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OPTIMIZATION AND IDENTIFICATION FOR THE NONLINEAR HYPERBOLIC SYSTEMS

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ABSTRACT. In this paper we consider the optimal control problem of both operators and parameters for nonlinear hyperbolic systems. For the identification problem, we show that for every value of the parameter and operators, the optimal control problem has a solution. Moreover we obtain the necessary conditions of optimality for the optimal control problem on the system.

1. Introduction

The optimal control problems have been extensively studied by many authors [1, 3, 5, 7, 10, 13, 14, 15 and the references cited therein] and also identification problem for damping parameters in the second order hyperbolic systems have been dealt with by many authors [4, 6, 8, 12, and the references cited therein].

In this paper, we consider the following control systems;

$$
(1.1)\qquad \begin{cases} y'' + A_2(t,q)y' + A_1(t,q)y + N^*g(Ny) + By = f(t,q) \\ y(q,B)(0) = y_0 \in V,\ y'(q,B)(0) = y_1 \in H, \\ q \in Q_m,\ B \in \mathcal{P}_{a,b} \end{cases}
$$

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and the cost functional given by the quadratic form

$$J(q,B) = \frac{1}{2}||Cy(q,B) - z_d||_M^2. \tag{1.2}$$

Here $A_1(t,q)$ and $A_2(t,q)$ are differential operators containing unknown parameter $q \in Q_m$ and there are given by some bilinear forms on Hilbert spaces, $N^*g(Ny)$ is a nonlinear term, $B \in \mathcal{P}_{a,b}$ is an operator, $\mathcal{P}_{a,b}$ is a suitable space, f is a forcing term, C is an observation operator defined on an observation space M and z_d is a desired value.

The optimal control problem subject to (1.1) with (1.2) is to find optimal pairs $(\bar{q}, \bar{B}) \in Q_m \times \mathcal{P}_{a,b}$ such that

$$\inf_{(q,B)\in Q_m \times \mathcal{P}_{a,b}} J(q.B) = J(\bar{q}, \bar{B}).$$

Recently, inspired by the optimal control theoretical studies of Euler-Bernoulli Beam Equations with Kelvin-Voigt Damping, and Love-Kirchhoff Plate Equations with various damping terms, these appeared numerous paper studying optimal control theory and identification problems. In Banks et al.[4], Banks and Kunisch [5], they treated the existence of the optimal control (or minimizing parameters) by using the methods of approximations, but they didn't deal with the necessary conditions (or characterizations) on them. When $A_1(t,q) \equiv \gamma A_2(t,q), \gamma > 0$ and $N^*g(Ny) = 0$ in (1.1), the identification problem estimating q via output least-square identification problem is studied by Ahmed [1,2] based on the transposition method.

In the nonlinear parabolic type case, Papageorgiou [11] treated with optimal control problems contained parameter and control. But we deal with the second order nonlinear hyperbolic systems.

In this paper we will study the identification problem to the system (1.1) with (1.2) and the existence of weak solution for the system (1.1). It is not easy to find the optimal control pairs $(\bar{q}, \bar{B})$ belonging to a general admissible set $Q_m \times \mathcal{P}_{a,b}$ of both parameters and operators subject to (1.1) with (1.2). Hence we will show the existence of such $(\bar{q}, \bar{B})$ when $Q_m \times \mathcal{P}_{a,b}$ is a compact subset of a topological space. Moreover, we obtain the necessary conditions of optimality for the optimal control problem.

2. Preliminaries

Let X be a real Hilbert spaces. $(\cdot,\cdot)_X$ and $||\cdot||_X$ denote the inner product and the induced norm on X. X^* the dual space of X and $\langle\cdot,\cdot\rangle_{X^*,X}$ denotes the dual pairing between X^* and X. Let us introduce underlying Hilbert spaces to describe nonlinear hyperbolic systems. Let H be a real pivot Hilbert space, its norm $||\cdot||_H$ by $|\cdot|_H$. Throughout this paper we assume there is a sequence of real separable Hilbert spaces V_1, V_2, V_1^*, V_2^* forming a Gelfand quintuple satisfying $V_1 \hookrightarrow V_2 \hookrightarrow H \equiv H^* \hookrightarrow V_2^* \hookrightarrow V_1^*$. And also we assume that the embedding $V_1 \hookrightarrow V_2$ is dense and continuous with $||\phi||_{V_2} \leq c||\phi||_{V_1}$ for $\phi \in V_1$ and $V_2 \hookrightarrow H$ is a densely compact embedding. From now on, we write $V_1 = V$ for convenience of notation. We assume that the equalities $\langle\phi,\varphi\rangle_{V^*,V} = \langle\phi,\varphi\rangle_{V_2^*,V_2}$ for $\phi \in V_2^*, \varphi \in V$ and $\langle\phi,\varphi\rangle_{V^*,V} = (\phi,\varphi)_H$ for $\phi \in H, \varphi \in V$. We shall give an exact description of nonlinear hyperbolic systems. We suppose that Q is algebraically contained in a linear topological vector space with topology τ_m and $Q_m = (Q, \tau_m)$ is compact. Let $\mathcal{L}(X, Z)$ denote the space of all bounded linear operators from X to Z and A^* the dual of the operator A. Consider the space of operators $\mathcal{L}(V, V_2^*)$ and suppose that it is given the strong strong (weak) operator topology which we denote by $\tau_{so}(\tau_{wo})$. Given this topology, $\mathcal{L}_s(V, V_2^*) \equiv (\mathcal{L}(V, V_2^*), \tau_{so})$ is a locally convex linear topological vector space which is sequentially complete. For some $b > 0$ and $a \in R$, let

$$\begin{aligned}\mathcal{P}_{a,b} = \{B \in \mathcal{L}(V, V_2^*) : ||B||_{\mathcal{L}(V,V_2^*)} \leq b,\\ \langle Bx, x\rangle_{V^*,V} + a|x|_H \geq 0, \forall x \in V\}.\end{aligned}$$

Note that $\mathcal{P}_{a,b}$ is compact in $\mathcal{L}(V, V_2^*)$. Let $I = [0, T]$, $T \geq 0$ be fixed and $t \in [0, T]$. Let $q \in Q_m$.

We will need following hypotheses on the data.

$H(A)$ $A_i : I \times Q_m \to \mathcal{L}(V_i, V_i)$ is an operator $(i = 1, 2)$.

(1) $a_i(t, q; \phi, \varphi) = a_i(t, q; \varphi, \phi)$, where $a_i(t, q; \phi, \varphi) = \langle A_i(t, q)\phi, \varphi\rangle_{V_i^*,V_i}$, $\forall\phi, \varphi \in V_i$.

(2) There exists $c_{i1} > 0$ such that $|a_i(t, q; \phi, \varphi)| \leq c_{i1}||\phi||_{V_i}||\varphi||_{V_i}$, $\forall\phi, \varphi \in V_i$.

(3) There exist $\alpha_i > 0$ and $\lambda_i \in R$ such that $a_i(t,q;\phi,\varphi) + \lambda_i|\varphi|^2_H \geq \alpha_i||\phi||^2_{V_i}, \forall\phi \in V_i$.

(4) The function $t \mapsto a_i(t,q;\phi,\varphi)$ is continuously differentiable on $[0,T]$.

(5) There exists $c_{i2} > 0$ such that $|a_i'(t,q;\phi,\varphi)| \leq c_{i2}||\phi||_{V_i}||\varphi||_{V_i}$, $\forall\phi,\varphi \in V_i$, where $' = \frac{d}{dt}$ and $a_i'(t,q;\phi,\varphi) = \langle A_i'(t,q)\phi,\varphi\rangle_{V_i^*,V_i}$.

$H(f)$ $f : I \times Q_m \to V_2^*$ is the forcing term such that $f(t,q) \in L^2(0,T;V_2^*)$.

$H(N)$ $N : V_i \to H$ is a linear operator such that $N \in \mathcal{L}(V_i, H)$ with $||N\varphi|| \leq \sqrt{k_i}||\varphi||_{V_2}$, k_i is constant and the range of N on V_i is dense in H.

$H(g)$ $g : H \to H$ is a continuous nonlinear mapping of real gradient(or potential) type such that

(1) $||g(\varphi)|| \leq c_1||\varphi|| + c_2, \varphi \in H$ and for some constant c_1, c_2.

(2) $||g(\varphi) - g(\phi)|| \leq c_3||\varphi - \phi||, \varphi, \phi \in H$ and for some constant c_3.

We consider the following problem for nonlinear hyperbolic systems of the form :

$$y'' + A_2(t,q)y' + A_1(t,q)y + N^*g(Ny) + By = f(t,q) \tag{2.1}$$

$$y(q,B)(0) = y_0 \in V, y'(q,B)(0) = y_1 \in H, \quad q \in Q_m, B \in \mathcal{P}_{a,b}, \tag{2.2}$$

where $y' = \dfrac{dy}{dt}, y'' = \dfrac{d^2y}{dt^2}$.

We define a Hilbert space, which will be a space of solutions, as following;

$$W(0,T) = \{y|y \in L^2(0,T;V), y' \in L^2(0,T;V_2), y'' \in L^2(0,T;V^*)\}$$

with an inner product
$(y_1,y_2)_{W(0,T)} = \int_0^T \{(y_1(t),y_2(t))_V + (y_1'(t),y_2'(t))_{V_2} + (y_1''(t),y_2''(t))_{V^*}\}dt$
and the induced norm

$$||y||_{W(0,T)} = (||y||^2_{L^2(0,T),V)} + ||y'||^2_{L^2(0,T;V_2)} + ||y''||^2_{L^2(0,T,V^*)})^{\frac{1}{2}}.$$

We denote by $\mathcal{D}(0,T)$ the space of distributions on $(0,T)$.

DEFINITION 2.1. A function y is said to be a weak solution of (2.1)-(2.2) if $y \in W(0,T)$ and y satisfies

$$\begin{aligned}&\langle y''(\cdot),\phi\rangle_{V^*,V} + a_2(\cdot,q;y'(\cdot),\phi) + a_1(\cdot,q;y(\cdot),\phi) + \langle g(Ny(\cdot)),N\phi\rangle_H \\ &+ \langle By(\cdot),\phi\rangle_{V^*,V} = \langle f(\cdot,q),\phi\rangle_{V_2^*,V_2}, \forall\phi\in V \text{ in the sense of } \mathcal{D}(0,T), \\ &y(q,B)(0) = y_0 \in V,\ \frac{dy}{dt}(q,B)(0) = y_1 \in H,\ q\in Q_m,\ B\in\mathcal{P}_{a,b}.\end{aligned}$$

By Definition 2.1 it is verified that a weak solution y of (2.1) satisfies

$$\begin{aligned}&\int_0^T \langle y''(t) + A_2(t,q)y'(t) + A_1(t,q)y(t) + N^*g(Ny(t)) \\ &+ By(t),\phi(t)\rangle_{V_2^*,V_2}dt = \int_0^T \langle f(t,q),\phi(t)\rangle_{V_2^*,V_2}dt,\ \forall\phi\in L^2(0,T;V_2).\end{aligned}$$

We state the existence and uniqueness results of a weak solution of (2.1)-(2.2).

THEOREM 2.1. *If $H(A)$, $H(f)$, $H(N)$ and $H(g)$ hold. Then the system*

$$\text{(2.3)}\qquad \begin{cases} y'' + A_2(t,q)y' + A_1(t,q)y + N^*g(Ny) \\ \qquad\qquad + By = f(t,q) \ \ \text{in } (0,T) \\ y(q,B)(0) = y_0 \in V,\ y'(q,B)(0) = y_1 \in H, \\ q\in Q_m,\ B\in\mathcal{P}_{a,b} \end{cases}$$

has a unique weak solution $y \in W(0,T)\cap C(0,T;V)\cap C^1(0,T;H)$. Here the concept of a weak solution for (2.3) is defined as

$$\begin{aligned}&\langle y''(\cdot),\phi\rangle_{V^*,V} + a_2(\cdot,q;y'(\cdot),\phi) + a_1(\cdot,q;y(\cdot),\phi) + \langle g(Ny(\cdot)),N\phi\rangle_H \\ &+ \langle By(\cdot),\phi\rangle_{V^*,V} = \langle f(\cdot,q),\phi\rangle_{V_2^*,V_2}, \forall\phi\in V \text{ in the sense of } \mathcal{D}'(0,T)\end{aligned}$$

with the initial conditions $y(q,B)(0) = y_0 \in V$, $y'(q,B)(0) = y_1 \in H$, $q\in Q_m$, $B\in\mathcal{P}_{a,b}$.

PROOF. We can prove by using the method Lions [9] and Ha [8].

3. Existence of both parameters and operators for optimality

In this section we consider the optimal control problem for the following system:

$$(3.1)\qquad \begin{cases} y'' + A_2(t,q)y' + A_1(t,q)y + N^*g(Ny) \\ \qquad\qquad + By = f(t,q) \ \text{ in } (0,T) \\ y(q,B)(0) = y_0 \in V,\ y'(q,B)(0) = y_1 \in H, \\ \qquad\qquad q \in Q_m,\ B \in \mathcal{P}_{a,b}. \end{cases}$$

Note that since there is a unique solution y to (3.1) for given $(q,B) \in Q_m \times \mathcal{P}_{a,b}$, we have a well-defined mapping $y = y(q,B)$ of $Q_m \times \mathcal{P}_{a,b}$ into $W(0,T)$.

We often call (3.1) the state equation and $y(q,B)$ the state with respect to (3.1). Let us consider a quadratic cost functional attached to (2.3) as

$$(3.2)\qquad J(q,B) = \frac{1}{2}||Cy(q,B) - z_d||_M^2,\ (q,B) \in Q_m \times \mathcal{P}_{a,b},$$

where M is a Hilbert space of observations, $C \in \mathcal{L}(W(0,T),M)$ is an observer and z_d is a desired value belonging to M. Our main aim is to find $(\bar{q},\bar{B}) \in Q_m \times \mathcal{P}_{a,b}$ satisfying

$$(3.3)\qquad J(\bar{q},\bar{B}) = \min_{(q,B)\in Q_m \times \mathcal{P}_{a,b}} J(q,B)$$

and to give a characterization of such $(\bar{q},\bar{B})$. We call $(\bar{q},\bar{B})$ the optimal pairs to the system (3.1) and (3.2). Furthermore, we will give an assumption to $a_i(t,q;\phi,\varphi), i = 1,2$ and f:

$H(A)_1$ $q \to a_i(t,q;\phi,\varphi) : Q_m \to R$ is continuous for all $t \in [0,T], \phi, \varphi \in V_i$.

$H(f)_1$ $q \to f(\cdot,q) : Q_m \to V_2^*$ is continuous.

Note that for each $q \in Q_m, \phi, \varphi \in V_i$ the following equalities hold :

$$\sup_{||\varphi||_{V_i}=1} |a_i(t,q;\phi,\varphi)| = \sup_{||\varphi||_{V_i}=1} |\langle A_i(t,q)\phi,,\varphi\rangle_{V_i^*,V_i}| = ||A_i(t,q)\phi||_{V_i^*},$$

whence the assumption $H(A)_1$ and the above equality imply that $||A_i(t,q)\phi|$ is continuous on q.

LEMMA 3.1. *If $H(A)$, $H(f)$, $H(N)$, $H(A)_1$ and $H(f)_1$ hold. Then $y(q,B) \in C(Q_m \times \mathcal{P}_{a,b}, W(0,T))$ is strongly continuous on (q,B).*

PROOF. It can be proved by using the method of Ahemd[2] and Ha[8].

THEOREM 3.1. *If $H(A)$, $H(f)$, $H(N)$, $H(A)_1$ and $H(f)_1$ hold. Then there is at least one optimal pairs $(\bar{q}, \bar{B})$ if $Q_m \times \mathcal{P}_{a,b}$ is compact.*

PROOF. It is clear from Lemma 3.1 and continuity of norm.

REMARK. We can the operator B to be function of time by taking for the admissible the set

$$\mathcal{P}^0_{a,b} = \{B \in L_\infty(I, \mathcal{L}(V, V_2^*)) : ess\, sup\{\|B(t)\|_{\mathcal{L}(V,V_2^*)},\ t \in I\ \} \le b, \\ \text{and } \langle B(t)\xi, \xi\rangle_{V_2^*,V} + a|\xi|^2_H \ge 0 \text{ a.e. on } I\},$$

where $b > 0$ and $a \in R$. In this case, replacing $\mathcal{P}^0_{a,b}$ instead of $\mathcal{P}_{a,b}$, we obtain the same results.

4. Necessary condition of optimality for both parameters and operators

Here we present the necessary conditions (the minimizing conditions) for optimal pairs $(\bar{q}, \bar{B}) \in Q_m \times \mathcal{P}_{a,b}$ to the system (3.1) with the cost functional $J(p,B)$ given by (3.2). If $J(p,B)$ is Gâteaux differentiable at $(\bar{q}, \bar{B})$ in the direction $(q-\bar{q}, B-\bar{B})$, the necessary condition on $(\bar{q}, \bar{B})$ is characterized by the following inequality

$$DJ(\bar{q}, \bar{B}; q-\bar{q}, B-\bar{B}) \ge 0, \ \forall (q,B) \in Q_m \times \mathcal{P}_{a,b}, \tag{4.1}$$

where $DJ(\bar{q}, \bar{B}; q-\bar{q}, B-\bar{B})$ denotes the Gâteaux derivative at $(\bar{q}, \bar{B})$ in the direction $(q-\bar{q}, B-\bar{B})$.

Note that since $J(q,B)$ composed of the term $y(q,B)$, the Gâteaux differentiability of $J(q,B)$ follows from that of $y(q,B)$. Hence to obtain that of $y(q,B)$ we will need the following condition:

$H(A)_2$ $q \to A_i(\cdot,q)$ is Gâteaux differentiable for all t and $DA_i(t,q)(p) \equiv DA_i(t,q;p) \in L^2(0,T;\mathcal{L}(V_i,V_i^*))$ for all $q \in Q_m$, where $DA_i(t,q;p)$ denotes the Gâteaux derivative at q in the direction of p.

$H(g)_1$ For any $\varphi \in H$ the Fréchet derivative of g exists and satisfies $g_\varphi(\varphi) \in \mathcal{L}(H,H)$ with $\|g_\varphi(\varphi)\|_{\mathcal{L}(H,H)} \le c_4$, where $g_\varphi(\varphi)$ is the Fréchet derivative of g at φ and c_4 is constant.

$H(f)_2$ $q \to f(t,q)$ is Gâteaux differentiable for all t and $f_q(t,q)p \equiv f_q(t,q;p) \in L^2(0,T,V_2^*)$, where $f_q(t,q;p)$ is Gâteaux derivative at q in the direction of p.

LEMMA 4.1. *Assume that the conditions in Theorem 2.1, $H(A)_1$, $H(A)_2$, $H(f)_1$, $H(f)_2$ and $H(g)_1$ are satisfied. Then $y(q,B)$ is weakly Gâteaux differentiable at (q,B) in the direction $(q-\bar{q},B-\bar{B})$, and if we denote the Gâteaux derivative of $y(q,B)$ by $z = Dy(\bar{q},\bar{B};q-\bar{q},B-\bar{B})$, it satisfies the following Cauchy problem:*

$$(4.2)\qquad \begin{cases} z'' + A_2(t,\bar{q})z' + A_1(t,\bar{q})z + N^* g_y(Ny(\bar{q},\bar{B}))Nz + \bar{B}z \\ = -DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{B}) - DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{B}) \\ \qquad + (\bar{B}-B)y(\bar{q},\bar{B}) + f_q(t,\bar{q};q-\bar{q}) \quad in\ (0,T) \\ z(0) = z'(0) = 0. \end{cases}$$

PROOF. We can prove by using the method of Ahemd [2] and Park et al. [12].

By Lemma 4.1, the cost functional $J(q,B)$ is Gâteaux differentiable at $(\bar{q},\bar{B})$ in the direction $(q-\bar{q},B-\bar{B})$, and so, the condition (4.1) is rewritten by

(4.3)

$$\begin{aligned} DJ(\bar{q},\bar{B};q-\bar{q},B-\bar{B}) &= \langle C^*\Lambda_M(Cy(\bar{q},\bar{B})-z_d), z\rangle_{W^*(0,T),W(0,T)} \\ &\quad + \langle C^*\Lambda_M(Cy(\bar{q},\bar{B})-z_d), y_B(\bar{q},\bar{B};B-\bar{B})\rangle_{W^*(0,T),W(0,T)} \ge 0, \\ &\forall\,(q,u) \in Q_m \times \mathcal{P}_{a,b}, \end{aligned}$$

where z is the unique weak solution to (4.2), $C^* \in \mathcal{L}(M^*, W^*(0,T))$ is the adjoint operator of C and Λ_M is the canonical isomorphism of M onto M^* in the sense that

(i) $\langle \Lambda_M \phi, \phi \rangle_{M^*,M} = ||\phi||_M^2$,
(ii) $||\Lambda_M \phi||_{M^*} = ||\phi||_M$ for all $\phi \in M$.

In order to avoid the complexity of setting up observation spaces, we consider the following two types of distributive and terminal value observations in time sense. that is, the following cases :

(i) we take $C_1 \in \mathcal{L}(L^2(0,T;V_2), M)$ and observer $z(q,B) = C_1 y(q,B)$;
(ii) we take $C_2 \in \mathcal{L}(H, M)$ and observer $z(q,B) = C_2 y(q,B)(T)$.

4.1. The case where $C_1 \in \mathcal{L}(L^2(0,T;V_2), M)$

In this case the cost functional is given by

$$J(q,B) = \frac{1}{2}||C_1 y(q,B) - z_d||_M^2, \forall q \in Q_m \times \mathcal{P}_{a,b},$$

and then the necessary condition (4.3) is equivalent to

$$\begin{aligned} &\int_0^T \langle C_1^* \Lambda_M (C y(\bar{q},\bar{B})(t) - z_d), y_B(\bar{q},\bar{B}; B - \bar{B})\rangle_{V_2^*,V_2} dt \\ &\qquad + \int_0^T \langle C_1^* \Lambda_M (C_1 y(\bar{q},\bar{B})(t) - z_d), z(t)\rangle_{V_2^*,V_2} dt \geq 0, \\ &\forall (q,B) \in Q_m \times \mathcal{P}_{a,b}, \end{aligned} \tag{4.4}$$

Let us introduce an adjoint state $\eta(\bar{q},\bar{B})$ satisfying

$$\begin{aligned} &\eta''(\bar{q},\bar{B}) - A_2(t,\bar{q})\eta'(\bar{q},\bar{B}) + [(A_1(t,\bar{q}) - A_2'(t,\bar{q})) \\ &\qquad + (N^* g_y(Ny(\bar{q},\bar{B})N)^* + \bar{B}^*]\eta(\bar{q},\bar{B}) \\ &\qquad = C_1^* \Lambda_M (C_1 y(\bar{q},\bar{B}) - z_d), \\ &\eta(\bar{q},\bar{B})(T) = 0, \quad \eta'(\bar{q},\bar{B})(T) = 0. \end{aligned} \tag{4.5}$$

Since $C_1^* \Lambda_M (C_1 y(\bar{q},\bar{B}) - z_d) \in L^2(0,T;V_2^*)$ and $A_2'(t,\bar{q}) \in L^\infty(0,T;$ $\mathcal{L}(V_2, V_2^*))$, the equation (4.5) is well-posed and permits a unique weak solution $\eta(\bar{q},\bar{B}) \in W(0,T)$ if we consider the change of the time variable as $t \to T-t$. Multiplying (4.5) by z, which is the weak solution to (4.2), integrating it by parts after integrating it on $[0,T]$, we obtain

$$\begin{aligned}
&\int_0^T \langle \eta(\bar{q},\bar{B})(t), z''(t) + A_2(t,\bar{q})z'(t) + [A_1(t,\bar{q}) \\
&\qquad + N^* g_y(Ny(\bar{q},\bar{B})(t))N + \bar{B}]z(t)\rangle_{V^*,V}dt \\
&= \int_0^T \langle \eta(\bar{q},\bar{B})(t), -DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{B})(t) \\
&\qquad - DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{B})(t)\rangle_{V^*,V}dt \\
&\quad + \int_0^T \langle \eta(\bar{q},\bar{B})(t), (\bar{B}-B)y(\bar{q},\bar{B})(t) \\
&\qquad + f_q(t,\bar{q};q-\bar{q})\rangle_{V^*,V}dt \geq 0, \forall (q,B) \in Q_m \times \mathcal{P}_{a,b}.
\end{aligned} \tag{4.6}$$

From (4.3) and (4.4), we obtain the inequality

$$\begin{aligned}
&\int_0^T \langle \eta(\bar{q},\bar{B})(t), z''(t) + A_2(t,\bar{q})z'(t) + [A_1(t,\bar{q}) \\
&\qquad + N^* g_y(Ny(\bar{q},\bar{B})(t))N + \bar{B}]z(t)\rangle_{V^*,V}dt \\
&\quad + \int_0^T \langle C_1 y_B(\bar{q},\bar{B};q-\bar{q})(t), C_1 y(\bar{q},\bar{B})(t) - z_d\rangle dt \\
&= \int_0^T \langle \eta(\bar{q},\bar{B})(t), -DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{B})(t) \\
&\qquad - DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{B})(t)\rangle_{V^*,V}dt \\
&\quad + \int_0^T \langle \eta(\bar{q},\bar{B})(t), (\bar{B}-B)y(\bar{q},\bar{B})(t) + f_q(t,\bar{q};q-\bar{q})\rangle_{V^*,V}dt \\
&\quad + \int_0^T \langle C_1 y_B(\bar{q},\bar{B};q-\bar{q})(t), C_1 y(\bar{q},\bar{B})(t) - z_d\rangle_{V^*,V}dt \geq 0, \\
&\qquad \forall (q,u) \in Q_m \times \mathcal{P}_{a,b}.
\end{aligned}$$

Here we used the inequality (4.4). Summarizing these we have the following theorem.

THEOREM 4.1. *Assume that* $H(A)$, $H(f)$, $H(N)$, $H(g)$, $H(A)_1$, $H(A)_2$, $H(f)_1$, $H(f)_2$, $H(g)_1$ *hold. Then the optimal pairs* $(\bar{q}, \bar{B})$ *is characterized by state and adjoint systems and inequality:*

$$\begin{cases} y''(\bar{q}, \bar{B}) + A_2(t, \bar{q})y'(\bar{q}, \bar{B}) + A_1(t, \bar{q})y(\bar{q}, \bar{B}) + N^*g(Ny(\bar{q}, \bar{B})) \\ \qquad + By(\bar{q}, \bar{B}) = +f(t, \bar{q}) \quad in\ (0, T) \\ y(\bar{q}, \bar{B})(0) = y_0 \in V, y'(\bar{q}, \bar{B})(0) = y_1 \in H, \end{cases}$$

$$\begin{cases} \eta''(\bar{q}, \bar{B}) - A_2(t, \bar{q})\eta'(\bar{q}, \bar{B}) + [(A_1(t, \bar{q}) - A_2'(t, \bar{q})) \\ \qquad\qquad + (N^*g_y(Ny(\bar{q}, \bar{B})N)^* + \bar{B}^*]\eta(\bar{q}, \bar{B}) \\ \qquad = C_1^*\Lambda_M(C_1y(\bar{q}, \bar{B}) - z_d) \ \ in\ (0, T), \\ \eta(T, \bar{q}) = 0, \eta'(T, \bar{q}) = 0, \end{cases}$$

$$\begin{cases} \int_0^T \langle \eta(\bar{q}, \bar{B})(t), (\bar{B} - B)y(\bar{q}, \bar{B})(t) + f_q(t, \bar{q}; q - \bar{q})\rangle_{V^*, V} dt \\ \qquad + \int_0^T \langle C_1 y_B(\bar{q}, \bar{B}; B - \bar{B})(t), C_1 y(\bar{q}, \bar{B})(t) - z_d\rangle_{V^*, V} dt \\ \quad \geq \int_0^T \langle \eta(\bar{q}, \bar{B})(t), DA_2(t, \bar{q}; q - \bar{q})y'(\bar{q}, \bar{B})(t) \\ \qquad + DA_1(t, \bar{q}; q - \bar{q})y(\bar{q}, \bar{B})(t)\rangle_{V^*, V} dt, \forall (q, u) \in Q_m \times \mathcal{P}_{a,b}. \end{cases}$$

4.2. The case where $C_2 \in \mathcal{L}(H, M)$

In this case the cost functional is given by

$$J(q, B) = \frac{1}{2}||C_2 y(q, B)(T) - z_d||_M^2, \ \forall\, (q, B) \in Q_m \times \mathcal{P}_{a,b},$$

the necessary condition (4.3) is equivalent to

$$\begin{aligned} &(C_2^*\Lambda_M(C_2 y(q, B)(T) - z_d), z(T))_H \\ &\quad + (C_2^*\Lambda_M(C_2 y(q, B)(T) - z_d), y_B(\bar{q}, \bar{B}; B - \bar{B})(T))_H \geq 0, \\ &\quad \forall (q, B) \in Q_m \times \mathcal{P}_{a,b}. \end{aligned} \tag{4.7}$$

Let us introduce an adjoint state $\eta(\bar{q},\bar{B})$ satisfying

$$(4.8)\qquad \begin{cases} \eta''(\bar{q},\bar{B}) - A_2(t,\bar{q})\eta'(\bar{q},\bar{B}) + [(A_1(t,\bar{q}) - A_2'(t,\bar{q})) \\ \qquad + (N^*g_y(Ny(\bar{q},\bar{u})N)^* + \bar{B}^*]\eta(\bar{q},\bar{B}) = 0, \\ \eta(\bar{q},\bar{B})(T) = 0, \\ \eta'(\bar{q},\bar{B})(T) = -C_2^*\Lambda_M(C_2y(\bar{q},\bar{B})(T) - z_d). \end{cases}$$

It follows by the same reason as the case 4.1 that there is a unique weak solution $\eta(\bar{q},\bar{B}) \in W(0,T)$, because $C_2^*\Lambda_M(C_2y(\bar{q},\bar{B})(T) - z_d) \in H$.

THEOREM 4.2. *We assume that H(A), H(f), H(N), H(g), $H(A)_1$, $H(A)_2$, $H(f)_1$, $H(f)_2$ and $H(g)_1$ hold. Then the optimal pairs $(\bar{q},\bar{B})$ is characterized by state and adjoint systems and inequality:*

$$\begin{cases} y''(\bar{q},\bar{B}) + A_2(t,\bar{q})y'(\bar{q},\bar{B}) + A_1(t,\bar{q})y(\bar{q},\bar{B}) + N^*g(Ny(\bar{q},\bar{B})) \\ \qquad + By(\bar{q},\bar{B}) = f(t,q) \quad in\ (0,T), \\ y(\bar{q},\bar{B}) = y_0 \in V, y'(\bar{q},\bar{B}) = y_1 \in H, \end{cases}$$

$$\begin{cases} \eta''(\bar{q},\bar{B}) - A_2(t,\bar{q})\eta'(\bar{q},\bar{B}) + [(A_1(t,\bar{q}) - A_2'(t,\bar{q})) \\ \qquad + (N^*g_y(Ny(\bar{q},\bar{B})N)^* + \bar{B}^*]\eta(\bar{q},\bar{B})(T) = 0, \\ \eta(\bar{q},\bar{B})(T) = 0, \\ \eta'(\bar{q},\bar{B})(T) = -C_2^*\Lambda_M(C_2y(\bar{q},\bar{B})(T) - z_d), \end{cases}$$

$$\begin{cases} (C_2^*(C_2y(\bar{q},\bar{B})(T) - z_d)_H \\ \quad + \int_0^T \langle(\bar{B} - B)y(\bar{q},\bar{B})(t) + f_q(t,\bar{q};q-\bar{q}), \eta(\bar{q},\bar{B})(t)\rangle_{V^*,V}dt \\ \geq \int_0^T \langle DA_2(t,\bar{q};q-\bar{q})y'(\bar{q},\bar{B})(t) \\ + DA_1(t,\bar{q};q-\bar{q})y(\bar{q},\bar{B})(t), \eta(\bar{q},\bar{B})(t)\rangle_{V^*,V}dt, \forall(q,B) \in Q_m \times \mathcal{P}_{a,b}. \end{cases}$$

PROOF. We prove the inequality condition of optimal control only. Multiplying (4.8) by z, which is a weak solution to (4.2), integrating it by parts after integrating it on $[0,t]$, we obtain

$$
\begin{aligned}
&(z(T), \eta'(\bar{q}, \bar{B})(T))_H + \int_0^T \langle \eta(\bar{q}, \bar{B})(t), z''(t) + A_2(t, \bar{q}) z'(t) + [(A_1(t, \bar{q}) \\
&\qquad\qquad + N^* g_y(N y(\bar{q}, \bar{B})(t) N + \bar{B}] z(t) \rangle_{V^*, V} dt \\
&= \int_0^T \langle \eta(\bar{q}, \bar{B})(t), -DA_2(t, \bar{q}; q - \bar{q}) y'(\bar{q}, \bar{B})(t) \\
&\qquad\qquad - DA_1(t, \bar{q}, q - \bar{q}) y(\bar{q}, \bar{B})(t) \rangle_{V^*, V} dt \\
&\quad + \int_0^T \langle \eta(\bar{q}, \bar{B})(t), (\bar{B} - B) y(\bar{q}, \bar{B})(t) + f_q(t, \bar{q}; q - \bar{q}) \rangle_{V^*, V} dt \\
&\quad + (z(T), -C_2^* \Lambda_M (C_2 y(\bar{q}, \bar{B})(T) - z_d))_H = 0, \\
&\qquad \forall (q, B) \in Q_m \times \mathcal{P}_{a,b}.
\end{aligned}
$$

Hence from (4.7) and (4.8) we conclude that

$$
\begin{aligned}
&(z(T), C_2^* \Lambda_M (C_2 y(\bar{q}, \bar{B})(T) - z_d))_H \\
&\qquad + (y_B(\bar{q}, \bar{B}; u - \bar{B})(T), C_2^* \Lambda_M (C_2 y(\bar{q}, \bar{B})(T) - z_d)_H \\
&= \int_0^T \langle \eta(\bar{q}, \bar{B})(t), -DA_2(t, \bar{q}; q - \bar{q}) y'(\bar{q}, \bar{B})(t) \\
&\qquad\qquad - DA_1(t, \bar{q}; q - \bar{q}) y(\bar{q}, \bar{B})(t) \rangle_{V^*, V} dt \\
&\quad + \int_0^T \langle \eta(\bar{q}, \bar{B})(t), (\bar{B} - B) y(\bar{q}, \bar{B})(t) + f_q(t, \bar{q}; q - \bar{q}) \rangle_{V^*, V} dt \\
&\quad + (y_B(\bar{q}, \bar{B}; B - \bar{B})(T), C_2^* \Lambda_M (C_2 y(\bar{q}, \bar{B})(T) - z_d)_H \geq 0, \\
&\qquad \forall \ (q, B) \in Q_m \times \mathcal{P}_{a,b}.
\end{aligned}
$$

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