論 文
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3단 비간섭 슬라이딩모드 제어

Three-Level Decoupled Sliding Mode Control

尉遲明* · 張成東** · 愼輝範***
(Ming Yuchi · Sung-Dong Chang · Hwi-Beom Shin)

Abstract - A three-level decoupled sliding mode controller is developed to achieve asymptotic stability for a class of sixth-order nonlinear systems. The sixth-order system is decoupled into three subsystems according to the structure of the whole system. Each subsystem has a separate control target in the form of a sliding surface. The information of the third sliding surface is transferred to the second one through an intermediate variable and the information of the second sliding surface is transferred to the first one through another intermediate variable. Consequently, the controller designed on the basis of the first sliding surface can make three subsystems move toward their sliding surfaces, respectively. The three-level decoupled sliding mode controller is applied to the double-inverted pendulum problem where the zero stable states are required.

Key Words : Decoupled sliding mode control, Double-inverted pendulum

1. Introduction

The sliding mode controller (SMC) is a powerful method to control nonlinear and uncertain systems. It is a robust control method which can be applied in the presence of model uncertainties and parameter disturbances. The SMC is derived from the variable structure control, which has a variable high-speed switching feedback path (for example, the gains in each feedback path switch between two values according to some switching rules) [1, 2]. The switching rules are generated to drive the state trajectory of nonlinear plant onto a user-chosen surface in the state space and to maintain it on this surface for all subsequent time. This surface is called a sliding surface because if the state trajectory is above or below the surface, a feedback path has one or a different gain, respectively. The plant dynamics restricted to the sliding surface represents the controlled systems behavior. By proper design of the sliding surface, different control goals such as stabilization, tracking, regulation, and the like, can be obtained. Some practical implementation of SMC can be found in [3-5].

* 準 會 員 : 慶尙大 電氣工學科 碩士課程
** 正 會 員 : 慶尙大 電氣工學科 博士課程
*** 正 會 員 : 慶尙大 電氣工學科 副敎授 · 工博
自動化및컴퓨터應用硏究所 硏究員
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In general, the SMC design breaks down into two phases. The first phase is to design a sliding surface so that the plant state restricted to the surface may have desired dynamics. The second phase is to design a switched control that will drive the plant state to the sliding surface and maintain it on this surface. In most cases, a Lyapunov approach is used to characterize the second design phase [1-6].

Recently, a two-level decoupled SMC model has been proposed for stabilizing a certain class of fourth-order nonlinear system such as the inverted pendulum [7]. The two-level decoupled SMC can attain good result but it cannot stabilize all the state variables of systems higher than fourth-order like a double-inverted pendulum.

In this paper, a three-level decoupled SMC is developed to achieve asymptotic stability for a class of sixth-order nonlinear systems and it is applied to the double-inverted pendulum problem where the zero stable states are required. The simulation result shows that all the controlled states can achieve asymptotic stability.

2. Sliding Mode Control

Consider the n-th order nonlinear system such as

$$\begin{aligned} x^{(n)} &= f(x, \dot{x}, \cdots, x^{(n-1)}) + b(x, \dot{x}, \cdots, x^{(n-1)})u + d(t) \\ y &= x \end{aligned} \quad (1)$$

where $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)^T = (x, \dot{x}, \cdots, x^{(n-1)})^T \in R^n$ is the state vector, f and b are nonlinear functions, u is the control input, and $d(t)$ is external disturbance. The disturbance is assumed to have upper bound D; that is, $|d(t)| \leq D$ for all $t \geq 0$. The control objective is to determine a feedback control $u = u(\boldsymbol{x})$ such that the state $\boldsymbol{x}$ of the closed-loop system will follow the desired state $\boldsymbol{x}_d = (x_d, \dot{x}_d, \cdots, x_d^{(n-1)})^T$; that is, the state error

$$\boldsymbol{e} = \boldsymbol{x} - \boldsymbol{x}_d = (e, \dot{e}, \cdots, e^{(n-1)})^T \tag{2}$$

should converge to zero, where $e = x - x_d$. In general, a sliding surface is defined by

$$s(\boldsymbol{e}) = \boldsymbol{c}\boldsymbol{e} \tag{3}$$

where $\boldsymbol{c} = [c_1, c_2, \cdots, c_{n-1}, 1]$ in which c_i's are real and all roots of polynomial $h(p) = p^{n-1} + c_{n-1}p^{n-2} + \cdots + c_1$ are in the open left half-plane where p is a Laplace operator.

Consider a Lyapunov function such as

$$V = \frac{1}{2}s^2. \tag{4}$$

Then, the plant trajectory will attract toward the sliding surface within finite time and remain on it if $\dot{V}$ is negative definite. Also, once the trajectory remains on the surface, the state error $\boldsymbol{e}(t)$ will converge to zero. Therefore, the control u should satisfy that

$$\dot{V} = \frac{1}{2}\frac{d}{dt}s^2 < 0. \tag{5}$$

Because u is designed on the basis of a second-order system for simplicity of the problem, we will only consider, in this paper, a second-order nonlinear system given by

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= f(x) + b(x)u + d(t) \\ y(t) &= x_1(t) \end{aligned} \tag{6}$$

where the desired state $\boldsymbol{x}_d = 0$. From (2) and (3), the sliding surface can be defined as

$$s = c_1x_1 + x_2. \tag{7}$$

Then, the dynamic behavior of (6) without disturbance is

$$\dot{x}_1 + c_1x_1 = 0 \tag{8}$$

and (5) will have the form such that

$$\begin{aligned} \dot{V} &= \frac{1}{2}\frac{d}{dt}s^2 \\ &= s(c_1\dot{x}_1 + \dot{x}_2) \\ &= s[c_1x_2 + f(x) + b(x)u + d(t)]. \end{aligned} \tag{9}$$

It can be easily shown from (9) that if u has the following form, $\dot{V}$ will be negative:

$$u = \hat{u} - K \cdot sgn(sb(x)), \qquad K > D/|b(x)| \tag{10}$$

where

$$\hat{u} = \frac{-c_1x_2 - f(x)}{b(x)} \tag{11}$$

$$sgn(\phi) = \begin{cases} 1 & \text{if } \phi > 0 \\ 0 & \text{if } \phi = 0. \\ -1 & \text{if } \phi < 0 \end{cases} \tag{12}$$

The controller shown in (10) is discontinuous across the sliding surface s and requires an infinitely fast switching mechanism in ideal case. Since the implementation of the control switching can not be perfect, we have to sample the signals in digital control systems, which causes chattering. Chattering is undesirable because it may excite unmodeled high-frequency plant dynamics resulting in unforeseen instability. A way to eliminate chattering is to introduce a thin boundary layer neighboring the sliding surface such as [6], [11]

$$B(t) = \{x \ |s(x, t)| \leq \Phi\} \tag{13}$$

where Φ is called the thickness of the boundary layer. The control u changes continuously within this boundary layer. Replacing $sgn(sb(x))$ with $sat(sb(x)/\Phi)$ in (10) yields that

$$u = \hat{u} - K \cdot sat(sb(x)/\Phi), \qquad \Phi > 0 \tag{14}$$

$$sat(\phi) = \begin{cases} sgn(\phi) & \text{if } |\phi| \geq 1 \\ \phi & \text{if } |\phi| < 1 \end{cases}. \tag{15}$$

In the contents followed, the control u in (14) will be used as the main controller for the three-level decoupled SMC.

3. Design of Three-level Decoupled SMC

The SMC in (14) can be applied to the second-order nonlinear systems in the form of (6) but cannot force all the state variables to zero for the higher-order systems like fourth-order or sixth-order systems such as [7]

$$\begin{aligned} \dot{x}_{2i-1} &= x_{2i} \\ \dot{x}_{2i} &= f_i(x) + b_i(x)u + d_i(t) \end{aligned} \tag{16}$$

where $f_i(x)$ and $b_i(x)$ are nonlinear functions, $d_i(t)$ is external disturbance, and $i=1,\cdots,N$. The state vector $\boldsymbol{x}$ has an appropriate dimension and the disturbance is assumed to be bounded as $|d_i(t)|\le D_i$ for all $t\ge 0$. If $N=2$, (16) becomes fourth-order system like an inverted pendulum and becomes sixth-order system such as a double-inverted pendulum if $N=3$.

Lo and Kuo have proposed a two-level decoupled SMC [7]. Applying it to the fourth-order nonlinear system in the form of (16), it is shown that the two-level decoupled SMC can make the state variable asymptotically zero. However, the two-level controller fails to stabilize the state variables at the origin when it is applied to the sixth-order nonlinear system.

Considering the sixth-order nonlinear system given in (16), we can decouple it into three subsystems A, B, C in the form of (6) where A contains x_1, x_2, B contains x_3, x_4, and C contains x_5, x_6. Hence, three sliding surfaces can be defined as

$$s_i = c_i x_{2i-1} + x_{2i} \tag{17}$$

where $i=1,2,3$. Then, three control laws can be separately chosen according to (17) as

$$u_i = \widehat{u_i} - K_i \cdot sat(s_i b_i(x)/\Phi_i),\ K_i > D_i/|b_i(x)| \tag{18}$$

$$\widehat{u_i} = \frac{-c_i x_{2i} - f_i(x)}{b_i(x)} \tag{19}$$

where $i=1,2,3$. Intuitively, when any u_i in (18) is adopted as u in (16), the corresponding s_i will asymptotically approach to zero but the other two sliding surfaces will not. In other words, such an SMC can only control one sliding surface of the three. To solve this problem, we develop a three-level decoupled SMC, which can make all the sliding surfaces converge to zero.

The main idea of the three-level decoupled SMC system model is stated as follows. Through a transferring mechanism, the information of subsystem C is first reflected to B and then the information of B is also reflected to A. In this way, the subsystem A will contain the information of both B and C. It is therefore expected that all the subsystems can be controlled by designing one SMC on the basis of A. An intermediate variable w is used for transferring the information of sliding surface s_3 to s_2 and another variable z is used for transferring the information of s_2 to s_1. This mechanism is to incorporate w, z into s_2, s_1, respectively, where w and z are a function of s_3 and s_2, respectively. Therefore, the sliding surface s_2 is modified to take the form $c_2(x_3-w)+x_4$ and s_1 is modified to take the form $c_1(x_1-z)+x_2$. Thus the control target of subsystem C, $s_3=0$, is embedded to s_2 and the target of B, $s_2=0$, is embedded to s_1. All the subsystems are controlled simultaneously.

From the above, the three-level decoupled SMC can be written as follows:

$$u = u_1 = \widehat{u_1} - K_1 \cdot sat(s_1 b_1(x)/\Phi_1),\ K_1 > D_1/|b_1(x)| \tag{20}$$

$$\widehat{u_1} = \frac{-c_1 x_2 - f_1(x)}{b_1(x)},$$

$$s_1 = c_1(x_1 - z) + x_2, \tag{21}$$

$$z = sat(s_2/\Phi_z)\cdot z_u, \tag{22}$$

$$s_2 = c_2(x_3 - w) + x_4, \tag{23}$$

$$w = sat(s_3/\Phi_w)\cdot w_u, \tag{24}$$

$$s_3 = c_3 x_5 + x_6, \tag{25}$$

where z_u $(0<z_u<1)$ and w_u $(0<w_u<1)$ denote the upper bounds of abs(z) and abs(w), respectively, Φ_z and Φ_w are the thickness of boundary layer of s_2 and s_3, respectively, and sat( ) is the same as (15). It is noted that z and w are decaying oscillation signals because z_u and w_u are factors between 0 and 1. Φ_z and Φ_w are adopted here to avoid chattering.

The control sequences are as follows. When $s_3\neq 0$, this information is transferred to s_2 through w, and then transferred to s_1 through z. Therefore, a corresponding control is generated to reduce s_3 to zero. When $s_2\neq 0$, this information is transferred to s_1 through z and a control signal is generated to reduce s_2 to zero. When $s_3\to 0$, then $w\to 0$, then $x_3\to 0$, then $s_2\to 0$, then $z\to 0$, then $x_1\to 0$, then $s_1\to 0$ and the control objective can be achieved. The structure of the three-level decoupled SMC is shown in Fig. 1.

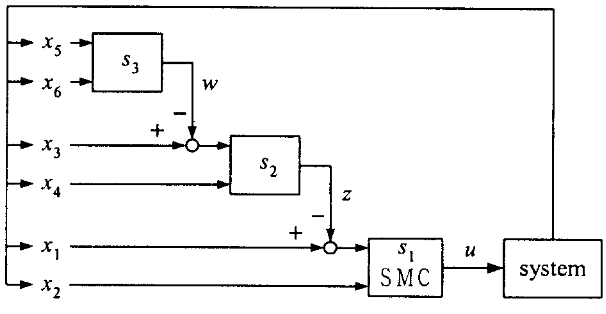

Fig. 1 Structure of three-level decoupled SMC

4. Simulation Results

In this section, the three-level decoupled SMC is applied to a balancing problem of the double-inverted pendulum whose structure is illustrated in Fig. 2. Pole 1 is connected to the cart and pole 2 is the one above pole 1. The control objective is to keep pole 1 and 2 balanced while the cart moves and stops at the original position, that is to say, make θ_1, $\dot{\theta}_1$, θ_2, $\dot{\theta}_2$, x, and $\dot{x}$ become zero after some balancing time. The dynamic equations of the double-inverted pendulum system are represented as

$$\begin{aligned} \dot{x} &= x_2 \\ \dot{x}_2 &= f_1 + b_1 u + d \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2 + b_2 u + d \\ \dot{x}_5 &= x_6 \\ \dot{x}_6 &= f_3 + b_3 u + d \end{aligned} \tag{26}$$

where

$x_1 = \theta_1$: angle of pole 1 with respect to vertical axis,

$x_2 = \dot{\theta}_1$: angular velocity of pole 1 with respect to vertical axis,

$x_3 = \theta_2$: angle of pole 2 with respect to vertical axis,

$x_4 = \dot{\theta}_2$: angular velocity of pole 2 with respect to vertical axis,

$x_5 = x$: position of the cart,

$x_6 = \dot{x}$: velocity of the cart,

and f_1, f_2, f_3, b_1, b_2, and b_3 are given in Appendix. In order to demonstrate the external disturbance rejection capability, a uniformly distributed disturbance is applied on the system.

The three-level decoupled SMC is designed in the form of (20)-(25) and the following system parameters are used in the simulation:

$$l_1 = 1[\mathrm{m}],\ l_2 = 1[\mathrm{m}],\ m_c = 1[\mathrm{kg}],\ m_1 = 1[\mathrm{kg}],\ m_2 = 1[\mathrm{kg}],\ g = 9.8[m/s^2].$$

The controller parameters are chosen as:

$$c_1 = 10,\ c_2 = 5,\ c_3 = 0.5,\ K_1 = 10,\ \Phi_1 = 5,\ \Phi_z = 5,\ \Phi_w = 5,\ z_u = 0.5,\ w_u = 0.5,\ |d| \le 0.0873.$$

The initial values are:

$$\theta_{10} = 30^\circ,\ \theta_{20} = 10^\circ,\ \dot{\theta}_{10} = 0,\ \dot{\theta}_{20} = 0,\ \dot{x}_0 = x_0 = 0.$$

Comments:

1) K_1 is the hitting control gain whose sole purpose is to make the sliding condition ($s\dot{s} < 0$) viable. Therefore, the value of K_1 should be large enough to overcome the effect of external disturbance but a too large K_1 may produce oscillation. We choose $K_1 = 10$.

2) Because the thin boundary layer Φ neighbors on the sliding surface, we can arbitrarily adjust the steady-state error by proper selection of Φ. However, a small Φ might produce a boundary layer so thin that it risks exciting high frequency dynamics. Here we select all the Φ equal to 5.

3) z_u and w_u are factors between 0 and 1, and they decide the decaying oscillation signal speed. Small values will increase the speed but may cause divergence. $z_u = w_u = 0.5$ are adopted here.

4) To avoid the situation where the cart never stops, c_3 must be properly chosen. When the cart moves toward the origin, a larger c_3 makes s_3 change it's sign at a position close to the origin and, accordingly, the force to slow down the cart will be exerted at a position closed to the origin. Nevertheless, the duration of the action may not be long enough to reduce the speed of the cart to zero as the cart passes through the origin. Also, the value of c_3 must not be too large, otherwise the cart will be always oscillating around the origin. In this simulation, $c_3 = 0.5$.

Figs. 3-5 show the simulation result of three-level decoupled SMC system. Comparing to Figs. 6-8 which are the simulation result of two-level decoupled SMC system in [7], We can find that θ_1 and θ_2 move back to the origin but the cart go away from the origin when applying the two-level controller to the double-inverted pendulum system. However, the three-level SMC can force all the state variables of the double-inverted pendulum converge

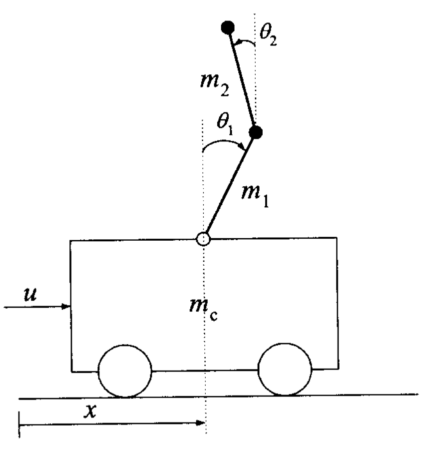

Fig. 2 Double inverted pendulum

to zero, i.e. the poles and the cart are stabilized to the equilibrium.

5. Conclusion

In this paper, a three-level decoupled SMC is developed for sixth-order nonlinear systems. By decoupling the whole system into three subsystems, each subsystem has a separate control target expressed in terms of a sliding surface. Two intermediate variables w and z are introduced to transfer two of the three control targets (converging the sliding surface to zero) to the other one. Through this mechanism, the main SMC can generate a control action to make all the three subsystems move toward their corresponding sliding surfaces. The usefulness of the proposed method is verified by applying it to a double-inverted pendulum system. Compared to the two-level SMC system, the three-level SMC succeeds in stabilizing the position of the cart to origin. The weak point of the three-level SMC system is that the stabilizing time is a little long, which would need some future work.

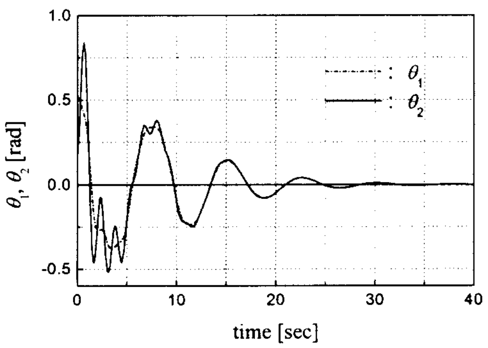

Fig. 3 Angle evolution of pole 1 and 2 with three-level decoupled SMC

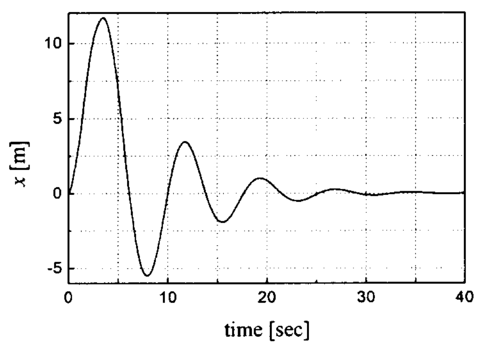

Fig. 4 Position evolution of cart with three-level decoupled SMC

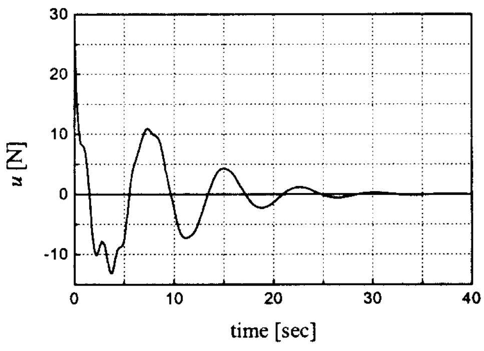

Fig. 5 Control input of three-level decoupled SMC

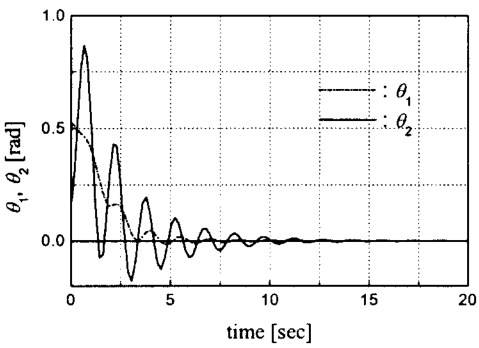

Fig. 6 Angle evolution of pole 1 and 2 with two-level decoupled SMC

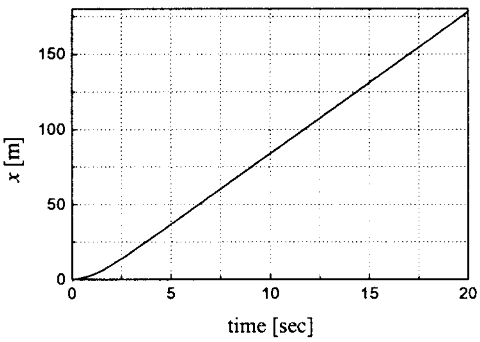

Fig. 7 Position evolution of cart with two-level decoupled SMC

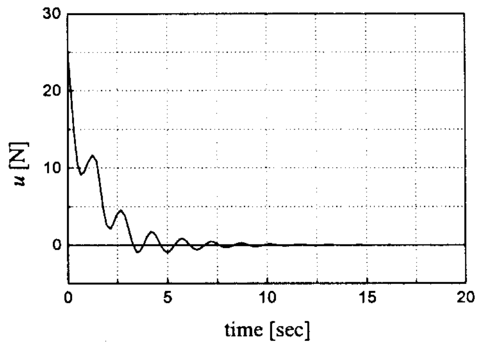

Fig. 8 Control input of two-level decoupled SMC

Appendix

System equation of double-inverted pendulum:

$$f_1=\frac{A_{21}}{l_1m_1}\sin(x_3-x_1)+\frac{1}{l_1}g\sin x_1-\frac{A_{11}}{l_1m_c}\cos x_1\sin x_1$$

$$f_2=\frac{A_{11}}{l_2m_1}\sin(x_3-x_1),\quad f_3=\frac{A_{11}}{m_c}\sin x_1$$

$$b_1=\frac{A_{22}}{l_1m_1}\sin(x_3-x_1)-\frac{\cos x_1}{l_1m_c}-\frac{A_{11}}{l_1m_c}\cos x_1\sin x_1$$

$$b_2=\frac{A_{12}}{l_2m_1}\sin(x_3-x_1),\quad b_3=\frac{1}{m_c}+\frac{A_{12}}{m_c}\sin x_1$$

$$A_{11}=\frac{a_{22}(l_1x_2^2-g\cos x_1)-a_{12}l_2x_4^2}{\Delta}$$

$$A_{22}=\frac{a_{12}\sin x_1}{\Delta\cdot m_c},\quad A_{12}=-\frac{a_{22}\sin x_1}{\Delta\cdot m_c}$$

$$A_{21}=\frac{-a_{12}(l_1x_2^2-g\cos x_1)+a_{11}l_2x_4^2}{\Delta}$$

$$a_{11}=\frac{1}{m_1}+\frac{\sin^2x_2}{m_c},\quad a_{22}=\frac{1}{m_1}+\frac{1}{m_2}$$

$$a_{12}=-\frac{\cos(x_3-x_1)}{m_1}$$

$$\Delta=a_{11}a_{22}-a_{12}^2$$

where

l_1, l_2: length of pole 1 and pole 2, respectively,

m_c, m_1, m_2: mass of the cart, pole1, and pole 2, respectively,

u : control force to move the cart,

g : acceleration of gravity.

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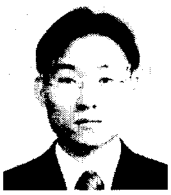
저 자 소 개

유 치 밍 (尉 遲 明)

1975년 5월 20일 생. 1997년 화중이공대학 자동제어공학과 졸업. 1998년 ~ 현재 경상대 전기공학과 석사과정

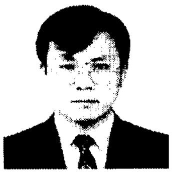
E-mail : mingming_yuchi@hotmail.com



장 성 동 (張 成 東)

1975년 1월 28일 생. 1997년 경상대 전기공학과 졸업. 2000년 동 대학원 전기공학과 졸업(석사). 2000년 ~ 현재 동 대학원 전기공학과 박사과정

E-mail : s_niar@gshp.gsnu.ac.kr



신 휘 범 (愼 輝 範)

1958년 10월 30일 생. 1982년 서울대 전기공학과 졸업. 1985년 한국과학기술원 전기및전자공학과 졸업(석사). 1992년 한국과학기술원 전기및전자공학과 졸업(공박). 1990년~1992년 (주)현대전자산업 과장. 1993년~현재 경상대 전기공학과 부교수

Tel : 055-751-5348, Fax : 055-759-2723

E-mail : hbshin@nongae.gsnu.ac.kr