Control of Active Suspension System by Using H_∞ Theory

Tan Tien Nguyen, Van Giap Nguyen, and Sang Bong Kim

Abstract : This paper presents a control of active suspension for quarter car model with two degree of freedom by using H_∞ method. Absolute velocity of car body is measured for feedback. The system parameter variations are treated with multiplicative uncertainty model Simulation results show that the H_∞ controller provides good trade-off between ride quality, suspension packaging and road holding constraints. The experiment with a front wheel suspension system was done to verify the simulation results.

Keywords : H_∞ control problem, H_∞ controller, uncertainty, weighting function

I. Introduction

Automotive suspension systems have considerable importance in vehicle design. The designed suspension must provides for a trade-off between several competing objectives: passenger comfort, small suspension stroke for packing, and small tire deflection for vehicle handling. The passenger comfort index is judged by the acceleration of the car body. The next ones are suspension and tire deflection indices. They usually consist of a spring and a damper. With only mechanical construction, the passive suspension systems have their limit properties. In the interest of improving the overall ride performances of automotive vehicle in recent years, suspensions incorporating active components have been developed and studied by many researchers (C. Yue et al., 1989 with LQR; A.G. Ulsoy and D. Hrovat, 1990 with LQG; M. Yamashita et al., 1994 with H_∞; W.Y. Jeong, et al., 1996 with H_∞/H_2 ; H.S. Kim et al., 1997 with PI Observer).

Because the active suspension systems must provide a trade-off between several competing objectives, in the early stages, linear quadratic regulator theory became one of the standard techniques for deriving optimal control laws based on full state feedback. It is based on the minimization of some weights combination of passenger comfort indices (acceleration and jerk), suspension deflection and tire deflection. In order to implement the active suspension system by using quadratic controller such as optimal controller, the full states information of the system should be required. But, all the states cannot be measured in the real system. Therefore, many kinds of observer are used to estimate the states of the system. The observer based control system must guarantee the robustness properties in the presence of the change of parameters. And the attention gradually turned to more realistic systems based on limited state feedback and recent research has looked at H_∞ and fuzzy logic approaches[8].

In this paper, we use the H_∞ control method to minimize the worst case of the transfer function from the disturbance to the vertical absolute velocity of the car body under the existence of the parameter variations. After obtaining the H_∞ controller, three interest transfer functions from road disturbance to car body acceleration, suspension deflection, and tire deflection are plotted: the plotted results are compared to the passive case to show the effectiveness of H_∞ controller. In the experimental apparatus, the road disturbance is generated by a cam shaft. By changing the revolution of cam shaft, the road disturbances are given with different frequencies. Measuring the suspension deflection, tire deflection and calculating the car body acceleration from measured car body displacement, then we can derive the gains of three interest transfer functions. These transfer functions are plotted to verify the simulation results. The results showed us that H_∞ controller reduces considerably the gains of three interest transfer functions at frequency region around 1 Hz.

II. Two degree of freedom quarter car model

The quarter car model is shown in the Fig. 1. It can be used to quantify the comparative ride performance of passive system.

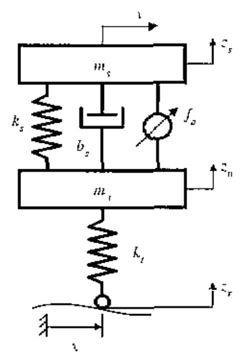

Fig. 1. Suspension system.

Here we defined parameters as follows

- m_s : sprung mass [Kg]
- m_u : unsprung mass [Kg]
- b_s : damping coefficient [Ns/m]
- k_s : spring stiffness coefficient [N/s]
- k_t : tire stiffness coefficient [N/m]
- f_a : active force [N]
- z_s : vertical position of sprung mass [m]
- z_u : vertical position of unsprung mass [m]
- z_r : vertical position of road [m]

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x : horizontal position of car [m]
v : horizontal velocity of car [m]

Assume that the suspension spring coefficient and tire stiffness coefficient are linear in their operation range; the tire does not leave the ground; and z_s and z_u are measured from the static equilibrium position. From the scheme of the system model in the Fig. 1, the equations of motion can be derived as the following

$$m_s\ddot{z}_s + b_s(\dot{z}_s - \dot{z}_u) + k_s(z_s - z_u) = f_a \quad (1)$$

$$m_u\ddot{z}_u + b_s(\dot{z}_u - \dot{z}_s) + k_s(z_u - z_s) + k_t(z_u - z_r) = -f_a \quad (2)$$

Let us define a state variables as follows

$x_1 = z_s - z_u$: suspension deflection
$x_2 = \dot{z}_s$: absolute velocity of sprung mass
$x_3 = z_u - z_r$: tire deflection
$x_4 = \dot{z}_u$: absolute velocity of unsprung mass
$u = f_a$: scalar active force
$d = \dot{z}_r$: road disturbance input

Then, equations (1) and (2) can be rewritten

$$\dot{x}_p = A_p x_p + B_p u + \Gamma d \quad (3)$$

and the measured output

$$y_p = C_p x_p \quad (4)$$

where, $x_p = [x_1\ x_2\ x_3\ x_4]^T$,

$$A_p = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_u} & \frac{b_s}{m_u} & -\frac{k_t}{m_u} & -\frac{b_s}{m_u} \end{bmatrix}$$

$$B_p = \begin{bmatrix} 0 \\ \frac{1}{m_s} \\ 0 \\ -\frac{1}{m_u} \end{bmatrix}, \Gamma = \begin{bmatrix} 0 \\ 0 \\ -1 \\ 0 \end{bmatrix}$$

$C_p = [0\ 1\ 0\ 0]$

III. Transfer functions of the system

Three interest performance variables are : body vibration isolation, measured by the sprung mass acceleration $\ddot{z}_s$; suspension travel, measured by the deflection of suspension $z_s - z_u$; and the tire load constancy, measured by tire deflection $z_u - z_r$. Then, three considered transfer functions from disturbance $\dot{z}_r$ to the acceleration of sprung mass $H_A(s)$, to the suspension deflection $H_{SD}(s)$, and to the tire deflection $H_{TD}(s)$ can be derived as the following

$$H_A(s) = \frac{\ddot{Z}_s(s)}{\dot{Z}_r(s)} = \frac{\dot{X}_2(s)}{\dot{Z}_r(s)} \quad (5)$$

$$H_{SD}(s) = \frac{Z_s(s) - Z_u(s)}{\dot{Z}_r(s)} = \frac{X_1(s)}{\dot{Z}_r(s)} \quad (6)$$

$$H_{TD}(s) = \frac{Z_u(s) - Z_r(s)}{\dot{Z}_r(s)} = \frac{X_3(s)}{\dot{Z}_r(s)} \quad (7)$$

We will consider these transfer functions in the case of using no feedback(passive) and in the case of H_∞ control using measurement unsprung mass velocity feedback.

1. Passive system

Three transfer function (5)-(7) can be written as

$$H_A(s) = sE_2(sI - A_p)^{-1}\Gamma \quad (8)$$

$$H_{SD}(s) = E_1(sI - A_p)^{-1}\Gamma \quad (9)$$

$$H_{TD}(s) = E_3(sI - A_p)^{-1}\Gamma \quad (10)$$

where $E_1 = [1\ 0\ 0\ 0]$, $E_2 = [0\ 1\ 0\ 0]$ and $E_3 = [0\ 0\ 1\ 0]$ are the controlled output matrices.

2. H_∞ control :

The augmented system for H_∞ control problem is given in the Fig. 2.

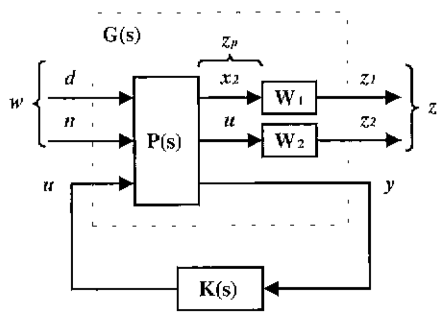


Fig. 2. Augmented system.

The state space expression of the plant $P(s)$ with adding measurement noise can be rewritten as follows

$$\dot{x}_p = A_p x_p + B_{p1} w + B_{p2} u \quad (11)$$

$$z_p = C_{p1} x_p + D_{p11} w + D_{p12} u \quad (12)$$

$$y_p = C_{p2} x_p + D_{p21} w + D_{p22} u \quad (13)$$

where,

$w = \begin{bmatrix} d \\ n \end{bmatrix}$ (d: disturbance, n: noise)

$z_p = \begin{bmatrix} x_2 \\ u \end{bmatrix}$ (u: control input-active force)

$B_{p1} = [\Gamma\ 0]$, $B_{p2} = B_p$

$C_{p1} = \begin{bmatrix} C_{p11} \\ C_{p12} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $C_{p2} = C_p$

$D_{p11} = \begin{bmatrix} D_{p111} \\ D_{p112} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$D_{p12} = \begin{bmatrix} D_{p121} \\ D_{p122} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$D_{p21} = [0\ 1]$, and $D_{p22} = [0]$

Assume that the weight W_1 on x_2 has dynamic equations

$$\dot{x}_w = A_w x_w + B_w x_2 \quad (14)$$

$$z_1 = C_w x_w + D_w x_2 \quad (15)$$

and the weight W_2 on u is scalar value α.

Straight forwardly, we can express the augmented system $G(s)$ which is shown in Fig. 2 with dotted line as follows

$$x = Ax + B_1 w + B_2 u \quad (16)$$

$$z = C_1 x + D_{11} w + D_{12} u \quad (17)$$

$$y = C_2 x + D_{21} w + D_{22} u \quad (18)$$

where,

$$x = \begin{bmatrix} x_p \\ x_w \end{bmatrix}, \quad z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}, \quad y = y_p$$

$$A = \begin{bmatrix} A_p & 0 \\ B_u C_{p11} & A_w \end{bmatrix}$$

$$B_1 = \begin{bmatrix} B_{p1} \\ B_w D_{p111} \end{bmatrix}, \quad B_2 = \begin{bmatrix} B_{p2} \\ B_w D_{p121} \end{bmatrix}$$

$$C_1 = \begin{bmatrix} D_w C_{p11} & B_w \\ \alpha C_{p12} & 0 \end{bmatrix}, \quad C_2 = [C_{p2} \ 0]$$

$$D_{11} = \begin{bmatrix} D_w D_{p111} \\ \alpha D_{p112} \end{bmatrix}, D_{12} = \begin{bmatrix} D_w D_{p121} \\ \alpha D_{p122} \end{bmatrix}$$

$D_{21} = D_{p21}$, and $D_{22} = D_{p22}$

The H_∞ control problem is formulated as follows: consider the two-port diagram in Fig. 2, and find an internal stabilizing controller, $K(s)$, for the augmented system, $G(s)$, such that the ∞-norm of the closed loop transfer function, T_{zw}, is below a given positive scalar γ:

$$\underset{K(s)\,stabilizing}{Find} \quad \| T_{zw} \|_\infty \le \gamma \quad (19)$$

For the problem to have a solution, the following condition must be satisfied [2]

i) (A, B_2) is stabilizable and (C_2, A) is detectable

ii) D_{12} is full column rank and D_{21} is full row rank

iii) $\begin{bmatrix} A - j\omega I & B_2 \\ C_1 & D_{12} \end{bmatrix}$ has full column rank for all ω

iv) $\begin{bmatrix} A - j\omega I & B_1 \\ C_2 & D_{21} \end{bmatrix}$ has full row rank for all ω

v) $D_{11} = 0$ and $D_{22} = 0$

And the H_∞ controller $K(s)$ is

$$K(s) = \begin{bmatrix} A_K & B_K \\ C_K & D_K \end{bmatrix} = \begin{bmatrix} \hat{A}_\infty & -Z_\infty L_\infty \\ F_\infty & 0 \end{bmatrix} \quad (20)$$

where,

$$\hat{A}_\infty = A + \gamma^{-2} B_1 B_1^T X_\infty + B_2 F_\infty + Z_\infty L_\infty C_2$$

$$F_\infty = -B_2^T X_\infty, \quad L_\infty = -Y_\infty C_2^T$$

$$Z_\infty = (I - \gamma^{-2} Y_\infty X_\infty)^{-1}$$

$$X_\infty = Ric\begin{bmatrix} A & \gamma^{-2} B_1 B_1^T - B_2 B_2^T \\ -C_1^T C_1 & -A^T \end{bmatrix} \ge 0$$

$$Y_\infty = Ric\begin{bmatrix} A^T & \gamma^{-2} C_1^T C_1 - C_2^T C_2 \\ -B_1 B_1^T & -A \end{bmatrix} \ge 0$$

$$\rho(X_\infty Y_\infty) < \gamma^2$$

The change of the parameters of the system is treated by multiplicative uncertainty model $\Delta(s)$. It is derived from the nominal plant $P_n(s)$ and the perturbed plant $P_p(s)$ as follows

$$\Delta(s) = \frac{P_p(s)}{P_n(s)} - 1 \quad (21)$$

The weighting function $W_1(j\omega)$ is chosen to satisfy

$$\sigma[\Delta(j\omega)] < | W_1(j\omega) |, \forall \omega \quad (22)$$

And from the small gain theorem, the robust stability is assured if

$$\| T_{zw} \|_\infty = \| W T_{z_p w} \|_\infty < 1 \quad (23)$$

Then, if the value of γ in the above solution satisfies $\gamma < 1$, the closed-loop system achieves robust stability in the presence of the possible perturbation.

The transfer function from disturbance to the state of the augmented system is

$$T_{xd} = \{sI - [A + B_2 K(s) C_2]\}^{-1} [B_1 + B_2 K(s) D_{21}] \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where the transfer function of the controller $K(s)$ is

$$K(s) = C_K (sI - A_K)^{-1} B_K + D_K$$

with A_K, B_K, C_K, D_K which are given by Eq. (20). And three transfer functions (5)-(7) become

$$H_A(s) = s[E_2 \ 0] T_{xd} \quad (24)$$

$$H_{SD}(s) = [E_1 \ 0] T_{xd} \quad (25)$$

$$H_{TD}(s) = [E_3 \ 0] T_{xd} \quad (26)$$

IV. Simulation and experimental results

The numerical values of system parameters used in simulation and experiment are given in Table 1.

Table 1. Numerical values of suspension system.

Parameters		Values	Units
Sprung mass	m_s	253	Kg
Unsprung mass	m_u	42	Kg
Damping coefficient	b_s	1,345	N.s/m
Spring stiffness coefficient	k_s	16,434	N/m
Tire stiffness coefficient	k_t	97,939	N/m

The transfer function using the parameters identified above is regarded as a nominal plant, and the transfer function using parameters that vary within some ranges are regarded as perturbed plant. Here the change of parameters is assumed to be :

1) the change of sprung mass (car body mass) includes passengers and luggage weights Δ_{m_s}

2) when the spring, damping, and tire stiffness coefficients are measured, usually these errors are about $\pm$ 10% of the given values

Then, we will assume those error ranges as follows

$$\Delta_{m_s} = 100 \text{ Kg}, \qquad \Delta_{k_s} = \pm 10\% \, k_s,$$

$$\Delta_{b_s} = \pm 10\% \, b_s, \qquad \Delta_{k_t} = \pm 10\% \, k_t.$$

With the changes of parameters as above, we calculate the nominal transfer function $P_n(s)$ and the perturbed transfer function $P_p(s)$. Then, the uncertainties $\Delta(j\omega)$ are calculated. The weighting function $W_1(j\omega)$ is chosen so as to satisfy Eq. (22).

The plots of the uncertainties $\Delta(j\omega)$ for the variations of parameters and the weighting function $W_1(j\omega)$ are given in the Fig. 3.

In this paper, after some troublesome calculations, the weighting function $W_1(j\omega)$ and W_2 are chosen as

$$W(j\omega)=\begin{bmatrix} W_1(j\omega) & 0 \\ 0 & W_2 \end{bmatrix}=\begin{bmatrix} \frac{15.6s+48.75}{4.65s+145} & 0 \\ 0 & 0.00035 \end{bmatrix}$$

and the controller $K(s)$ is calculated with the value of $\gamma=0.99$.

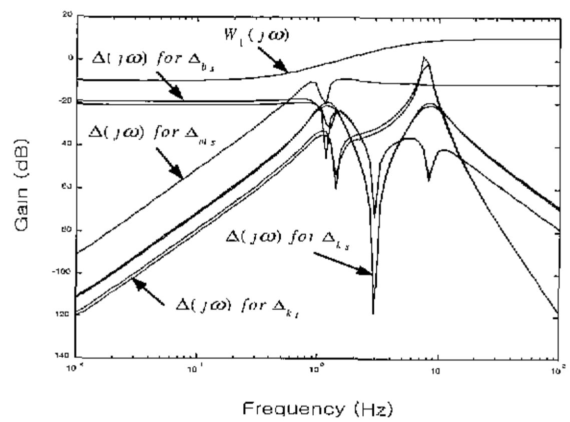
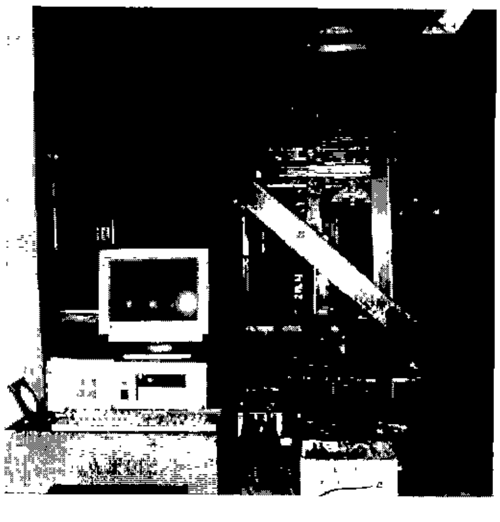


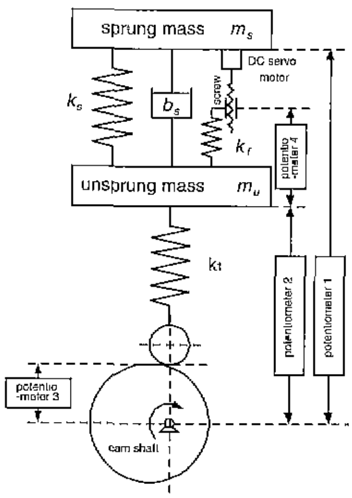
Fig. 3. Plots of uncertainties and weighting function.

The schematic diagram of experimental apparatus and the photograph are given in the Fig. 4.

The road disturbance is generated by cam shaft. Road disturbance input is assumed in the interval 0.5Hz-12Hz and is made by changing the velocity of the cam shaft. Four potentiometers are used to measure the displacements of sprung mass, unsprung mass and road variation, screw



(a)



(b)

Fig. 4. The experimental apparatus : (a) photograph and (b) schematic diagram.

displacement as shown in the Fig. 4. The actuator is driven by a DC servo motor(LG, Model FMD-E30TA, 300W, 75V, 4.8A). The actuating force for active suspension is generated by rotating the screw.

At each frequency, we measure displacement of sprung mass and unsprung mass. The vertical change of the road is measured by vertical displacement of cam follower and then road disturbance is derived. From the displacement of sprung mass, car body velocity and acceleration are derived. From sprung mass and unsprung mass displacement, we have suspension deflection. Tire deflection is calculated from unsprung mass and road displacement. For example, the road disturbance, acceleration of car body, suspension

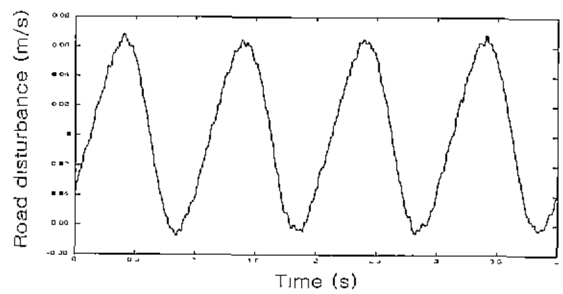


Fig. 5. Road disturbance at frequency 1Hz.

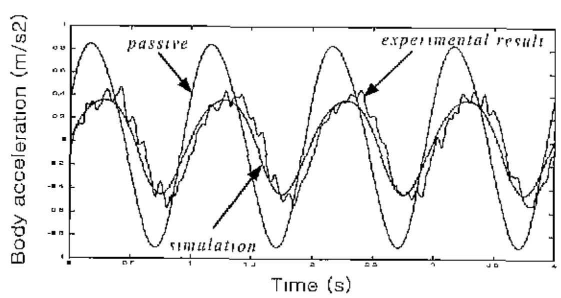


Fig. 6. Body acceleration at frequency 1Hz.

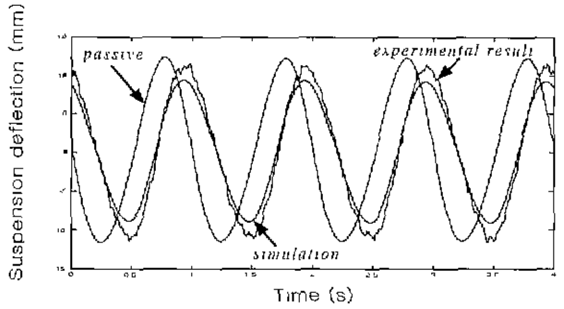


Fig. 7. Suspension deflection at frequency 1Hz.

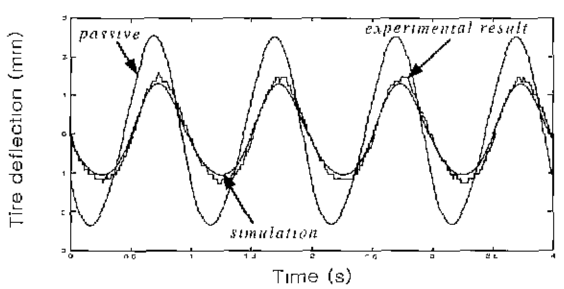


Fig. 8. Tire deflection at frequency 1Hz.

and tire deflection at frequency 1Hz are given in Fig. 5-8. In these figures, the simulation values of passive and H_∞ controlled system are plotted together with experimental value of controlled system. Then, we can derive the gain plots of three interest transfer functions (5)-(7).

The Fig. 9-11 show these transfer functions in the case of passive system, H_∞ controller with simulation and experimental results. These figures show us that the H_∞ controller with sprung mass velocity feedback achieves the good performance properties. It reduces considerably the gains from the road disturbance to car body acceleration, to suspension deflection, and to tire deflection at frequency around 1 Hz. The experimental results is not as good as the simulation results because the system is modeled ignoring the friction, the error of linearization and also because of the system parameters measurement errors.

V. Conclusions

In this paper, the H_∞ controller is used to control the

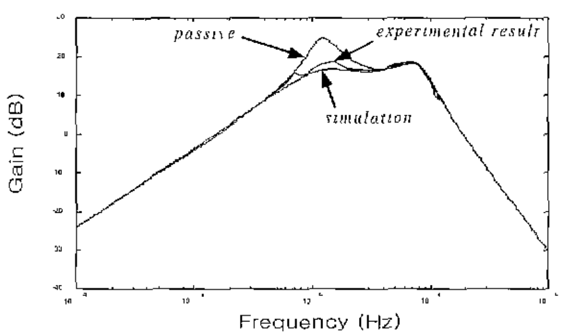


Fig. 9. Gain plots for body acceleration transfer functions.

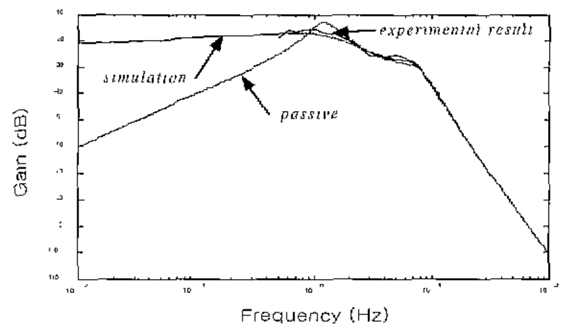


Fig. 10. Gain plots for suspension deflection transfer functions.

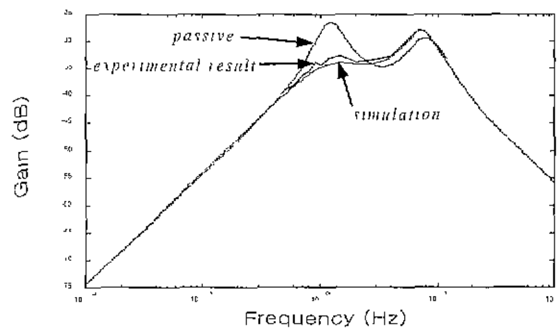


Fig. 11. Gain plots for tire deflection transfer functions.

active suspension system with measured car body velocity for feedback. The preliminaries for H_∞ controller design is set up to solve the controller. The system parameters variations are treated with multiplicative uncertainty model and the robustness property of the system is guaranteed by small gain theorem. The simulation results are shown to verify the effectiveness of the H_∞ controller comparing to the passive system. Also, the experiment is done to prove the applicability of the adopted H_∞ theory.

The simulation and experimental results show that the H_∞ controller can reduce considerably the gains from road disturbance to car body acceleration, to suspension deflection and to tire deflection at the frequencies around 1 Hz.

From the above results, the H_∞ controller can be used usefully to control active suspension system because it satisfies two requirements .

1) Good performance: small gain from road disturbance to car body acceleration, to suspension deflection and to tire deflection.

2) Robustness property.

It is expected that the active suspension system with H_∞ controller can be applied in car industry as an objective to increase the quality of car.

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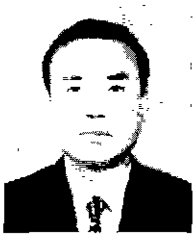

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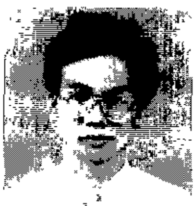

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