Guaranteed Cost Control of Parameter Uncertain Systems with Time Delay※

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Abstract : In this paper, we deal with the problem of designing guaranteed cost state feedback controller for the generalized time-varying delay systems with delayed state and control input. The generalized time delay system problems are solved on the basis of LMI(linear matrix inequality) technique considering time-varying delays. The sufficient condition for the existence of controller and guaranteed cost state feedback controller design methods are presented. Also, using some changes of variables and Schur complements, the obtained sufficient condition can be reformulated as LMI forms in terms of transformed variables. Therefore, all solutions of LMIs, guaranteed cost controller gain, and guaranteed cost are obtained at the same time. The proposed controller design method can be extended into the problem of robust guaranteed cost controller design method for parameter uncertain systems with time-varying delays easily.

Keywords : guaranteed cost control, state feedback, delayed system, LMI

I. Introduction

The stability analysis and control of dynamic systems with time delay are problems of recurring interest as time delays are frequently encountered in many physical processes and very often are the causes for instability and poor performance of control systems[3,6-9]. Also, control system design that can handle model uncertainties has been one of the most challenging problems and received considerable attention from control engineers and scientists in the past decades. Much effort has been directed towards finding a controller in order to guarantee robust stability in spite of parameter uncertainty and time delay.

Since the work of Chang and Peng[2], this issue has been considered extensively in continuous time case[11,13] and discrete time case[5,12], respectively. In particular, Petersen and McFarlane[10] introduced a notion of quadratic guaranteed cost control which extends the notion of quadratic stability to allow for a quadratic performance index and presented a Riccati equation approach for designing quadratic guaranteed cost controllers. However, most of works do not consider time delay. Recently some works considered the guaranteed cost control of non-delay system to uncertain time delay systems and proposed a guaranteed cost control design method. However, the papers do not deal with the time delay and parameter uncertainty simultaneously. Especially, Yu and Chu[14] proposed an LMI approach to guaranteed cost control of linear uncertain time delay systems. However, Yu and Chu[14] considered just state delay and time invariant delay. Therefore, our objective is to design a guaranteed cost controller for parameter uncertain systems with time-varying delays in both state and control input using LMI technique.

In this paper, we consider the guaranteed cost control of parameter uncertain systems with time-varying delays in state and control input. So, we propose the design method on the guaranteed cost control of generalized time-varying delay systems with parameter uncertainties using LMI approach. Also the sufficient condition for the existence of controller is presented. Finally, an example is given to verify the results of proposed method.

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II. Guaranteed cost controller design

Consider a linear system with time-varying delays

$$\begin{aligned} \dot{x}(t) &= Ax(t)+A_d x(t-d_1(t))+Bu(t)+B_d u(t-d_2(t)) \\ x(t) &= \phi(t), \quad t\in[-d,0], \quad d=\max\{d_1(0),d_2(0)\} \end{aligned} \tag{1}$$

where $x(t)\in\mathbf{R}^n$ is the state and $u(t)\in\mathbf{R}^m$ is the control input, And we assume that all states are measurable. In here, time-varying delays are satisfied with

$$0\le d_i(t)<\infty, \quad \dot{d}_i(t)\le\beta_i<1, \quad i=1,2. \tag{2}$$

As a guaranteed cost controller of the time delay system (1), we propose a state feedback law

$$u(t) = Kx(t). \tag{3}$$

Associated with the system (1) is the cost function

$$J=\int_0^\infty [x(t)^T Qx(t)+u(t)^T Ru(t)]dt, \tag{4}$$

where Q and R are positive definite matrices.

Definition 1 : Consider the time delay system (1). if there exist a control law $u^*(t)$ and a positive scalar J^* such that the closed loop system is asymptotically stable and the closed loop value of the cost function (4) satisfies $J\le J^*$, then J^* is said to be a guaranteed cost and $u^*(t)$ is said to be a guaranteed cost control law for the time-varying delay system (1).

When we apply the control (3) to the system (1), the resulting closed loop system is given by

$$\dot{x}(t) = A_K x(t)+A_d x(t-d_1(t))+B_d Kx(t-d_2(t)) \tag{5}$$

where, $A_K=A+BK$.

Theorem 1 : The controller (3) is a guaranteed cost controller if there exist positive definite matrices P, Q, R, S_1 and S_2 such that

$$\begin{bmatrix} \Pi & PA_d & PB_d \\ A_d^TP & -(1-\beta_1)S_1 & 0 \\ B_d^TP & 0 & -(1-\beta_2)S_2 \end{bmatrix} < 0 \tag{6}$$

holds for time delays (2). Here,

$$\Pi = A_K^TP + PA_K + Q + K^TRK + S_1 + K^TS_2K.$$

Proof : Firstly, we define a Lyapunov functional as

$$V(x(t)) := x(t)^TPx(t) + \int_{t-d_1(t)}^{t} x(\tau)^TS_1x(\tau)d\tau + \int_{t-d_2(t)}^{t} x(\tau)^TK^TS_2Kx(\tau)d\tau. \tag{7}$$

Taking the derivative of the Lyapunov functional (7) along the solution of (5) yields

$$\begin{aligned} \dot V(x(t)) &= \dot x(t)^TPx(t) + x(t)^TP\dot x(t) \\ &+ x(t)^TS_1x(t) + x(t)^TK^TS_2Kx(t) \\ &-(1-\dot d_1(t))x(t-d_1(t))^TS_1x(t-d_1(t)) \\ &-(1-\dot d_2(t))x(t-d_2(t))^TK^TS_2Kx(t-d_2(t)), \end{aligned} \tag{8}$$

which is negative definite when the matrix[15]

$$\begin{aligned} \dot V_a(x(t)) &= x(t)^TPx(t) + x(t)^TPx(t) \\ &+ x(t)^TS_1x(t) + x(t)^TK^TS_2Kx(t) \\ &-(1-\beta_1)x(t-d_1(t))^TS_1x(t-d_1(t)) \\ &-(1-\beta_2)x(t-d_2(t))^TK^TS_2Kx(t-d_2(t)) < 0. \end{aligned} \tag{9}$$

The matrix inequality (6) implies that

$$\dot V(x(t)) \le \dot V_a(x(t)) < x(t)^T(-Q-K^TRK)x(t) < 0. \tag{10}$$

Therefore, we have

$$\begin{bmatrix} x(t) \\ x(t-d_1(t)) \\ Kx(t-d_2(t)) \end{bmatrix}^T \begin{bmatrix} \Pi & PA_d & PB_d \\ A_d^TP & -\tilde S_1 & 0 \\ B_d^TP & 0 & -\tilde S_2 \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d_1(t)) \\ Kx(t-d_2(t)) \end{bmatrix} < 0 \tag{11}$$

which ensures the asymptotical stability of the closed loop system (5). Here, $\tilde S_i = (1-\beta_i)S_i$, $i=1,2$.

Furthermore, by the integrating both sides of the inequality (10) from 0 to T and using the initial condition, we obtain

$$\begin{aligned} -\int_0^T x(t)^T(Q+K^TRK)x(t)dt & \\ > x(T)^TPx(T) - x(0)^TPx(0) & \\ + \int_{T-d_1(T)}^{T} x(\tau)^TS_1x(\tau)d\tau & \\ - \int_{-d_1(0)}^{0} x(\tau)^TS_1x(\tau)d\tau & \\ + \int_{T-d_2(T)}^{T} x(\tau)^TK^TS_2Kx(\tau)d\tau & \\ - \int_{-d_2(0)}^{0} x(\tau)^TK^TS_2Kx(\tau)d\tau. & \end{aligned} \tag{12}$$

As the closed loop system (5) is asymptotically stable, when $T \to \infty$,

$$\begin{aligned} & x(T)^TPx(T) \to 0, \\ & \int_{T-d_1(T)}^{T} x(\tau)^TS_1x(\tau)d\tau \to 0, \\ & \int_{T-d_2(T)}^{T} x(\tau)^TK^TS_2Kx(\tau)d\tau \to 0. \end{aligned} \tag{13}$$

Hence. we get

$$\begin{aligned} & \int_0^\infty x(t)^t(Q+K^TRK)x(t)dt \\ & \le \phi(0)^TP\phi(0) + \int_{-d_1(0)}^{0} \phi(\tau)^TS_1\phi(\tau)d\tau \\ & + \int_{-d_2(0)}^{0} \phi(\tau)^TK^TS_2K\phi(\tau)d\tau. \end{aligned} \tag{14}$$

It follows from the Definition 1 that the result of Theorem 1 is true. This completes the proof. ■

In the following, we prove that the above sufficient condition for the existence of guaranteed cost controllers is equivalent to the solvability of a system of LMIs.

Theorem 2 : Consider the time delay system (1). If there exist positive definite matrices X, Y_1, Y_2, Z, U and a matrix M such that

$$\begin{bmatrix} \Gamma & M^T & M^T & X & X \\ * & -Z & 0 & 0 & 0 \\ * & * & -Y_2 & 0 & 0 \\ * & * & * & -Y_1 & 0 \\ * & * & * & * & -U \end{bmatrix} < 0 \tag{15}$$

holds for the time delays (2), then the state feedback control law

$$u^*(t) = MX^{-1}x(t) \tag{16}$$

is a guaranteed cost control law and

$$J_1^* = \phi(0)^TX^{-1}\phi(0) + \int_{-d_1(0)}^{0} \phi(\tau)^TY_1^{-1}\phi(\tau)d\tau + \int_{-d_2(0)}^{0} \phi(\tau)^TK^TY_2^{-1}K\phi(\tau)d\tau \tag{17}$$

is a guaranteed cost for the time delay system (1). Here, $\Gamma = XA^T + AX + M^TB^T + BM + A_d\tilde Y_1A_d^T + B_d\tilde Y_2B_d^T$, $\tilde Y_i = (1-\beta_i)^{-1}Y_i$, $i=1,2$ and $*$ mean symmetric terms.

Proof : Using Schur complements and some changes of variables, the proof is completed. The inequality of (11) is equivalent to

$$\begin{bmatrix} A_K^TP+PA_K & PA_d & PB_d & K^T & K^T & I & I \\ * & -\tilde S_1 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\tilde S_2 & 0 & 0 & 0 & 0 \\ * & * & * & -R^{-1} & 0 & 0 & 0 \\ * & * & * & * & -S_2^{-1} & 0 & 0 \\ * & * & * & * & * & -S_1^{-1} & 0 \\ * & * & * & * & * & * & -Q^{-1} \end{bmatrix} < 0 \tag{18}$$

$$\Leftrightarrow \begin{bmatrix} \Theta & K^T & K^T & I & I \\ * & -R^{-1} & 0 & 0 & 0 \\ * & * & -S_2^{-1} & 0 & 0 \\ * & * & * & -S_1^{-1} & 0 \\ * & * & * & * & -Q^{-1} \end{bmatrix} < 0 \tag{19}$$

$$\Leftrightarrow \begin{bmatrix} \Upsilon & P^{-1}K^T & P^{-1}K^T & P^{-1} & P^{-1} \\ * & -R^{-1} & 0 & 0 & 0 \\ * & * & -S_2^{-1} & 0 & 0 \\ * & * & * & -S_1^{-1} & 0 \\ * & * & * & * & -Q^{-1} \end{bmatrix} < 0 \tag{20}$$

where, I is identity matrix with proper dimension, $\Theta = A_K^TP + PA_K + PA_d\ \tilde S_1^{-1}A_d^TP + PB_d\ \tilde S_2^{-1}B_d^TP$ and

$\Upsilon = P^{-1}A_K^T + A_K P^{-1} + A_d\ \tilde{S}_1^{-1}A_d^T + B_d\ \tilde{S}_2^{-1}B_d^T$. Using some changes of variables, $M=KP^{-1}$, $X=P^{-1}$, $Y_i=S_i^{-1}$, $i=1,2$, $Z=R^{-1}$ and $U=Q^{-1}$, (20) is changed to (15). ■

The matrix inequality (15) is an LMI form in terms of X, Y_1, Y_2, Z, U and M. Therefore the guaranteed cost state feedback controller gain K can be calculated from the $M=KP^{-1}$ after finding the LMI solutions subject to minimization of guaranteed cost J^*. Using LMI Toolbox[4], the solutions can be easily obtained at the same time because (15) is an LMI form in terms of variables. In particular, the optimal guaranteed control law which minimizes the value of the guaranteed cost for the closed loop system can be determined by solving the following optimization problem.

Theorem 3 : Consider the time-varying delay system (1). If the following optimization problem

$$min\{\alpha + tr(G_1) + tr(N_2^T G_2 N_2)\} \text{ subject to} \tag{21}$$

$$i)\ \text{LMI (15)},$$

$$ii)\ \begin{bmatrix} -\alpha & \phi(0)^T \\ \phi(0) & -X \end{bmatrix} < 0,$$

$$iii)\ \begin{bmatrix} -G_1 & N_1^T \\ N_1 & -Y_1 \end{bmatrix} < 0,$$

$$iv)\ \begin{bmatrix} -G_2 & M^T \\ M & -Y_2 \end{bmatrix} < 0,$$

$$v)\ X > I$$

has a solution positive definite matrices(or scalar) X, Y_1, Y_2, Z, U, G_1, G_2, α, and a matrix M, then (16) is a guaranteed cost control law which ensures the minimization of the guaranteed cost satisfying $J \le J^*$ for the time delay system (1). Here, $tr(\cdot)$ denotes the trace of the matrix $(\cdot)$ and $\int_{-d_i(0)}^{0} \phi(\tau)\phi(\tau)^T d\tau = N_i N_i^T$, $i=1,2$.

Proof : The procedures of proof are summarized as follows:

i) By Theorem 2, the proof is completed.

ii) The (ii) in (21) is equivalent to $\phi(0)^T X^{-1}\phi(0) < \alpha$.

iii) The second term of right hand side in (17) has the following relations.

$$\begin{aligned} &\int_{-d_1(0)}^{0} \phi(\tau)^T Y_1^{-1}\phi(\tau)d\tau \\ &= \int_{-d_1(0)}^{0} tr(\phi(\tau)^T Y_1^{-1}\phi(\tau))d\tau \\ &= tr(N_1 N_1^T Y_1^{-1}) = tr(N_1^T Y_1^{-1} N_1) < tr(G_1) \end{aligned} \tag{22}$$

Therefore $-G_1 + N_1^T Y_1^{-1} N_1 < 0$ is equivalent to (iii) in (21).

(iv) The third term of right hand side in (17) has the following relations.

$$\begin{aligned} &\int_{-d_2(0)}^{0} \phi(\tau)^T K^T Y_2^{-1} K\phi(\tau)d\tau \\ &= \int_{-d_2(0)}^{0} tr(\phi(\tau)^T K^T Y_2^{-1} K\phi(\tau))d\tau \\ &= tr(N_2 N_2^T K^T Y_2^{-1} K) = tr(N_2^T K^T Y_2^{-1} K N_2) \\ &< tr(N_2^T P G_2 P N_2). \end{aligned} \tag{23}$$

Therefore $-PG_2P + K^T Y_2^{-1} K < 0$ is equivalent to

$$\begin{bmatrix} -PG_2P & K^T \\ K & -Y_2 \end{bmatrix} < 0 \tag{24}$$

$$\Leftrightarrow \begin{bmatrix} -G_2 & P^{-1}K^T \\ KP^{-1} & -Y_2 \end{bmatrix} < 0. \tag{25}$$

Therefore, (25) is changed into (iv) in (21) by $M=KP^{-1}$.

(v) So, it follows from (17) that

$$\begin{aligned} J_1 &< \alpha + tr(G_1) + tr(N_2^T P G_2 P N_2) \\ &< \alpha + tr(G_1) + tr(N_2^T G_2 N_2) \\ &:= J^* \end{aligned} \tag{26}$$

because of the condition (v) in (21). In other words,

$$\begin{aligned} &PG_2P < G_2 \\ \Leftrightarrow\ &XG_2X > G_2\ (\text{ by } P^{-1}=X) \\ \Leftrightarrow\ &(X-I)G_2(X+I) > 0\ (\text{ by } X > I \text{ and } G_2 > 0). \end{aligned} \tag{27}$$

Until now, we just discuss generalized time delay system without parameter uncertainties. In the following theorem, the guaranteed cost controller design method of parameter uncertain system with time delays is given using the proposed method.

Theorem 4 : Consider a parameter uncertain system with time delay

$$\begin{aligned} \dot{x}(t) &= [A+\Delta A(t)]x(t) + [A_d + \Delta A_d(t)]x(t-d_1(t)) \\ &\quad + [B+\Delta B(t)]u(t) + [B_d + \Delta B_d(t)]u(t-d_2(t)) \\ x(t) &= \phi(t),\ t\in[-d,0],\ d=\max\{d_1(0), d_2(0)\} \end{aligned} \tag{28}$$

and parameter uncertainties are defined as

$$\begin{aligned} \Delta A(t) &= D_1F_1(t)E_1 \\ \Delta A_d(t) &= D_2F_2(t)E_2 \\ \Delta B(t) &= D_3F_3(t)E_3 \\ \Delta B_d(t) &= D_4F_4(t)E_4 \end{aligned} \tag{29}$$

and unknown matrix is defined as

$$\begin{aligned} F_i(t)\in\Omega := \{&F_i(t):\ F_i(t)^T F_i(t) \le I, \\ &\text{the elements of } F(t) \\ &\text{are Lebesgue measurable},\ i=1,2,3,4\}. \end{aligned} \tag{30}$$

If there exist positive definite matrices(or scalars) X, Y_1, Y_2, Z, U, ε_i, $i=1,2,3,4$, and a matrix M such that

$$\begin{bmatrix} \Phi_1 & XE_1^T & M^TE_3^T & M^T & M^T & X & X & A_d\ \tilde{Y}_1E_2^T & B_d\ \tilde{Y}_2E_4^T \\ * & -\varepsilon_1 I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Y_2 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -Z & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -Y_1 & 0 & 0 & 0 \\ * & * & * & * & * & * & -U & 0 & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I + E_2\ \tilde{Y}_1E_2^T & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_1 I + E_1\ \tilde{Y}_2E_4^T \end{bmatrix} < 0 \tag{31}$$

holds for time-varying delay (2) and parameter uncertaintics

(29), then (16) is a guaranteed cost control law and (17) is a guaranteed cost for the time delay system (28). Here,

$$\Phi_1 = XA^T + AX + M^T B^T + BM + \sum_{i=1}^{4} \varepsilon_i D_i D_i^T + A_d \tilde{Y}_1 A_d^T + B_d \tilde{Y}_2 B_d^T.$$

Proof : Using the cost function (4), Lyapunov functional (7), and the following lemma

$$2x(t)^T PDF(t)Ex(t) \le \varepsilon x(t)^T PDD^T Px(t) + \frac{1}{\varepsilon} x(t)^T E^T Ex(t) \quad (32)$$

then the sufficient condition is obtained similarly to the proof of theorem 2 as follows:

$$\begin{bmatrix} \Phi_2 & PA_d & PB_d \\ * & \frac{1}{\varepsilon_2} E_2^T E_2 - \tilde{S}_1 & 0 \\ * & * & \frac{1}{\varepsilon_4} E_4^T E_4 - \tilde{S}_2 \end{bmatrix} < 0 \quad (33)$$

where,

$$\Phi_2 = (A+BK)^T P + P(A+BK) + S_1 + K^T S_2 K + Q + K^T RK + \varepsilon_1 PD_1 D_1^T P + \frac{1}{\varepsilon_1} E_1^T E_1 + \varepsilon_3 PD_3 D_3^T P + \frac{1}{\varepsilon_3} K^T E_3^T E_3 K + \varepsilon_2 PD_2 D_2^T P + \varepsilon_4 PD_4 D_4^T P.$$

And then, using Schur complements, some changes of variables, $M = KP^{-1}$, $X = P^{-1}$, $Z = R^{-1}$, $U = Q^{-1}$, $Y_i = S_i^{-1}$, $i = 1, 2$, and matrix inversion lemma

$$\begin{aligned} &(\tilde{S}_1 - \frac{1}{\varepsilon_2} E_2^T E_2)^{-1} \\ &= \tilde{S}_1^{-1} + \tilde{S}_1^{-1} E_2^T (\varepsilon_2 I - E_2 \tilde{S}_1^{-1} E_2^T)^{-1} E_2 \tilde{S}_1^{-1}, \\ &(\tilde{S}_2 - \frac{1}{\varepsilon_4} E_4^T E_4)^{-1} \\ &= \tilde{S}_2^{-1} + \tilde{S}_2^{-1} E_4^T (\varepsilon_4 I - E_4 \tilde{S}_2^{-1} E_4^T)^{-1} E_4 \tilde{S}_2^{-1}, \end{aligned} \quad (34)$$

the matrix inequality (33) is changed to LMI (31) in terms of X, Y_1, Y_2, Z, U, ε_i, $i = 1, 2, 3, 4$, and M. ■

Also, we can get the optimal control law which minimizes the value of guaranteed cost for the parameter uncertain closed loop system by solving the following optimization problem.

Theorem 5 : Consider the parameter uncertain time-varying delay systems (28). If the following optimization problem

$$\min\{\alpha + tr(G_1) + tr(N_2^T G_2 N_2)\} \text{ subject to} \quad (35)$$

i) LMI (31),

ii) $\begin{bmatrix} -\alpha & \phi(0)^T \\ \phi(0) & -X \end{bmatrix} < 0,$

iii) $\begin{bmatrix} -G_1 & N_1^T \\ N_1 & -Y_1 \end{bmatrix} < 0,$

iv) $\begin{bmatrix} -G_2 & M^T \\ M & -Y_2 \end{bmatrix} < 0,$

v) $X > I$

has a solution positive definite matrices(or scalars) X, Y_1, Y_2, Z, U, G_1, G_2, α, ε_1, ε_2, ε_3, ε_4, and a matrix M, then (16) is a guaranteed cost control law which ensures the minimization of the guaranteed cost satisfying $J \le J^*$ for the parameter uncertain time-varying delay system (28).

Proof. The procedures of proof are the same as those of Theorem 3. ■

Example : Consider a parameter uncertain system with time-varying delays in state and control input

$$\begin{aligned} \dot{x}(t) &= \left\{\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} F_1(t) [1 \ 1]\right\} x(t) \\ &+ \left\{\begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix} + \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} F_2(t) [1 \ 1]\right\} x(t - d_1(t)) \\ &+ \left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} F_3(t)\right\} u(t) \\ &+ \left\{\begin{bmatrix} 0.1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} F_4(t)\right\} u(t - d_2(t)), \\ d_1(t) &= 3 + 0.1 \sin t, \ d_2(t) = 5 + \sin(0.4t), \\ \phi(t) &= [e^{t+1} \ 0]^T. \end{aligned} \quad (36)$$

All solutions are obtained in (35) simultaneously as follows:

$$\begin{aligned} X &= 10^7 \times \begin{bmatrix} 2.2897 & -0.8203 \\ -0.8203 & 2.7302 \end{bmatrix}, \\ Y_1 &= 10^8 \times \begin{bmatrix} 2.4326 & -1.5914 \\ -1.5914 & 1.7524 \end{bmatrix}, \\ Y_2 &= 1.7511 \times 10^8, \\ Z &= 4.6929 \times 10^8, \\ U &= 10^3 \times \begin{bmatrix} 2.5934 & -1.2321 \\ -1.2321 & 2.0896 \end{bmatrix}, \\ M &= 10^7 \times [0.0000 \ -9.7925], \\ \varepsilon_1 &= 1.0174 \times 10^8, \quad \varepsilon_2 = 1.6253 \times 10^8, \\ \varepsilon_3 &= 4.3608 \times 10^8, \quad \varepsilon_4 = 5.1448 \times 10^8, \\ \alpha &= 3.6180 \times 10^{-7}, \\ G_1 &= 3.7556 \times 10^{-8}, \\ G_2 &= 10^7 \times \begin{bmatrix} 0.0000 & -0.0000 \\ -0.0000 & 6.4650 \end{bmatrix}. \end{aligned} \quad (37)$$

By applying Theorem 4 and Theorem 5, the guaranteed cost state feedback law is

$$u^*(t) = [-1.4400 \ -4.0194] x(t), \quad (38)$$

and the guaranteed cost of parameter uncertain system with time-varying delays is

$$J^* = 3.9981 \times 10^{-7}. \quad (39)$$

III. Conclusions

In this paper, we proposed guaranteed cost controller design methods for the generalized time-varying delay systems with delayed state and control input. The existence condition and the design methods of guaranteed cost controller were given. Also, the guaranteed cost bound was presented. Through some techniques, the sufficient condition was changed into an LMI form in terms of finding variables. A guaranteed cost state feedback controller could be easily obtained using LMI toolbox. Furthermore, we presented the robust guaranteed cost controller design method and guaranteed cost bound for parameter uncertain system with time-varying delays in state and control input using the proposed method. Also, all solutions of LMIs, guaranteed cost controller gain, and upper bound of guaranteed cost were obtained at the same time. Through a numerical example, we checked the validity of the proposed method.

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