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CN AND *CH* BAND STRENGTHS OF BRIGHT GIANTS IN THE GLOBULAR CLUSTER M15

SANG-GAK LEE
Astronomy Program, SEES, Seoul National University, Seoul 151-742, Korea
E-mail: sanggak@astrosp.snu.ac.kr
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ABSTRACT

CN and *CH* band strengths for ten new bright giants in the globular cluster M15 have been measured from archival spectra obtained with the Multiple Mirror Telescope. Using published indices for other bright M15 giants, a *CN-CH* band strength anticorrelation is found for bright red giants. However, stars that do not follow the *CN-CH* anticorrelation are also found. They seem to show a positive correlation between the two indices. Among them, all the AGB and HB stars of the sample are included. Stars I-38 and X6, which are located near the RGB fiducial line in the CMD, have low measured *CH*(G) indices compared with other RGB stars. Stars IV-38, S4, and S1, which are all near the RGB tip, have strong measured *CH*(G) indices. Therefore, most of their evolutionary states are suspected to be different from those of a normal single RGB star.

Key words: globular clusters: individual(M15), stars: late-type-stars: Population II

I. INTRODUCTION

Stars on the red giant branch in globular clusters have been found to show variations in the surface abundances of carbon, nitrogen, oxygen, sodium, magnesium, and aluminium. These abundances are found to be strongly correlated. The origin of the variation of these light elemental abundances could be due either to extra mixing during the RGB evolutionary state or *ab initio* abundances of primordial material.

M15 is one of the most metal-poor clusters ([Fe/H]≤ -2.0) among the Galactic globular clusters. In the thorough early work of Trefzger et al. (1983), the atmospheric *C* and *N* abundances were determined to vary from star to star for M15 giant stars. However, they were not correlated negatively as the evolutionary state becomes more advanced. Large star-to-star fluctuations in the abundances of *C* and *N* at any point in the color-magnitude diagram of giants have been found. However, it has been noticed that although $<C/C_o>$ declines with advancing evolutionary state, $<N/N_o>$ exhibits no change. These unusual features seen in M15 were also found earlier in M92 (Carbon et al., 1982). The lack of negative correlation between *C* and *N* abundances constituted an outstanding abundance puzzle requiring explanation in the cases of M15 as well as M92 (Carbon et al., 1982). However, Pilachowski (1988) found convincing evidence that the dredge-up of $O \rightarrow N$ processed material is occurring in some M92 stars near the tip of the giant branch, which had been confirmed for a larger sample of stars by Sneden et al. (1991) who found an *O* versus *N* anticorrelation among both M92 and M15 giants. What caauses the lack of anticorrelation between carbon and nitrogen abundances in M15 and M92 discovered in earlier works?

Recent high resolution spectroscopic study of giants in M15 (Sneden et al. 1997) has proved that the anticorrelation of *Na* with *O* and *Al* with *Mg* for giants is as strong in M15 as in M13, M10, and M92, and except for one unusual giant S8 with a very high *N* abundance, the sum of the *CNO* abundances of five M15 giants is constant as in the case of six M92 giants (Pilachowski 1988). The constancy of $C+N+O$, together with the *Na* and *Al* enhancements and corresponding *O* and *Mg* depletions, strongly suggests that the abundance differences are likely due to variation in the degree of mixing and nucleosynthesis, not to initial differences in the amount of $C+N+O$, proton-capture elements with which the stars were born. Nevertheless, until *C*, *N*, and *O* abundances can be obtained for a large sample of stars with a wider range of evolutionary phase, this conclusion can not be generalized to the cluster as a whole.

Moreover the barium and europium variations in M15 giants (Sneden et al. 1997) are unlikely to be the result of nucleosynthesis within the stars that we have observed. They provide strong evidence for primordial neutron-capture abundance variations in a cluster other than ω Cen.

In this study, we intend to find a relation between *CN* and *CH* band strengths among the red giants of M15, although spectra of a limited number of giants are available in archival spectra. The *CN* band strength is an indicator of nitrogen abundance while the *CH* band strength provides clues to the carbon abundance.

We have investigated the *CN* and *CH* band strengths of ten giant stars in the globular cluster M15 by measuring indices from archival spectra obtained with the Multiple Mirror Telescope. These data are combined with other data in the literature to discuss the relation between *CN* and *CH* band strengths.

II. OBSERVATIONAL DATA AND INDICES

Spectra of ten M15 giants were obtained with the Multiple Mirror Telescope on two nights each in Feb., 1987 and March, 1988 for a study of the "Distant Halo" (Croswell, 1990).

The blue channel of the MMT spectrograph with an 832 l/mm grating blazed at 4300 Å in second order was used. The resolution was 1 Å with 5 pixels per resolution element, and the wavelength coverage ranged from 3600 Å to 4400 Å. The observed data were reduced using the standard NOVA package at the CFA(Center for Astrophysics). I used the wavelength calibrated data for ten giants for this program.

The ten bright stars of M15 are listed in Table 1. Their designations in column 1 are taken from Arp (1955) or Sandage (1970) and the star numbers from the original Kuster (1921) study are listed in the next column. Columns 3 and 4 list the V and $(B-V)$ values of Cudworth (1976) and those of Sandage (1969). The absolute magnitudes are also listed. The previously published values of the distance modulus to M15 and the interstellar reddening toward it show a significant variation. We adopted $E(B-V)=0.10$ and a distance modulus $(m-M)_V=15.40$ (Durrell and & Harris 1993).

Two stars IV-38 and S4, are listed in the next two rows. Their indices are transformed from the published indices of Shetrone et al. (1999). Seven stars whose indices are transformed from the abundances determined by Trefzger et al. (1983) are listed in the subsequent rows of Table 1. Figure 1 is the color-magnitude diagram of M15 with the giant branch fiducial line taken from Table 2 of Durrell & Harris (1993).

Among the original ten program stars, I-51 and IV-44

Table 1. Data for Bright Giants in M15

Star ID	Kuster No	V	$(B-V)$	M_v	$CN_{(3883)}$	$CN_{(4215)}$	<CN>	CH(G)	Remark
I-38	K943	14.24	0.94	−1.47	0.040	0.080	0.100	0.073	RGB
I-43	K928	13.83	1.03	−1.88	0.068	0.088	0.122	0.130	RGB
I-50	K919	13.57	1.11	−2.14	0.150	0.136	0.211	0.140	RGB
I-51	–	15.80	0.27	0.09	−0.200	0.011	−0.089	–	HB
II-49	K460	15.12	0.88	−0.59	0.090	0.090	0.135	0.142	RGB
III-14	K260	13.90	0.73	−1.81	−0.100	0.080	0.030	0.031	AGB
IV-44	–	15.79	0.25	0.08	−0.100	0.068	0.019	0.051	HB
–	K152	15.26	0.80	−0.45	0.022	0.011	0.022	0.146	RGB
–	K714	15.21	0.67	−0.82	0.045	0.080	0.103	0.047	AGB
S6	K1040	13.40	1.29	−2.31	0.300	0.170	0.320	0.138	RGB
IV-38	K757	12.88	1.43	−2.70	0.445	0.194	0.417	0.229	RGB[a]
S4	K825	12.79	1.34	−2.79	0.151	0.173	0.249	0.195	RGB[a]
II-75	K144	13.06	1.34	−2.52	–	–	0.191	0.130	RGB[a]
–	K341	12.81	1.37	−2.77	–	–	–	0.195	RGB[a]
S1	K431	13.03	1.30	−2.55	–	–	0.223	0.244	RGB[a]
S3	K387	13.44	1.13	−2.14	–	–	0.207	0.148	RGB[a]
S7	K146	13.57	1.12	−2.01	–	–	0.113	0.147	RGB[a]
S8	K969	13.45	1.16	−2.13	–	–	0.722:	0.130	RGB[a]
X6	K29	14.11	1.01	−1.18	–	–	0.158	0.097	RGB[a]

[a]Photometry, V. B-V, and M_v are taken from Sneden et al. (1997, AJ. 114, 1964)
[b]Photometry, V. B-V, and M_v are taken from Trefzger et al. (1983, ApJ. 266, 144)
Note.-Absolute magnitude of S6 is −2.18 in Sneden et al. (1997) which is fainter by 0.13 than that of this study.

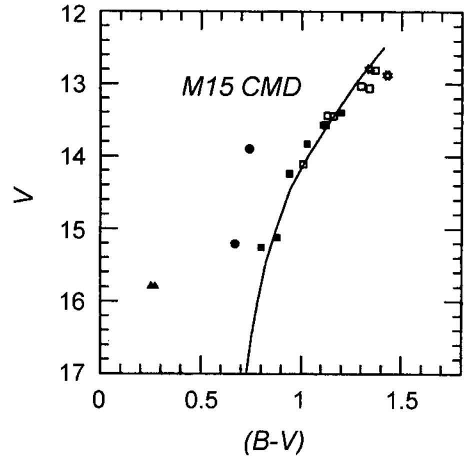

Fig. 1. V vs. $B-V$ color-magnitude diagram of the globular M15. Fiducial line of the giant branch is taken from Durrell & Harris (1993). Filled squares, filled circles, and filled triangles are RGB, AGB, and HB original program stars. Open star symbols and open squares are stars whose indices are taken from Shectrone et al. (1999) and Trefzger et al. (1983), respectively.

are horizontal branch stars (HB), III-14 and K714 are asymptotic giant branch stars (AGB), and the rest of the stars are red giants (RGB) according to their positions in the color-magnitude diagram. They are represented as filled triangles for HB stars, filled circles for AGB stars, and filled squares for RGB stars, respectively.

Two stars taken from Shetrone et al. (1999) are represented as open star symbols and seven stars from Trefzger et al. (1983) are marked as open squares. All RGB stars among the original program stars are found to be members of M15 with 99% probability by Cudworth (1976). While IV-38, II-75, S3, and S7 have a membership probability exceeding 90%, S4, K341, S1, and S8 have a membership probability between 50 % and 90% . Zero % probability of membership for X6 is given by Cudworth (1976). However, on the basis of its kinematics as observed by Drukier et al. (1998), we assume X6 is a member.

The definitions of $CN_{(3883)}$, $CN_{(4215)}$, and CH(G) indices are the same as in Lee (1999)'s study of M3. These indices are measurements of the strength of the CN and CH bands. The CN indices are the same as those in smith et al. (1996, 1997) and the CH index is almost identical to the CH index in Smith et al. (1996, 1997) except for the continuum section. An additional $<CN>$ index was defined as an average of the $CN_{(3883)}$ index and twice the $CN_{(4215)}$ index in Lee (1999). This implies that in M3 the two indices are strongly correlated in such a way that the $CN_{(4215)}$ index has a value of one-half the $CN_{(3883)}$ index.

However in M15, which is a more metal-poor globular cluster than M3, the correlation between $CN_{(3883)}$ and $CN_{(4215)}$ indices of red giants is found to be $CN_{(3883)} = 1.98 \times CN_{(4215)} - 0.08$. So the slope is approximately the same as in the case of M3, but overall the $CN_{(3883)}$ index is shifted by 0.08 to a lower value. To derive this relation, red giant stars from the original program and two red giants whose indices are transformed from Shetrone et al. (1999) are used but HB stars and AGB stars are excluded. A similar correlation has been found in M71 stars except for the overall shift of the $CN_{(3883)}$ index by 0.10 to a higher value (Lee 2000). Although the zero point of the $CN_{(3883)}$ index for a given $CN_{(4215)}$ index seems to vary in each globular cluster, the overall shift produces only small effects on the $<CN>$ index (a systematic shift of 0.04 to a lower index value for each star in the case of M15) and does not affect a study of comparisons of $<CN>$ indices in a globular cluster. Therefore, for the sake of consistency of index definition, we use the $<CN>$ index as defined in Lee (1999). However, in the case of a study of several globular clusters of different metallicites, the zero point of the $CN_{(3883)}$ index in each globular cluster should be taken into account for comparison between CN indices of red giants in different globular clusters, especially for the index comparison with abundance. The zero point shift seems to be due to the different abundance effects on $CN_{(3883)}$ and $CN_{(4215)}$ indices. As the globular cluster metallicity increases, the $CN_{(3883)}$ index becomes greater for a given $CN_{(4215)}$ index, while the $CN_{(4215)}$ index becomes weaker for a given $CN_{(3883)}$ index. Although the $CN_{(3883)}$ index contains mainly the (0-0) band of CN $B^2\Sigma - X^2\Sigma$ while the $CN_{(4215)}$ index includes the (0-1) band, many other lines contained in both indices seem to have a complicated dependency on abundance. Therefore, even in a globular cluster, if the carbon and nitrogen abundances of a star fall outside the abundances

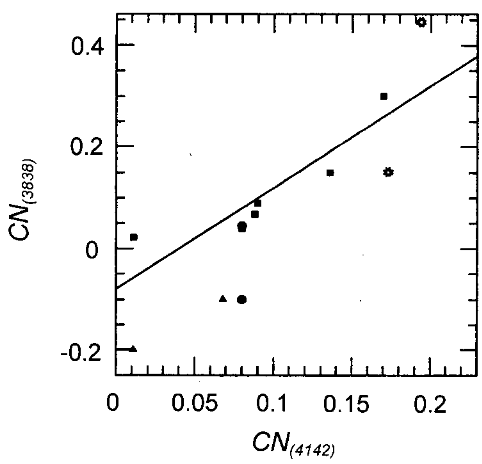

Fig. 2. Plot of the $CN_{(3883)}$ index vs. the $CN_{(4215)}$ index for original program stars including two stars with open star symbols, whose indices are transformed from S(3839) and S(4142) of Shectrone et al. (1999). The correlation between the two indices for RGB stars is quite similar to that found in a previous study of M3 (Lee, 1999). Symbols used are the same as for Fig. 1.

range of typical giant stars of a cluster, the star would not satisfy the correlation between the $CN_{(3883)}$ and $CN_{(4215)}$ indices found for those typical red giant stars. Such stars are found in Figure 2, in which the solid line represents the relation between $CN_{(3883)}$ and $CN_{(4215)}$ for typical red giants. They are HB stars, I-51 and IV-44, and an AGB star III-14. Their low <*CN*> and *CH*(G) indices confirm our speculation. However, for another asymptotic giant branch star, K714, which seems to satisfy the correlation between *CN* indices for RGB stars, it is suggested that its carbon and nitrogen abundances may be larger than those of III-14. We can not compare the carbon and nitrogen abundances of two stars quantitatively. However the larger <*CN*> and *CH*(G) indices of K714 compared to those of III-14 are confirmed in Figures 3-a and 3-b, and Figures 4-a and 4-b. The same

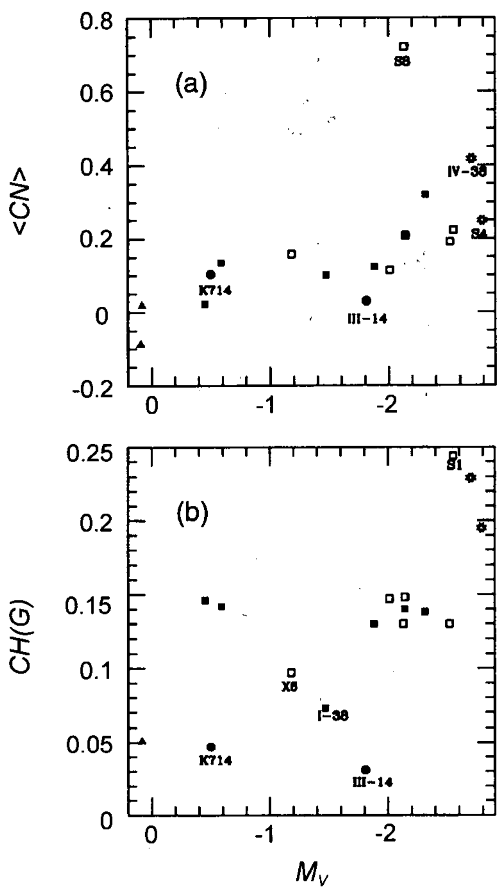

Fig. 3. (a) <*CN*> index is plotted as a function of absolute magnitude for all stars listed in Table 1. An overall increase in the <*CN*> index toward higher luminosity is found, especially for stars brighter than about −1.5. (b). *CH*(G) index is plotted as a function of absolute magnitude for all stars listed in Table 1. The *CH*(G) index of RGB stars declines gently toward higher luminosity, except for two stars with open star symbols, III-38 and S4, taken from Shetrone et al. (1999) and one original program star indicated by a filled square, I-38, and two stars with open squares added from Trefzger et al. (1983), X6 and S1. Symbols used are the same as for Fig. 1.

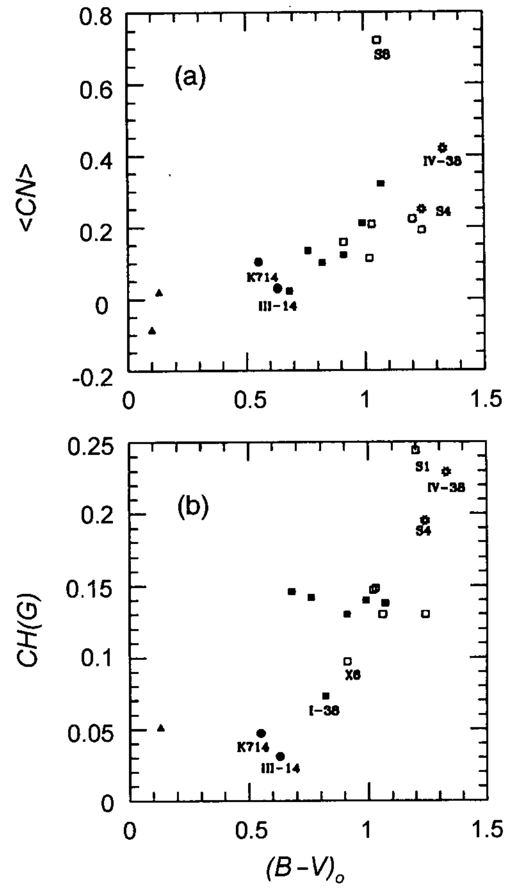

Fig. 4. (a). <*CN*> index is plotted as a function of $(B-V)_0$ for all stars listed in Table 1. Overall increase of <*CN*> index toward red is found. (b). *CH*(G) index is plotted as a function of $(B-V)_0$ for all stars listed in Table 1. The *CH*(G) index of RGB stars declines gently toward red, except for two stars with open star symbols, III-38 and S4, taken from Shetrone et al. (1999) and one original program star with filled squares, I-38, and two stars with open squares added from Trefzger et al. (1983), X6 and S1. However, *CH*(G) index increases toward red for these excluded stars together with AGB and HB stars. Symbols used are the same as for Fig. 1.

symbols in Figure 1 are used in all the figures in this study. Each index for the original program stars was measured five times and the error of each index is less than 0.01.

Shetrone et al. (1999) provide values of the S(3839), S(4142), and M_{CH} indices of Smith et al. (1996, 1997) for IV-38 and S4. We adopted the values of the differences between the S(3839), S(4142), and M_{CH} indices of Smith et al. (1996) and the corresponding $CN_{(3883)}$, $CN_{(4215)}$, and *CH*(G) indices of this study from previous work (Lee, 1999) to obtain the indices $CN_{(3883)}$, $CN_{(4215)}$, *CH*(G), and <*CN*> for IV-38 and S4; these are listed in Table 1.

The indices <*CN*> and *CH*(G) for seven other stars are transformed from the abundances of N/N_o and C/C_o in Table 3 of Trefzger et al. (1983). Three stars among the original ten program stars are among the stars of Trefzger et al. (1983). The relationships between <*CN*> and N/N_o and between *CH*(G) and C/C_o give <*CN*> and *CH*(G) indices for those stars whose N/N_o and C/C_o are available. These are <*CN*>$=0.08+0.02\times(N/N_o)$ and *CH*(G)$=0.03+1.63\times(C/C_o)$.

However, the ranges of nitrogen and carbon abundances for these relationships are so limited that for some cases extrapolation is applied to determine the <*CN*> and *CH*(G) indices too. Indices for seven stars are listed in Table 1, and the values with colons are the ones which have been extrapolated. The evolutionary states of stars based on their positions in the CMD are listed in the last column of Table 1. For non-original program stars *V*, *B-V*, and M_V magnitudes are taken either from Sneden et al. (1997) or Trefzger et al. (1983). One of the original program stars, S6, has an absolute magnitude of −2.18 in Sneden et al. (1997), which is fainter by 0.13 than that of Table 1.

III. THE *CN*-CH BAND STRENGTH ANTICORRELATION

The behavior of the <*CN*> and *CH*(G) indices as a function of M_V is shown in Figures 3-a and 3-b, respectively, and that as a function of (*B-V*) is presented in Figures 4a and 4-b, respectively.

RGB stars brighter than about −1.50 in Figure 3-a show that the <*CN*> index increases with luminosity. S8 shows an extremely high <*CN*> index, which was transformed from N/N_o of Trefzger et al. (1983). While two stars, IV-38 and S4(star-marked), and one star S1 with a transformed index by extrapolation show unusually strong *CH* band strengths in Figure 3-b, for the rest of all the RGB stars, the *CH*(G) index declines gently with luminosity except for I-38 and X6. I-38 and X6 have *CH*(G) index values lower than those of RGB stars but larger than those of AGB and HB stars.

In Figure 4-a, the overall increase of the <*CN*> index toward the red is also noticed. However, it is difficult to confirm the existence of the bidmodal distribution of the <*CN*> index as seen in M3 (Lee 1999).

If we exclude three stars IV-38, S4, and S1, which show unusually large *CH*(G) index values, and I-38 and X6, whose *CH*(G) index values are in between those of RGB and AGB, an overall *CH*(G) decline toward the red for RGB stars can be noticed. However, these five stars together with two known AGB stars show an increase of *CH*(G) toward the red.

A plot of the <*CN*> indices versus the *CH*(G) indices in Figure 5 shows that there is an anticorrelation between <*CN*> and *CH*(G) indices for RGB stars excluding the aforementioned five stars IV-38, S4, S1, I-38, and X6, together with HB and AGB stars. However, these five stars together with AGB and HB stars show a completely opposing trend in which both indices are positively correlated. Does this imply that these five stars are somehow related to either the AGB or HB? Does this mean that they are all in an evolutionaty state after the RGB tip? At this moment, there is no way to confirm this, but it is suspected that they may be different from normal RGB stars. Among the five stars, I-38 and IV-38 have a membership probability of more than 90%, while S4 and S1 have a value exceeding 50%. Although a zero % probability of membership for X6 is determined from proper motion data by Cudworth (1976), the radial velocity of X6 is acceptable for membership (Drukier et al. 1998). Therefore, we do not doubt they are all members of M15. Is it possible that they are all AGB stars overlapping with the RGB in the color-magnitude diagram? Nevertheless, if we consider the evolution time scales in the RGB and AGB states, seven out of seventeen stars seems to be too large a fraction since it is known that the AGB state lasts only 1/5 of the duration of the RGB evolutionary state. However, if we consider the fact that this sample of stars is selected from a bright section of the RGB state, this fraction of 7/17 may not be too unlikely.

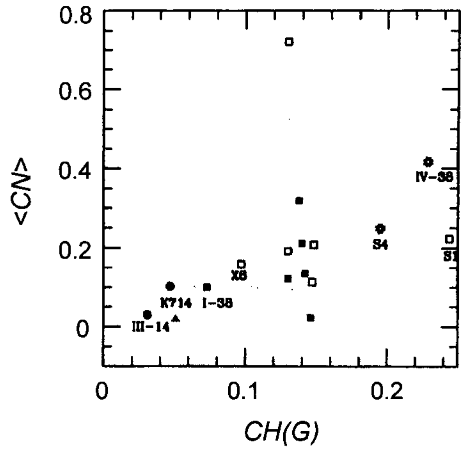

Fig. 5. Plot of the <*CN*> index vs. the *CH*(G) index. Excluding two stars with open star symbols, III-38 and S4, taken from Shetrone et al. (1999) and one original program star with filled squares, I-38, and two stars with open squares added from Trefzger et al. (1983), X6 and S1, and original AGB and HB stars, a *CN-CH* band strength anticorrelation is shown for remaining RGB stars. However, a *CN-CH* correlation is shown for those excluded stars. Symbols used are the same as Fig. 1.

If we assume that they are all AGB stars, why are the two indices positively correlated? One idea is that it is the opposite of the "weak *CN* effect". This means that during the AGB stage, the surface carbon abundance increases slightly according to evolution via a process similar to becoming a carbon star for intermediate mass stars. Then the opposite "weak *CN* effect" can explain the positive relation between the two indices. Since the strength of *CN* bands depends on both carbon and nitrogen abundances, the "weak *CN* effect" is caused by the dependence of not only nitrogen abundance but also carbon abundance. Therefore the weakness of *CN* bands in AGB stars is coupled with the weakness of *CH* bands. Since an AGB star has passed the RGB tip evolutionary state, its surface carbon abundance has been decreased, while its nitrogen abundance has been increased via a mechanism of extra mixing. However, the low carbon abundance of HB and AGB stars causes the weak *CN* bands, even though they may have a high nitrogen abundance. Therefore the carbon abundance increase during AGB evolution will strengthen not only the *CH* band but also *CN* band. If this speculation is true then we would expect that a star in a more advanced AGB stage will have stronger *CN* and *CH* bands. This is what we see in Figure 5, namely that IV-38, S4, and S1 have larger <*CN*> and *CH*(G) indices than I-38 and X6 do, although the sample size is small.

Therefore an anticorrelation between <*CN*> and *CH*(G) indices is found for red giants in M15 and stars not following that relation are suspected to be in different evolutionary states.

IV. DISCUSSION AND CONCLUSION

We have found an anticorrelation between *CN* and *CH* band strengths among giants in M15, although several stars are found not to follow this trend. The existence of an anticorrelation looks somewhat inconsistent with the previous work of Trefzger et al. (1983), which found that the carbon abundance declines with advancing evolutionary state without a corresponding increase in nitrogen abundance. However, if stars brighter than M_V=−0.5 for M15 and M_V=−1.5 for M92 in Figure 10 of Trefzger et al. (1983) are considered only, the mean nitrogen abundance is found to increase with luminosity. Since extra mixing, first proposed by Sweigart & Mengel (1979), can occur only at the bright red giant stage, surface nitrogen enhancement may appear effectively only for the brighter section of the RGB stage. Therefore an anticorrelation between *CN* and *CH* bands is expected for bright red giants in M15. However, stars that do not follow the anticorrelation seem to exist and show an opposite correlation between them. Among them, all AGB and HB stars of the original sample, even though they are small in number, are included. This fact leads to some speculation that the remainder of the stars are also in a different evolutionary state from the RGB. The locations of I-38 and X6 in the color-magnitude diagram indicate that they are more likely to be RGB stars. Hovever their low values for both indices of *CN* and *CH* bands, comparing with those of other RGB stars suggest a hint of the possibility of differences from normal RGB stars. Stars IV-38, S4, and S1 are among the reddest and brightest stars of the sample. Two stars IV-38 and S4 (star-marked) have been found to have faint satellite lines in their spectra indicating that these stars are binaries (Sneden et al. 1997). Pilachowski & Sneden (2000) have detected Hα emission in IV-38 and S4 and also in the AGB star III-14. These findings surely suggest that IV-38 and S4 are different from normal single RGB stars.

In conclusion, we have found an anticorrelation between <*CN*> and *CH*(G) indices among red giants in M15. However, there are some stars which do not follow that relation and are suspected to be in a different evolutionary state from the RGB single star state.

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