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The Characteristic Analysis of E.M.F. Induced by Moving Normal Spot in a Cylindrical Superconducting Foil

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Abstract

The e.m.f. induced by a normal spot moving in a superconducting foil has been investigated by using the simulation and the experiment of a simple superconducting Power Supply. The induced e.m.f. has been derived theoretically from the magnetic field distribution within the spot. It is the sum of a DC component induced constantly by the Faraday's law during the spot's movement and a pulse component induced periodically by the flux conservation law at every electrical degrees 2π radians. The DC component of the output voltage appears slightly nonlinear to the rotating speed, having values greater than the linear approximation values. The theoretical interpretation has been verified through experiment.

Keywords : normal spot, superconducting foil, superconducting power supply, Faraday's law, flux conservation law

I. Introduction

Superconducting power supply are among the known classes of superconducting generators that can supply large currents to superconducting load at low voltage. They have many merits such as persistent current mode at rest, their compact size, high efficiency, convenient change in rotating direction, and easy construction. Because of these advantages, several investigators have studied the machines such as exciters for brushless superconducting generators.

Despite all the efforts, there is still a definite need for a better understanding of their mechanisms. This is of paramount importance towards developing reliable design and construction criteria for these machines.

Therefore, In this paper has been made an attempt to reach these goals by looking in some details at the electromagnetic mechanism taking place in the normal spot. In the last section of the paper our theoretical derivations are compared with experimental observations.

II. Magnetic field distribution in the normal spot

Fig. 1 shows the scheme of a superconducting power supply. Several permanent magnets (M_1~M_n) are arranged with the same pole direction around a cylindrical superconducting foil, and each magnet makes a normal spot on the foil.

Current leads are arrayed at the same distance over the top and the bottom of the superconducting foil. They are connected to the superconducting loads after being put together in other to form the output terminals of the superconducting power supply. A superconducting power supply basically uses electro-

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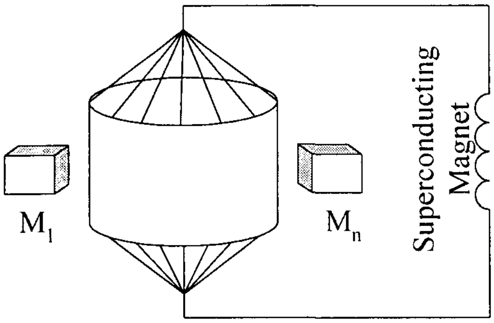

Fig. 1. The scheme of a superconducting power supply

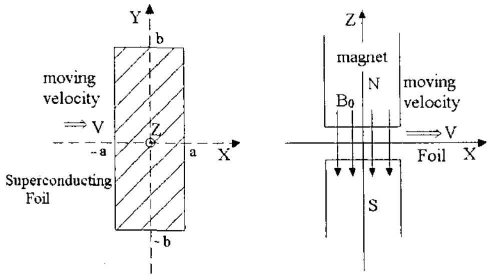

Fig 2. The mathematical model for the analysis

magnetic phenomena induced by the moving magnets above the superconducting foil. For this reason, the capability of a superconducting power supply is strongly dependent upon the physical phenomena in the normal spot.

In particular, the output voltage of the superconducting power supply is a function of magnetic field distribution in the normal spot. Therefore, in order to optimize the design condition of a superconducting power supply, it is necessary to determine the magnetic field distribution in the normal spot. The first step is to simplify its model in order to solve the problem by using electromagnetic mechanism in the superconducting foil at the presence of an applied field.

It is assumed that the superconducting foil is infinitely larger than the normal spot, the magnet is rectangular in shape, and the two sides perpendicular to the moving direction of the normal spot are much longer than the other two. Consequently, relative change in the y-direction can be neglected, if we confine the moving direction of the normal spot to x, and set the axis perpendicular to the foil to z-axis, forming a three dimensional coordinate system.

The flux from the N-pole flows through the superconducting foil into the S-pole of the same magnet. Accordingly, if the geometrical center of the foil is assumed to be the origin of the coordinate system in the applied field, the field is perfectly symmetric about the plane, z=0. The superconducting foil in the coordinate system moves in the positive direction of the x-axis at speed V.

In this case, because only the relative velocity of the superconducting foil to the magnet is important, the normal spot is assumed to be time-independent, expressed as a fixed point in the coordinate system. It is assume that the applied field is uniform and it has the negative direction of z-axis. Moreover, if the air gap between N-pole and S-pole is supposed to be very small, the fringing effect in the gap could be negligible.

On the other hand, resistivity below the critical temperature is independent of temperature. Although resistivity depends on the field, it can be approximated as a step function of magnetic field from kir-schenbaum's measurement. That is, conductivity is constant above the critical magnetic field, and can be neglected otherwise. The mathematical model made based upon these assumptions is shown in Fig. 2.

Magnetic field distribution is obtained from Ohm's law about moving and Maxwell equations.

$$\vec{J} = \sigma(\vec{v} \times \overrightarrow{B_N}) \tag{1}$$

$$\nabla \times \vec{B} = \mu \vec{J} \tag{2}$$

$$\nabla \cdot \vec{B} = 0 \tag{3}$$

$$\nabla \times \vec{E} = 0 \tag{4}$$

$$\nabla \cdot \vec{E} = 0 \tag{5}$$

For above, $\vec{J}$ is the eddy current induced from the moving normal spot, σ is the conductivity of superconducting foil, and μ is the permeability. If $\overrightarrow{B_0}$ is the magnetic field applied from out side, and $\vec{B}$ is the magnetic field induced in the normal spot, the total field in the normal spot $\overrightarrow{B_N}$ must be as follows.

$$\overrightarrow{B_N} = \overrightarrow{B_0} + \vec{B} \tag{6}$$

Taking curl of equation (2) and simplifying leads to the equating for the magnetic field distribution to be expressed as following.

$$\nabla^2 \vec{B} = \mu\sigma(\vec{v} \cdot \nabla)\vec{B} \tag{7}$$

Supposed that the superconducting foil is very thin, the current in the normal spot has only the J_y component. This means the magnetic field induced by the eddy current has only z component, and it can be assumed that there is no relative change in z direction. It is reminded that there is no relative change in y direction. And it is convenient to introduce the magnetic Reynolds's number ($R = \mu\sigma v a$), which determines the characteristics of magnetic field and affects the compression of magnetic field. Thus, equation (7) can be expressed as below.

$$\frac{\partial^2 B_z}{\partial x^2} - \frac{R}{a}\frac{\partial B_z}{\partial x} = 0 \tag{8}$$

Then, the solution of equation (8) is given by

$$B_z = A_1 + A_2 \exp\left(\frac{Rx}{a}\right) \tag{9}$$

Here, a is half of the normal spot's width, and A_1 and A_2 are constants. Also, equations (1) and (2) induce the following condition.

$$\frac{\partial B_x}{\partial x} = \mu\sigma v B_{Nz} \tag{10}$$

By using equation (6), (9) and (10), $A_1 = B_0$ is obtained.

Also, if x_0 is set to the position of $B_z = 0$, the total magnetic field at x_0 consists of only $-B_0$. Therefore, x_0 can be obtained as follows.

$$x_0 = \frac{a}{R}\ln\left(-\frac{B_0}{A_2}\right) \tag{11}$$

Compression of the field made by the moving normal spot can be determined by equation (11). The value of constant A_2 is obtained from the condition that the flux induced by eddy current on the left side of x_0 in the normal spot is equal to the flux induced on the right side.

$$\int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-a}^{x_0} B_z dxdy = \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{x_0}^{a} B_z dxdy$$

$$A_2 = -\frac{B_0}{\cosh R} \tag{12}$$

Therefore, the equation of the total field in the normal spot is as follows.

$$B_{Nz} = -\frac{B_0 R}{\sinh R}\exp\left(\frac{Rx}{a}\right) \tag{13}$$

III. Output Voltage Characteristics

The output voltage of a superconducting power supply is obtained "flux conservation law" that the total magnetic flux threading a closed resistanceless circuit cannot change so long as the circuit remains resistanceless, and "Faraday's law" that the electromotive force is induced in a conductor when a magnetic field of certain magnitude is in motion within it. In a superconducting power supply, the magnet rotates about the foil periodically.

This means, if the pole pitch is expressed as the electrical degree of 2π radians, switching of the current path occurs at every 2π radians. And this, in turn, generates the induced electromotive force can be approximated as following.

$$|emf| = \frac{\Delta\Phi(t)}{\Delta t} \propto \frac{\Phi_0}{T} = \Phi_0 \times f_s \tag{14}$$

Here, Φ_0 is the flux linked at every 2π radians, and T is the revolution period of poles, and f_s is

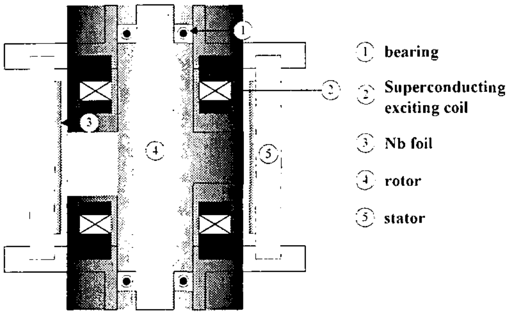

Fig. 3 Schematic of the superconducting power supply

the revolution frequency of poles. For equation (14) represents the voltage at $t = kT$, this is extended to the case of an arbitrary time to be represented as

below.

$$V_p(t) = \sum_{k=0}^{\infty} \Phi_0 f_s \delta(t - kT) \tag{15}$$

Here, $\delta(t)$ is Dirac delta function. And, a constant field rotates over the superconducting foil during operation. Consequently, the foil cuts the field at a constant speed, and this causes motional emf(V_d) to remain constant as the emf of the Faraday disk generator.

$$V_d = \int_h^b (\vec{v} \times \overrightarrow{B_N}) \cdot \overrightarrow{dl} = vB_N l \tag{16}$$

l means length of the normal spot in the y-direction. According to the statement above, eddy current circulates about the normal spot by this electromotive force. The eddy current flows through the mixed-state region at the front side of the moving normal spot, and in this case, magnetic coupling is made with the load current flowing through this region. Therefore, power generated in the normal spot is transferred steadily to the load as in transformers. As a result, the output voltage(V_s) of the superconducting power supply is shown below.

$$V_s(t) = \sum_{k=0}^{\infty} \Phi_0 f_s \delta(t - kT) + vB_N l \tag{17}$$

IV. Construction and Characteristic Experiment

Table I. Specification of the Nb foil

Material	Niobium
Thickness	50 μm
Size	100×377 mm
Critical magnetic field	$0.2\,T$ (at 4.2K)
$1/\sigma$	4.83×10^{-5} μΩm

Table II. Specifications of the Rotor, the Stator

Scection	Contents	
Rotor	Material	Wrought iron
	Outer diameter	110 mm
	Pole's cross-section shape	Round rectangular
	The number of pole	1
	Pole's width	14 mm
	Longitudinal length	50 mm
Stator	Material	Silicon steel plate
	Inner diameter	120 mm
	Outer diameter	150 mm
	Laminated thickness	110 mm

For evaluating the validity of the theoretical prediction, a small-scale superconducting power supply is constructed and characteristic experiment is carried out. Fig. 3 is the scheme of the superconducting power supply. The superconducting power supply is composed of a cylindrical Nb foil and a rotating pole. A couple of superconducting wire are spot-welded across Nb foil and then connected to superconducting load. Table I is the specification of the used Nb foil, and Table II is specification of the constructed stator and rotor. In addition, a hall sensor was put at the center of Nb foil, and a couple of voltages are recorded on the analog tape recorder after passing the insulating amplifier to measure the magnetic field in the normal spot.

The experiment has been performed through the following process. After applying above the critical magnetic field of Nb foil using two superconducting excitation coils installed over and under the rotor.

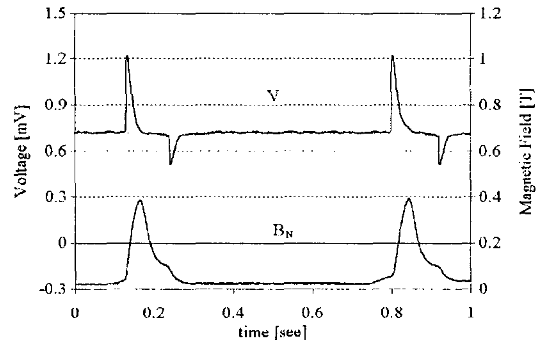

Fig. 4 output voltage and magnetic field distributio (N=90rpm)

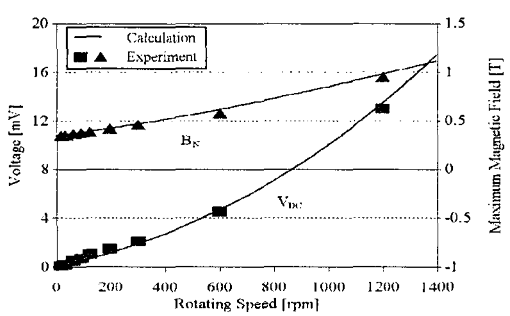

Fig. 5 magnetic field and DC output voltage versus th rotating speed

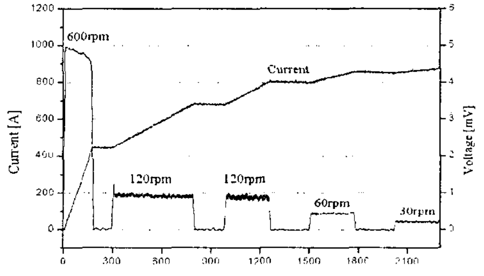

Fig. 6 Pumping current and output voltage

Output parameters such as voltage, magnetic filed, pumping current are measured at various rotating speed

V. Results and Discussion

Fig. 4 represents output voltage and vertical magnetic field distribution near the normal spot at applied field 0.35T, load inductance 2mH, and rotating speed 90rpm. As expected in equation (17), it is observable that the output voltage has DC component of 0.7mV, and pulse voltage appears precisely at every 0.67 seconds. On the other hand, maximum magnetic field in the normal spot was 0.4T, and the width of the normal spot was measured in 28mm units, which is two times as that of a real pole. Fig. 5 shows magnetic field in front of the normal spot and DC component of the output voltage with respect to rotating speed. Fig. 4 and Fig. 5 shows the compression of magnetic field in front of the normal spot resulting from increasing rotating speed. This is consistent with theoretical prediction in equation (13). Fig. 6 is the waveforms of output voltage and pumping current with respect to the rotating speed. Through equation (17), Fig. 5 and Fig. 6, it can be seen that the DC component of the output voltage appears to be slightly non-linear to the rotating speed, having values greater than the linear approximation values. Also, it is found that analytic results are well consistent with experimental data as the rotating speed increases.

VI. Conclusion

The conclusions obtained from this research are as follows.

a. The output voltage of the superconducting power supply is the sum of the DC component induced continuously during operation, and the pulse component generated at every electrical degree 2π radians.
b. The magnetic field in front of the normal spot is compressed with the increment of rotating speed.
c. The DC component of the output voltage appears to be slightly nonlinear to the rotating speed, having values greater than the linear approximation values.
d. As the rotating speed increases, the charging rate of the superconducting load becomes more dependent on the DC component of the output voltage.

Accordingly, the optimized design condition of system could be found, if the results from this study was used for the exciter design of the superconducting load requiring fast charging such as field windings of the superconducting generator.

Acknowledgments

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