group charts are set to 0.5, a half of the standard deviation of an individual variable for the cumulative sum statistics under our assumption of the normalized covariance matrix. As shown in Choi and Lee [1], the ARL performance of MCZ is best when only a single component of the multivariate mean vector is shifted, and MCW gives the shortest ARL for the equal amount of noncentrality distance when all components of the mean vector is equally shifted. In this study, we consider MCW for Equal Shift and MCZ for Only Shift as a group chart in this section. All the results are obtained by simultaneously operating the chart to an identical process in each run of independent 10,000 simulations respectively for the six distance levels. Tables 11 and 12 display the comparison results for the out-of-control process in the steady and initial states respectively. As shown in these table, the group charts MCW and MCZ appear to offer better protection to detect relatively larger shifts in the mean vector of the process than the MEWMA charts in the steady state for some situations: MCW if all the variables are positively correlated and simultaneously changes, and MCZ if the shift occurs in one of the variables. Otherwise, one of the nondiagonal MEWMA chart schemes gives the shortest ARL for the simulated identical processes. It is seen that the ARL performance of the MEWMA charts can be improved more dramatically for higher dimensions when initially shifting along the direction of just one variable in the process mean.

Table 9. h values of in-control ARL = 300 in steady state (in-control ARL = 100 in initial state) for the group CUSUM charts for multivariate normal process.

	MCZ		MCW	
p	N-5	P-5	N-5	P-5
2	5.19(4 08)	5 20(4.08)	5.25(4.14)	5.25(4.13)
4	5 90(4.77)	5.90(4.76)	5.90(4.81)	5.90(4.78)
10	6.81(5.68)	6 81(5.68)	6.83(5.69)	6.81(5.69)

Table 10. h values of in-control ARL = 300(in-control ARL = 100 in initial state) for the general MEWMAs of r = 0.1 for multivariate normal processes.

	$c = 0.0$		$c = 0.25$		$c = 0.5$		$c = 0.75$	
p	N-5	P-5	N-5	P-5	N-5	P-5	N-5	P-5
2	9 59	9.60	9.20	9.21	8.77	8.79	8.25	8 24
	(7.14)	(7.14)	(6.77)	(6 76)	(6.39)	(6.43)	(6.06)	(6 09)
10	24 09	24.09	20.32	20 36	17.37	17.35	13 82	13.84
	(20.29)	(20.29)	(17.06)	(17 03)	(15.55)	(15 59)	(14 59)	(15.00)

Table 11. ARL performance comparison using h of in−control ARL = 300 for identical multi−variate normal processes in steady state.

p	η_c	MCW	c =0	c =0 25	c =0 5	c =0 75	MCW	c = 0	c =0.25	c =0.5	c =0.75
		Equal Shift									
		N - 0.5					P - 5				
			MEWMA of r = 0 1					MEWMA of r = 0.1			
	0.1	259.5	229.5	226.9	223 8	*220.4	254.9	230.7	228.5	225.1	*219.7
	0.2	169.9	133 0	129 6	124 8	*121.8	167.8	134.0	130.3	127.0	*121.3
2	0.4	61 4	47 8	46.7	45.5	*44.3	60.5	47.8	46.6	45.6	*44.2
	0.8	15 9	15 2	15.0	14.7	*14.3	15.9	15.4	15.1	14.9	*14.4
	1.6	*5 5	6.0	5.9	5.8	5.7	*5 5	6 1	6.0	5.9	5.7
	3.2	*2.5	2.9	2.8	2.8	2.7	*2 5	2 9	2.9	2.8	2.7
	0.1	265 7	247.1	242.2	240.0	*235.3	264 4	247 9	241.8	239.2	*234.3
	0.2	196 4	159.2	153.9	147.6	*142.1	187 3	159 1	153.3	148.2	*141.6
4	0.4	88.3	61.3	59.2	57.3	*54.3	72 0	61 4	58.9	57.4	*53.7
	0.8	24 5	18.6	18.3	17.9	*16.9	17 8	18 6	18.3	18.0	*16.9
	1.6	8.2	7.1	7.0	6.8	*6 4	*6 1	7 1	7.0	6.8	6 4
	3.2	3 7	3.3	3.3	3.2	*3 0	*2 7	3.3	3 3	3.2	3.0
	0.1	284.8	272.2	265.5	264.4	*256 5	278 0	270 3	267.4	264.7	*259.5
	0.2	238.3	201.0	195.2	192.5	*180 6	221 1	202 9	198 2	193.0	*182.1
10	0.4	136.6	88.4	88.4	89 4	*79.3	90.0	88.4	89.1	89.2	*80 2
	0.8	44.2	25.3	27.2	26 8	*22.9	*20 6	25 1	27 1	26.7	22.7
	1 6	13.5	9.0	9.3	8 8	*7 8	*6 9	9.0	9.3	8.8	7 8
	3.2	5.8	4.2	4.1	3.9	*3 5	*3 1	4.2	4 1	3.9	3.5
		Only Shift									
		N - 0 5					P - 5				
			MEWMA of r = 0 1					MEWMA of r = 0 1			
p	η_c	MCW	c = 0	c =0.25	c =0 5	c =0 75	MCW	c = 0	c =0.25	c =0 5	c =0.75
	0.1	246.7	230.7	225.6	219 5	*213 6	249.3	230.2	219.8	210.8	*199.5
	0.2	156.9	134.0	126.8	120.6	*115.8	156.3	134.0	121.4	112.7	*108.7
2	0.4	56.7	47 8	45.7	44.7	*45.2	57.4	47.9	44.3	*44.3	48.1
	0.8	*15.4	15 4	15.4	15.6	16.1	15 4	*15.3	15.8	17.3	20.5
	1.6	*5.4	6.1	6 2	6.3	6.5	*5 5	6.1	6.7	7.6	9.1
	3.2	*2.5	2.9	3.0	3.0	3.1	*2.5	2.9	3.3	3 7	4.3
	0.1	265 0	248.8	229 3	218.0	*212.3	263.1	247.1	226.4	213.7	*208.7
	0.2	183 6	157 6	135.8	*125.4	127 5	184.1	159.0	131.0	*119 4	124.3
4	0.4	70 9	62 1	*53.1	53.9	60 6	69.2	62.0	*51.2	53 0	63.6
	0.8	*17 7	18.7	19.5	22.0	25.9	*17.7	18.6	19.9	23.6	30.8
	1.6	*6 1	7.1	8.2	9.5	10.8	*6.1	7.1	8.8	11.1	14.8
	3.2	*2 8	3.3	4.0	4.5	4.9	*2.8	3.3	4.3	5.5	7.2
	0 1	277.3	270.0	234.9	*233.6	235.3	272.3	267 6	235.0	*229.1	234 5
	0.2	218.5	202.6	148.2	*147.9	162.0	214.1	196.3	*144 3	145.3	161.6
10	0 4	89.2	88.4	*66.9	76.9	92.1	88 1	88.7	*65.4	76.0	93.3
	0.8	*20.6	25.1	29.0	37 4	46.2	*20.5	25 2	29.0	38.3	49 8
	1.6	*6.9	9.0	13.4	17 5	21 1	*6.9	9 0	13 9	19.2	25 5
	3.2	*3.1	4.2	6.5	8 2	9.3	*3.1	4 2	7 0	9 8	12 9

Table 12. ARL performance comparison using h of in−control ARL = 100 for identical multi−variate normal processes in initial state.

		Equal Shift									
		N - 0.5					P - 5				
			MEWMA of r = 0.1					MEWMA of r = 0.1			
p	η_c	MCW	c = 0	c = 0.25	c = 0.5	c = 0.75	MCW	c = 0	c = 0.25	c = 0.5	c = 0.75
2	0.1	90.0	85.0	84.3	81.6	*80.1	90.3	85.4	84.5	83.8	*81.6
	0.2	69.3	57.6	56.5	54.1	*51.7	69.7	57.5	56.2	54.8	*52.7
	0.4	35.5	26.9	25.7	24.4	*23.1	35.3	26.8	25.6	24.6	*23.4
	0.8	12.0	9.8	9.4	9.0	*8.6	12.2	9.9	9.5	9.1	*8.7
	1.6	4.5	3.7	3.6	3.5	*3.3	4.5	3.8	3.6	3.5	*3.4
	3.2	2.1	1.7	1.6	1.6	*1.5	2.1	1.7	1.6	1.6	*1.5
4	0.1	94.0	88.5	86.3	83.5	*80.7	92.6	88.7	86.1	84.4	*81.9
	0.2	78.3	65.9	61.8	57.9	*53.4	76.1	66.2	62.3	58.1	*53.1
	0.4	45.6	32.9	29.6	26.5	*23.8	41.3	33.0	29.6	26.7	*24.1
	0.8	17.5	12.1	10.9	9.9	*9.1	13.8	12.0	10.9	9.9	*9.1
	1.6	6.6	4.5	4.1	3.9	*3.6	5.1	4.5	4.2	3.9	*3.7
	3.2	3.0	2.0	1.9	1.8	*1.7	2.3	2.0	1.9	1.8	*1.7
10	0.1	95.8	91.6	90.0	87.4	*82.8	96.6	92.6	90.4	88.8	*84.7
	0.2	86.4	74.4	69.2	62.2	*56.2	85.0	75.4	69.1	63.5	*56.2
	0.4	60.5	41.6	35.7	30.2	*25.8	50.3	41.9	35.8	30.5	*26.1
	0.8	26.6	16.0	13.1	11.3	*10.2	16.6	15.9	13.0	11.3	*10.2
	1.6	10.3	5.8	5.0	4.5	*4.2	5.9	5.8	5.0	4.5	*4.2
	3.2	4.7	2.5	2.2	2.1	*2.0	2.7	2.5	2.2	2.1	*2.0
		Only Shift									
		N - 0.5					P - 5				
			MEWMA of r = 0.1					MEWMA of r = 0.1			
p	η_c	MCW	c = 0	c = 0.25	c = 0.5	c = 0.75	MCW	c = 0	c = 0.25	c = 0.5	c = 0.75
2	0.1	89.9	85.2	83.9	80.8	*79.3	89.1	84.3	82.1	81.2	*79.1
	0.2	67.9	57.6	55.9	52.7	*50.9	67.6	57.7	54.9	52.4	*50.3
	0.4	33.8	26.4	25.2	23.8	*22.8	33.6	26.3	24.7	23.9	*23.2
	0.8	11.6	9.8	9.4	9.0	*8.6	11.6	9.8	9.5	9.2	*8.9
	1.6	4.4	3.7	3.6	3.5	*3.4	4.4	3.7	3.6	3.6	*3.5
	3.2	2.1	1.6	1.6	1.6	*1.5	2.1	1.6	1.6	1.6	*1.6
4	0.1	93.0	89.0	85.7	82.5	*79.2	92.6	89.0	84.7	81.9	*78.6
	0.2	74.8	67.0	59.9	55.9	*51.6	74.9	66.6	59.0	55.0	*50.9
	0.4	40.5	32.9	28.7	26.4	*24.3	40.0	33.0	28.3	25.9	*24.2
	0.8	13.7	12.1	11.0	10.2	*9.5	13.6	12.1	11.0	10.3	*9.6
	1.6	5.1	4.5	4.2	4.0	*3.8	5.1	4.5	4.2	4.0	*3.8
	3.2	2.3	2.0	1.9	1.8	*1.7	2.3	2.0	1.9	1.8	*1.8
10	0.1	96.1	91.8	85.8	81.6	*75.0	95.2	91.3	84.7	81.3	*74.1
	0.2	83.5	74.5	61.1	54.3	*48.7	82.5	74.4	60.0	53.8	*48.2
	0.4	49.1	41.3	31.0	27.1	*24.4	48.7	41.8	30.9	27.2	*24.3
	0.8	16.4	15.9	13.0	11.5	*10.4	16.3	15.8	12.9	11.5	*10.5
	1.6	5.9	5.8	5.1	4.7	*4.4	5.8	5.8	5.1	4.7	*4.4
	3.2	2.7	2.5	2.3	2.2	*2.1	2.7	2.5	2.3	2.2	*2.1

5. CONCLUSIONS

This study suggests a new approach that uses a multivariate EWMA technique for controlling the mean of multivariate normal processes. It is an extension of the MEWMA chart procedure of Lowry, Woodall, Champ and Rigdon [9] by using a general matrix for the smoothing weight coefficient instead of a diagonal one. Our results show that the MEWMA chart with a full smoothing matrix has superior ARL performance over the diagonal chart scheme for a small shift in the mean vector when departures from control are delayed. It is also attractive in many real applications that process initially breaks out into the out-of-control condition. In this situation, the improvement of the ARL performance is shown to be more pronounced with using the nondiagonal smoothing components in the MEWMA chart procedure not only for a small shift, but for a large change from the in-control mean. The MEWMA chart scheme with a smoothing matrix of appropriate nondiagonal elements demonstrates outstanding performance especially for a small mean shift and is very receptive if the process is initially out-of-control.

The group CUSUM charts MCW and MCZ performs better than the MEWMA charts under a certain QC environment with a large shift in mean. Since the physical meaning of the EWMA vector is not clear, it is difficult to find an interpretation in terms of the original variables for a given out-of-control signal. If one is interested in detecting a shift in the mean of only a particular variable or identifying the out-of-control variable when the shift in the process mean occurs along one of the original variables, the MCZ chart scheme is recommended.

Whereas the diagonal scheme of MEWMA is directionally invariant, the ARL performance of the nondiagonal smoothing scheme is sensitive on the directional change of the shift and has a variation in different correlation measurement characteristics, thereby the resulting in complicating the chart design. Using nondiagonal components for the smoothing matrix also requires computational complexity, however, it offers a practical advantage of improving the performance in detecting a shift in the process mean.

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