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Linear Temperature Dependence of Magnetic Penetration Depth Length at Low T in an Isotropic Superconductor

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Abstract

The notion of the finite pairing interaction energy range Td is shown to result in a linear temperature dependence of the London magnetic penetration depth length, $\Delta\lambda/\lambda(0) = (T/Td)(2/\pi)\ln 2$ at low T in the case of the s-wave pairing state, accounting for data of high Tc superconductor by Hardy et al.

Keywords : Superconductivity, magnetic penetration depth length, superfluid density

The fluxoid quantization [1] in high Tc superconductor (HTS) indicates that pairings of carriers are responsible for superconductivity in HTS as in low Tc superconductor (LTS).

The oxygen isotope effect [2], Tc $\propto M^{-\alpha}$ with $\alpha = 0.40 \sim 0.49$ in LaSrCuO single crystal, suggests the electron-phonon interaction would play an important role for understanding superconductivity in cuprate materials. Thus, one expects the Bardeen, Cooper, and Schrieffer (BCS) pairing theory [3,4] would account for data of HTS as well as LTS. However, the symmetry of pairing state in HTS, s-wave or d-wave, is still unsettled.

The data on interference associated with two weakly coupled superconductors made with YBCO epitaxial films [5], YBCO-Pb tunnel junction along the c-axis direction [6] and microwave induced steps [7] suggest the s-wave is the proper symmetry for the order parameter in HTS. But the data on YBCO-Pb SQUIDs and on tunneling junction along ab-direction [8], also known as the π-phase shift data, and half fluxoid quantum observed in a YBCO ring [9] are considered as evidences for the d-wave pairing state.

However, the notion of multi-connected superconductor with the s-wave pairing state [10] also can account for data of [8] and [9].

The recent BiSrCaCuO bicrystal c-axis twist Josephson junction experiment [11] indicates the dominant order parameter contains the s-wave and not d-wave component. Moreover, no node in the order parameter is observed in the angular dependence of the non-linear transverse magnetic moment of YBCO in the Meissner state [12]. On the other hand, the scanning tunneling microscope imaging the effects of individual zinc impurity atoms on superconductivity in BiSrCaCuO [13] shows the four fold symmetric quasiparticle cloud, indicating the d-wave component. But no four fold is observed in the same system [14]. The observation of [13] may be a reflection of the Fermi surface.

In this paper, I discuss, from the finite Td point of view [15], the temperature dependence of the London magnetic penetration depth length $\lambda(T)$ which reflects the condensed carrier density, superfluid density,

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$\rho_s(T) / \rho_s(0) = [\lambda(0) / \lambda(T)]^2$. (1)

The $\rho_s(T)$ plays an important role for understanding the nature of condensation. In the two fluid picture [16], $\rho_s(T)$ varies as $1 - (T/Tc)^4$. But the BCS - $\rho_s(T)$ has an activation form at low T via the order parameter Δ, which indicates the excitation energy gap.

The measurements of $\lambda(T)$ at low T in HTS are compatible with neither the BCS result nor the two fluid picture. The data of $\lambda(T)$ in a single crystal YBCO [17] has a linear T dependence. This linear T dependence in fact is taken as providing evidence that the order parameter has nodes, suggesting the d-wave pairing state [18].

Contrary to the general belief, I show here that it is not necessary to have nodes in the order parameter, to account for a linear T dependence of $\rho_s(T)$ at low temperature.

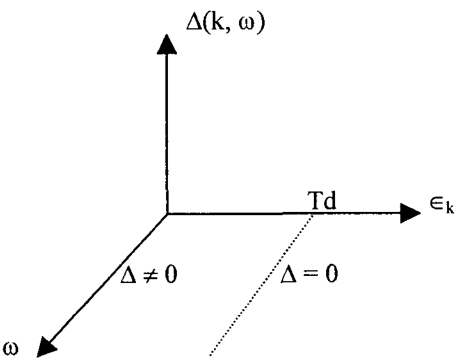
The fact is that to have a finite value of Tc, a finite pairing interaction energy range Td [15] is required within the pairing theory, since Tc is scaled with Td. In other words, the order parameter $\Delta(k, \omega)$ may be given as [15,19]

$$\Delta(k, \omega) = \begin{cases} \Delta & \text{for } |\epsilon_k| < Td \\ 0 & \text{for } |\epsilon_k| > Td \end{cases} \qquad (2)$$

for all frequencies ω. Here ϵ_k is the usual normal state excitation energy with momentum k, measured with respect to the Fermi level.

Units of = c = k_B = 1 are used here.

Physically, pairs are to be formed within a finite Td, $|\epsilon_k| < Td$. In other words, the states outside Td would not participate in pairings. The dynamical frequency ω does not have any constraint. Thus, one expects the density of states $N(\omega)$ would have low energy states resulted from states not participating in pairings [15], not like the BCS density of states $N_{BCS}(\omega)$. Mathematically, for a given ω, the sum of the spectral weights outside Td yields low energy states in $N(\omega)$.



Thus, for a finite Td, the condensation is not complete at zero temperature. Naturally, the multi-connected superconductors are realized [10], which can account for data of [8] and [9] as stated before.

Let us consider in a moment why $N_{BCS}(\omega)$ does not have low energy states. The reason is as follows : For $N_{BCS}(\omega)$, Td was considered as infinite. That is, all states are to participate in pairings. In other words, all low energy states are to be pushed up to high energy states. The condensation is complete at zero temperature, 100%. But for Tc (BCS), Td was treated as a finite value, to have a finite value of Tc (BCS). Thus, the BCS results are reliable only for Td >> Tc such as in LTS, and have several defects [15,19].

For a finite Td, pair breaking $\Delta = 0$ for $|\epsilon_k| > Td$ makes the states, which were pushed up to high energy states, go hack to where they came from. In other words, some low energy states are not pushed up to high energy states and remain as low energy states.

The density of states $n(\omega) = N(\omega)/N(0)$ is obtained via carrying out the ϵ_k-integration of the imaginary part of the usual Green's function [4], consistent with Eq (2), as [15,19]

$$n(\omega) = q(\omega/Td) + n_{BCS}(\omega)\, r(\omega/Td), \qquad (3)$$

$$q(\omega/Td) = (2/\pi) \tan^{-1}(\omega/Td), \qquad (4)$$

$$r(\omega/Td) = (2/\pi) \tan^{-1}[n_{BCS}(\omega) Td/\omega], \qquad (5)$$

$$n_{BCS}(\omega) = \mathrm{Re}\, \{\omega/(\omega^2 - \Delta^2)^{1/2}\}. \qquad (6)$$

The $q(\omega/Td)$ is resulted from mathematically the ϵ_k-integration of the Green's function with

$$\Delta(k, \omega) = 0 \quad \text{for } |\epsilon_k| > Td,$$

and physically the states not participating in pairings.

For the temperature dependence of the London penetration depth length $\Delta\lambda = \lambda(T) - \lambda(0)$, one may consider the temperature dependence of the normal component in the two fluid model of superconductivity. For this, it is worthy to recapitulate the spirit of Bardeen [20].

Let us first consider the superfluid at rest (corresponding to a zero momentum pairing condition) and supose that the excitations (normal component) have a net momentum $\boldsymbol{J}_n$. Their distribution function $f(\boldsymbol{p})$, is determined to minimize the free energy, F, subject to the subsidiary condition

$$\boldsymbol{J}_n = \sum_p \boldsymbol{p}\, f(\boldsymbol{p}). \tag{7}$$

Note that only single excitations in the BCS sense contribute to Eq. (7). Introducing _ as the Lagrange multiplier for $\boldsymbol{J}_n$, the minimization condition,

$$\delta F - \_ \cdot \delta \boldsymbol{J}_n = 0 \tag{8}$$

yields, via the BCS procedure,

$$f(\boldsymbol{p}) = 1 / [1+\exp \widetilde{E}(\boldsymbol{p})/T]\,, \tag{9}$$

$$\widetilde{E}(\boldsymbol{p}) = E(\boldsymbol{p}) - \_ \cdot \boldsymbol{p}\,, \tag{10}$$

where E($\boldsymbol{p}$) is the BCS spectrum.

For small _, $\boldsymbol{J}_n$ is proportional to _, and the coefficient of proportionality is defined as the normal density, ρ_n . This gives,

$$\rho_n = \lim_{\upsilon\to 0} (J_n/\_) = \sum_p pp \frac{d}{dE} f(E). \tag{11}$$

which coincides with the BCS result of $\rho_n = \rho - \rho_s$.

Now if the whole system is displaced in momentum space and moves with a velocity $\_s$, the pairs have a common velocity $\_s$. Defining $\_n = \_ + \_s$, it follows quite generally that for small _, the total momentum (mass current) is

$$\boldsymbol{J} = \rho\_s + \rho_n\_ = \rho_s\_s + \rho_n\_n\,, \tag{12}$$

and the associated increase in free energy is

$$\Delta F = \frac{1}{2}\rho_n \_n^{\,2} + \frac{1}{2}\rho_s \_s^{\,2}\,. \tag{13}$$

Thus, the two fluid model of superconductivity is microscopically realized.

Via $\omega = E(\boldsymbol{p})$ in Eq. (11), we may write the temperature dependence of $\lambda(T)$ as [18]

$$\Delta\lambda/\lambda(0) = \int_0^\infty d\,(\omega/T)\; n(\omega) f(\omega/T)[1-f(\omega/T)], \tag{14}$$

where $f(x)$ is the Fermi function as before.

Physically, the factor $f(\omega/T)$ in Eq. [14] is the occupation probability, say, of the state $|\uparrow\rangle$, and the $[1-f(\omega/T)]$ factor is the unoccupation probability of the partner state, say, $|\downarrow\rangle$, or vice versa, since the unpaired states contribute to the normal fluid density. The factor 2 of spin sum is cancelled with 2 via

$$\Delta\rho_s / \rho_s(T) = 2\Delta\lambda / \lambda(T).$$

For the BCS density of states $n_{BCS}(\omega)$, the well known result at low T follows

$$[\Delta\lambda/\lambda(0)]_{BCS} = (2\pi\Delta/T)^{1/2} \exp(-\Delta/T). \tag{15}$$

By inserting q(ω/Td) of Eq. (4) into Eq. (14), at low temperature, we get [21]

$$[\Delta\lambda/\lambda(0)]_q = (T/Td)(2/\pi)\ln 2\,, \tag{16}$$

similar to that resulted from the order parameter of the d-wave symmetry [18]

$$[\Delta\lambda/\lambda(0)]_d = (T/\Delta_o)\ln 2 \tag{17}$$

via $n_d(\omega) = \omega/\Delta_o$, where Δ_o is the maximum value (anti-node) of the order parameter.

The linear T dependence of $\lambda(T)$ of Eq. (16) is a reflection of low energy states, q(ω/Td). In LTS, the pairing interaction energy range Td ~ Tc exp(1/g) in the weak coupling g limit, is large compared to Tc, and makes the linear T dependence of $\lambda(T)$ hardly observable. On the orther hand, in HTS, even though the exact nature of the pairing interaction is not known, Td appears to be of the order of Tc. Thus, the linear T dependence of $\lambda(T)$ is observed [17]. In fact, the data of [17] results in Td = 2Tc via Eq. (16).

In conclusion, from the finite Td point of view, the condensation is not complete at zero temperature. The states not participating in pairings yield a linear T dependence of $\lambda(T)$ at low T. A linear T of $\rho_s(T)$ does not imply nodes in the order parameter.

I hope the notion of a finite pairing interaction energy range may be able to resolve other unsolved problems in HTS.

Recently, in the spirit of a finite Td, the spinless impurity isotropic scattering is shown [22] to reduce Tc and destroy superconductivity, accounting for data of Zn-doped YBCO [23].

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