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BETTER ASTROMETRIC DE-BLENDING OF GRAVITATIONAL MICROLENSING EVENTS BY USING THE DIFFERENCE IMAGE ANALYSIS METHOD

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ABSTRACT

As an efficient method to detect blending of general gravitational microlensing events, it is proposed to measure the shift of source star image centroid caused by microlensing. The conventional method to detect blending by this method is measuring the difference between the positions of the source star image point spread function measured on the images taken before and during the event (the PSF centroid shift, $\delta\theta_{c,\rm PSF}$). In this paper, we investigate the difference between the centroid positions measured on the reference and the subtracted images obtained by using the difference image analysis method (DIA centroid shift, $\delta\theta_{c,\rm DIA}$), and evaluate its relative usefulness in detecting blending over the conventional method based on $\delta\theta_{c,\rm PSF}$ measurements. From this investigation, we find that the DIA centroid shift of an event is always larger than the PSF centroid shift. We also find that while $\delta\theta_{c,\rm PSF}$ becomes smaller as the event amplification decreases, $\delta\theta_{c,\rm DIA}$ remains constant regardless of the amplification. In addition, while $\delta\theta_{c,\rm DIA}$ linearly increases with the increasing value of the blended light fraction, $\delta\theta_{c,\rm PSF}$ peaks at a certain value of the blended light fraction and then eventually decreases as the fraction further increases. Therefore, measurements of $\delta\theta_{c,\rm DIA}$ instead of $\delta\theta_{c,\rm PSF}$ will be an even more efficient method to detect the blending effect of especially of highly blended events, for which the uncertainties in the determined time scales are high, as well as of low amplification events, for which the current method is highly inefficient.

Key words : gravitational lensing – astrometry – photometry

I. INTRODUCTION

Searches for Galactic dark matter by detecting flux variations of source stars caused by gravitational microlensing have been and are being carried out by several groups (MACHO: Alcock et al. 1993; EROS: Aubourg et al. 1993; OGLE: Udalski et al. 1993; DUO: Alard & Guibert 1997). To increase the event rate, these searches are being conducted towards very dense star fields such as the Galactic bulge and the Magellanic Clouds. While searches towards these dense star fields result in an increased event rate, it also implies that the observed light curves are affected by the unwanted blended flux from unresolved nearby stars.

The light curve of a microlensing event with an isolated source star is represented by

$$F = A_0 F_0, \tag{1}$$

where F_0 is the un-blended flux of the source star (baseline flux). The gravitational amplification is related to the lens-source separation u normalized by the angular Einstein ring radius by

$$A_0 = \frac{u^2+2}{u\sqrt{u^2+4}}; \qquad u = \left[{\beta_0}^2 + \left(\frac{t-t_0}{t_{\rm E,0}} \right)^2 \right]^{1/2}, \tag{2}$$

where the lensing parameters β_0, t_0, and $t_{\rm E,0}$ represent the lens-source impact parameter, the time of maximum amplification, and the Einstein ring radius crossing time scale (Einstein time scale), respectively. These lensing parameters are determined by fitting theoretical light curves to the observed one. One can obtain information about the lens mass M because the Einstein time scale is proportional to the square root of the lens mass, i.e. $t_{\rm E,0} \propto M^{1/2}$.

When an event is affected by blended light, on the other hand, its light curve differs from that of the unblended event by

$$F_{\rm PSF} = A_0 F_0 + B, \tag{3}$$

where B represents the flux from blended stars. Then, to fit the observed light curve of a blended event, one should include B as an additional parameter in addition to the three fitting parameters (β_0, t_0, and $t_{\rm E,0}$) of an unblended event. As a result, the uncertainties in the determined Einstein time scale and the corresponding lens mass for a blended event are significantly larger compared to those for an unblended event (Di Stefano & Esin 1995; Woźniak & Paczyński 1997; Han 1997; Alard 1997).

To resolve blending problem, a newly developed technique to detect and measure light variations caused by microlensing was proposed by Tomaney & Crotts (1996) and Alard & Lupton (1998). This so-called Difference Image Analysis (DIA) method measures the variation of source star flux by subtracting observed images from a normalized reference image. Then the light curve obtained by using the DIA method is represented by

$$F_{\rm DIA} = F_{\rm obs} - F_{\rm ref} = F_0(A_0 - 1), \qquad (4)$$

where $F_{\rm obs} = A_0 F_0 + B$ and $F_{\rm ref} = F_0 + B$ represent the source star fluxes measured from the image obtained during the progress of the event and from the reference image, respectively. Since not only the baseline flux of the lensed source star but also the flux from blended stars, are subtracted by the DIA method, the light variation measured from the subtracted image is free from the effect of blending. Since photometric precision is improved by removing the blended light, the DIA method was adopted by the MACHO group and actually applied to microlensing searches (Alcock et al. 1999a, 1999b).

However, even with the DIA method dramatic reduction of the uncertainties in the determined Einstein time scales of microlensing events will be difficult. This is because the DIA method, by its nature, has difficulties in determining the baseline flux of the lensed source star. As a result, one still has to include an additional fitting parameter of F_0.* Therefore, detecting blending effect and estimating the blended light fraction in the observed source star flux is still an important issue to be resolved (see more discussion in § 2).

There have been several methods proposed for the detection of blending. For a high amplification event, one can determine the baseline flux of the source star from the shape of the light curve obtained by using the DIA method itself (Han 2000). If the color difference between the lensed and blended stars is large, blending can also be detected by measuring the color changes during the event (Buchalter, Kamionkowski, & Rich 1996; Han, Park, & Jeong 2000). One can also identify the lensed source among blended stars from high resolution images obtained by using the *Hubble Space Telescope* observations (Han 1997). In addition, blending can be detected from the astrometric observations of an event by using high resolution interferometers such as the *Space Interferometry Mission* (Han & Kim 1999). However, these methods either have limited applicability only for several special cases of microlensing events, e.g. high amplification event, or impractical due to the requirement of using highly-demanding instrument for space observations.

A much more practical method for the detection of blending that can be applicable for general microlensing events is provided by measuring the linear shift of the source star image caused by microlensing. The conventional method to detect blending by this method is measuring the difference between the positions of the source star image point spread function (PSF) measured on the images taken before and during the event (the PSF centroid shift, $\delta\theta_{\rm c,PSF}$). In this paper, we investigate the difference between the centroid positions measured on the reference and the subtracted images obtained by using the DIA method (DIA centroid shift, $\delta\theta_{\rm c,DIA}$), and evaluate its relative usefulness in detecting blending over the conventional method based on $\delta\theta_{\rm c,PSF}$ measurements. From this investigation, we find that the DIA centroid shift of an event is always larger than the PSF centroid shift. We also find that while $\delta\theta_{\rm c,PSF}$ becomes smaller as the event amplification decreases, $\delta\theta_{\rm c,DIA}$ remains constant regardless of the amplification. In addition, while $\delta\theta_{\rm c,DIA}$ linearly increases with the increasing value of the blended light fraction, $\delta\theta_{\rm c,PSF}$ peaks at a certain value of the blended light fraction and then eventually decreases as the fraction further increases. Therefore, measurements of $\delta\theta_{\rm c,DIA}$ instead of $\delta\theta_{\rm c,PSF}$ will be an even more efficient method to detect the blending effect of especially of highly blended events, for which the uncertainties in the determined time scales are high, as well as of low amplification events, for which the current method is highly inefficient.

II. DEGENERACY PROBLEM

Even with the blending-free flux variations of a microlensing event measured by using the DIA method, it will be difficult to know whether the event is affected by blending or not. This because the best-fit light curve obtained under the wrong determination of the baseline flux can match well with the observed light curve. In this section, we show that the still existing degeneracy in the DIA light curve by an example.

The relations between the best-fit lensing parameters of a microlensing event resulting from the wrong determination of F_0 and their corresponding true values are provided by the analytic equations derived by Han (1999). If a blended event is misunderstood as an un-blended event, the baseline flux of the source star is overestimated into $F_0 + B$, causing mis-normalization of the light curve. Then the best-fit impact parameter $\beta_{\rm fit}$ determined from the mis-normalized light curve differs from the true value β_0 by

$$\beta_{\rm fit} = \left[2(1 - A_{\rm p}^{-2})^{-1/2} - 2\right]^{1/2};$$
$$A_{\rm p} = \frac{A_{\rm p,0} + \eta}{1 + \eta}, \qquad (5)$$

where $A_{\rm p,0} = (\beta_0^2 + 2)/(\beta_0\sqrt{\beta_0^2 + 4})$ and $A_{\rm p}$ represent the peak amplifications of the true and the mis-normalized light curves and $\eta = \Delta F_0/F_0 = B/F_0$ is the

*However, we note that since higher photometric precision is expected by using the DIA method, the uncertainties of determined lens parameters will be smaller than those of lens parameters determined by using the current method based on PSF photometry. Han (2000) showed that for $\sim 30\%$ of high amplification events, one can determine F_0 with uncertainties less than 50%.

fractional deviation of the mis-determined baseline flux. Mis-normalization of the light curve makes the best-fit Einstein time scale also differ from the true value by

$$t_{E,fit} = t_{E,0} \left(\frac{{\beta_{th}}^2 - {\beta_0}^2}{{\beta_{th,0}}^2 - \beta^2} \right)^{1/2}, \quad (6)$$

where $\beta_{th,0} = 1.0$ is the threshold impact parameter for event detection applied under the assumption that the event is not affected by blending, while β_{th} is the lowered value because of the requirement of higher amplification due to blending. The value of β_{th} is related to the fractional deviation of the baseline flux η by

$$\beta_{th} = \left[2(1 - A_{th}^{-2})^{-1/2} - 2 \right]^{1/2};$$

$$A_{th} = A_{th,0}(1+\eta) - \eta \quad (7)$$

where $A_{th,0} = 3/\sqrt{5}$ is the threshold amplification corresponding to β_0 (see more detail in Han 1999).

Figure 1 shows the degeneracy problem in the light curve obtained by using the DIA method. In the figure, the solid curve represents the light variation curve of an example event with $\beta_0 = 0.5$ obtained by using the DIA method. The event has a baseline flux of $F_0 = 0.5$ and it is affected by the blended flux of $B = 0.5$. The dotted curve represents the best-fit light curve obtained under the assumption that the event is not affected by blending. The best-fit lensing parameters of the mis-normalized light curve determined by using the relations in equations (5)–(8) are $\beta_{fit} = 0.756$ and $t_{E,fit} = 0.742 t_{E,0}$, respectively. One finds that the two light curve matches very well, implying that the degeneracy problem in determining the lensing parameters still exists even with the blending-free light variation curve obtained by using the DIA method.

III. CENTROID SHIFT MEASUREMENTS BY USING THE PSF AND DIA METHODS

A better method to detect blending is provided by measuring the source image centroid shifts caused by microlensing. In the section, we investigate the properties of the DIA centroid shift. In addition, by comparing these properties to those of the PSF centroid shift, we evaluate the usefulness of $\delta\theta_{c,DIA}$ measurements in detecting blending over the conventional method based on the measurements of $\delta\theta_{c,PSF}$.

(a) PSF Centroid Shifts

Let us consider a blended source star image within which multiple stars with their individual positions x_i and fluxes $F_{0,i}$ are located. For Galactic microlensing events, the typical lens-source separation is of the order of milli-arcsec, while the average separation between stars is $(\mathcal{O})10^{-1}$ arcsec. Therefore, gravitational amplification occurs only for one of the stars in the blended image. Since the position of the lensed star differs from the centroid of the blended image, the centroid of the source star image is shifted towards the lensed source during the event. If the lensed star with a baseline flux $F_{0,j}$ is located at a position x_j, the amount of the PSF centroid shift is computed by

$$\delta\theta_{c,PSF} = \mathcal{D}\,(\bar{x} - x_j);$$

$$\mathcal{D} = \frac{(1-f)(A_0 - 1)}{(1-f)(A_0-1)+1}, \quad (8)$$

where $f = 1 - F_{0,j}/\sum_i F_{0,i}$ represents the fractional flux of the blended light out of the total flux of all stars including $F_{0,j}$ within the integrated seeing disk and $\bar{x} = \sum_i x_i F_{0,i} / \sum_i F_{0,i}$ is the position of the blended image centroid before the gravitational amplification (Goldberg 1998).

In Figure 2, we illustrate the PSF centroid shift of an example microlensing event. On the left side of the figure, the inner two dotted circles with their centers marked by '+' represent the contours of the individual two blended stars (measured at an arbitrary flux level) and the outer solid curve centered at 'x' represents the integrated images before (upper panel) and during (lower panel) gravitational amplification. The two stars have the same flux, and the lensed star is the right one.

(b) DIA Centroid Shifts

On the other hand, if the position of the source centroid is measured on the subtracted image obtained by using the DIA method, its position will be identical to that of the lensed star itself, i.e. x_j. Therefore, the DIA centroid shift is represented by

$$\delta\theta_{c,DIA} = \bar{x} - x_j. \quad (9)$$

From the comparison of equations (8) and (9), one finds that the DIA centroid shift of an event is always bigger than the PSF centroid shift. This is because within the allowed ranges of the blended light fraction and the amplification of $0 \leq f \leq 1$ and $A_0 > 1$, the factor $\mathcal{D}$ is always less than 1. This can be seen also in Figure 2, in which we present $\delta\theta_{c,DIA}$ and compare with $\delta\theta_{c,PSF}$. To better show both centroid shifts, we expand the region enclosed by a dot-dashed line and present on the right side.

(c) PSF versus DIA Centroid Shifts

To see the dependencies of the PSF and DIA centroid shifts on the amplification and the blended light fraction, we compute $\delta\theta_{c,PSF}$ and $\delta\theta_{c,DIA}$ as functions of f_b for events with various impact parameters and the resulting relations are presented in Figure 3. Note that the presented centroid shifts are average values, $\langle\delta\theta_{c,DIA}\rangle$ and $\langle\delta\theta_{c,PSF}\rangle$, under the assumption that stars are randomly distributed within

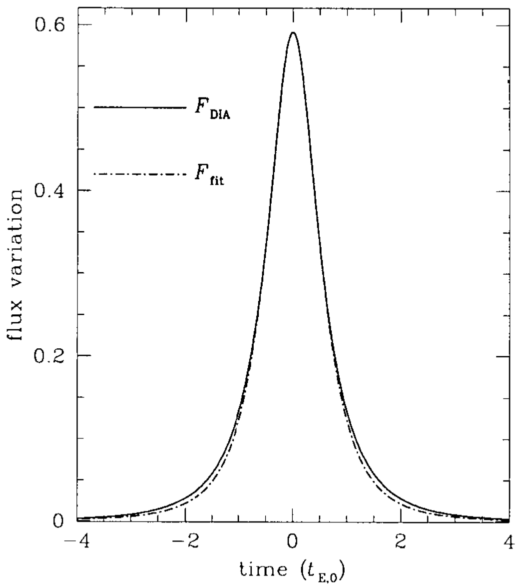

Fig. 1.— The degeneracy problem in the light curve of a microlensing event obtained by using the DIA method. The solid curve represents the light variation curve of an example event with $\beta_0 = 0.5$ obtained by using the DIA method. The event has a baseline flux of $F_0 = 0.5$ and it is affected by the blended flux of $B = 0.5$. The dot-dashed curve represents the best-fit curve obtained under the assumption that the flux of the source star is not affected by blending. The x-axis represents the time normalized by the Einstein time scale(t_0) and y-axis is the light variation measured from the subtracted image obtained by using the DIA method.

the effective seeing disk (the undistinguishable separation between stellar images). The adopted radius of the effective seeing disk is $\theta_{\rm eff} = 1''.5/2$. We also assume that detected events have observed amplifications, $A_{\rm obs} = (AF_0 + B)/(F_0 + B)$, greater than $3/\sqrt{5}$, which is the threshold amplification of an unblended event for a source to be located within the Einstein ring. Due to this restriction of detectable events, each curve becomes discontinuous at a certain value of f. In the figure, the discontinuity of each curve is represented by a dashed line.

From the dependencies of the centroid shifts on the impact parameter (and thus the event amplification), one finds that as long as an event is detected by satisfying the condition of $A_{\rm obs} \geq 3/\sqrt{5}$, for a given blended light fraction $\delta\theta_{\rm c,DIA}$ is the same regardless of β_0, implying that the DIA centroid shift does not depend on the event amplification. On the other hand, $\delta\theta_{\rm c,PSF}$ decreases as β_0 increases, implying that the PSF centroid shift becomes smaller as the amplification decreases. For low amplification events with $\beta_0 \lesssim 0.3$, the expected values of $\delta\theta_{\rm c,PSF}$ is less than the current threshold centroid shift of $0''.2$ (Goldberg & Woźniak 1998). On the other hand, the expected values of $\delta\theta_{\rm c,DIA}$ for a substantial fraction of these events are greater than the threshold shift, implying that measurents of the DIA centroid shifts will be an efficient especially in detecting blending for low amplification events.

In addition, from the dependencies of the PSF and DIA centroid shifts on the blended light fraction, one finds that while $\langle\delta\theta_{\rm c,DIA}\rangle$ linearly increases as the blended light fraction increases, $\langle\delta\theta_{\rm c,PSF}\rangle$ peaks at a certain value of f and eventually decreases as the blended light fraction continues to increase. The linear dependency of the DIA centroid shift on f can be explicitly seen by transforming equation (9) into the form

$$\delta\theta_{\rm c,DIA} = f(x_{\rm b} - x_j), \qquad (10)$$

where $x_{\rm b} = \sum_{i\neq j} x_i F_{0,i} / \sum_{i\neq j} F_{0,i}$ is the centroid of blended stars only, and thus $|x_{\rm b} - x_j|$ represents the separation between the lensed star and the centroid of the blended stars. These dependencies of $\delta\theta_{\rm c,DIA}$ and $\delta\theta_{\rm c,PSF}$ on the blended light fraction can be qualitatively understood in the following way. For an event affected by small amount of blended light, the position of the source centroid on the reference image will be very close to the position of the lensed star itself due to the dominance of its flux over other blended stars, resulting in a small value of $\langle\delta\theta_{\rm c,DIA}\rangle = \langle|\bar{x} - x_j|\rangle$. By contrast, if the lensed source is one of faint stars in the effective seeing disk and thus affected by large amount of blended light, the flux from the source has relatively little effect on $\bar{x}$, resulting in large value of $\langle\delta\theta_{\rm c,DIA}\rangle$. However, since the lensed source is too faint to draw

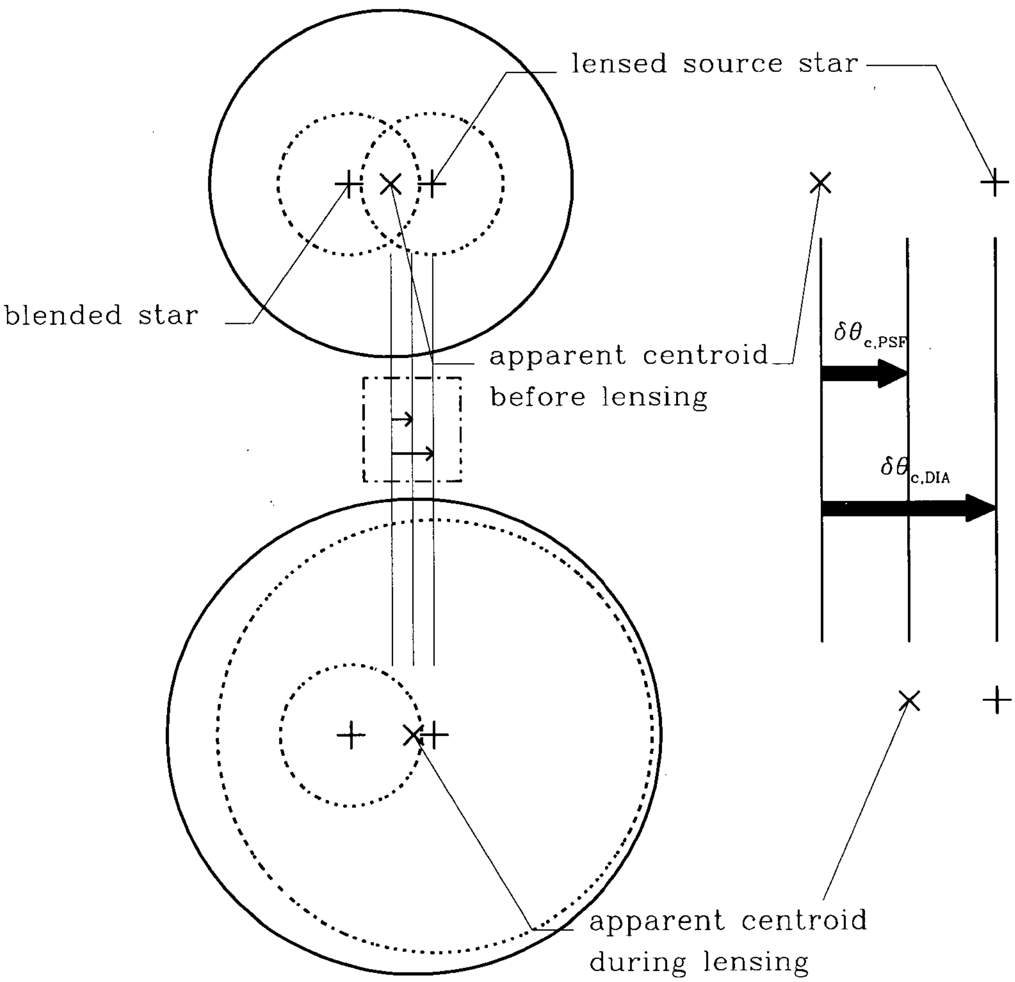

Fig. 2.— Illustration of the PSF and DIA centroid shifts. On the left side, the contours (measured at an arbitrary flux level) of two blended source stars with identical baseline fluxes (the inner two dotted circles with their centers marked by '+') and their integrated images (the outer solid curve centered at 'x') before (upper part) and during (lower part) gravitational amplification are presented. Among the two stars within the blended image, the right one is lensed. Both centroid shifts are marked between the two left contours of integrated image. To better show the centroids shifts, the region enclosed by a dot-dashed line is expanded and presented on the right side.

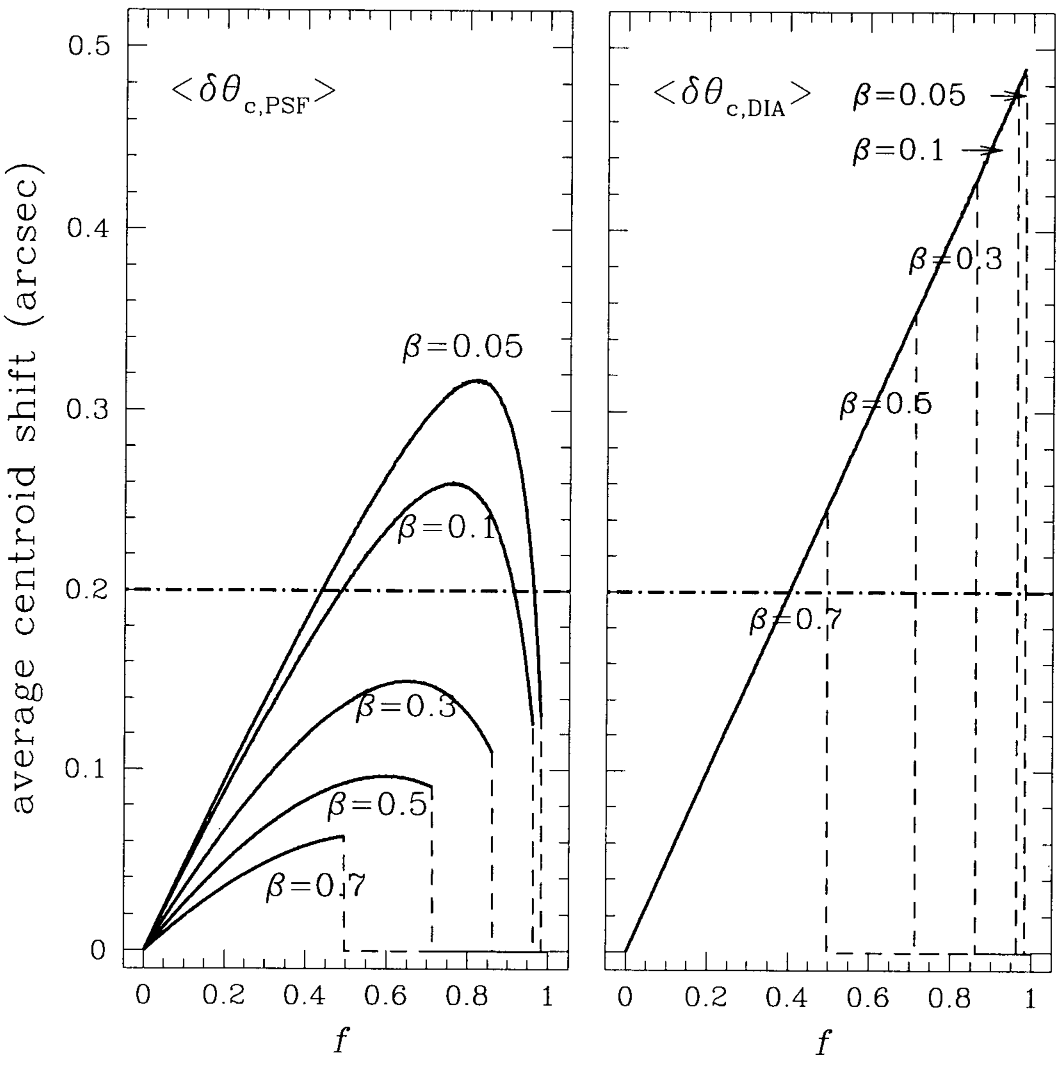

Fig. 3.— The dependencies of the PSF and DIA centroid shifts on the blended light fraction for events with various impact parameters. Note that the presented centroid shifts are averaged values under the assumption that stars are randomly distributed within the effective seeing disk with the adopted radius of $\theta_{\rm eff} = 1''.5/2$. We also assume that detected events have observed amplifications, $A_{\rm obs} = (AF_0 + B)/(F_0 + B)$, greater than $3/\sqrt{5}$. Due to this restriction of detectable events, each curve becomes discontinuous at a certain value of f.

the PSF centroid shift towards it, $\delta\theta_{c,\rm PSF}$ will be small despite the large value of $\langle|\bar{x} - x_j|\rangle$. Therefore, measurements of DIA centroid shifts will also be an efficient method for highly blended events. Detecting blending effect for these events is important because the uncertainties of the lensing parameters determined from the light curves will be large.

IV. SUMMARY

The findings from investigating the properties of DIA centroid shift and the evaluation of its usefulness in detecting blending over the conventional method based on the PSF centroid shift measurements are summarized as follows.

1. For a given event, the DIA centroid shift of an event is always larger than the PSF centroid shift.
2. While the amount of the PSF centroid shift becomes smaller as the event amplification decreases, the DIA centroid shift remains constant regardless of the amplification.
3. While the DIA centroid shift linearly increases with the increasing value of the blended light fraction, the PSF centroid shift peaks at a certain fraction of blended light fraction and then eventually decreases as the fraction continues to increase.

Due to these combined properties of the DIA centroid shift, measurements of $\delta\theta_{c,\rm DIA}$ will be an efficient method to detect blending especially of highly blended events, for which the uncertainties in the determined Einstein time scales are large, as well as of low amplification events, for which the current method is highly inefficient.

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