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Development of a Stochastic Group Replacement Model for Two Independent Equipments†

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설비의 일괄교체를 위한 확률모형 개발

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A system consisting of two continuously and independently operating equipment subject to breakdown and repair, is considered. It is assumed that both equipment age only when in operation, and a group replacement policy is in effect, that is, both equipment are replaced simultaneously by new identical ones as soon as either of them reaches a specified replacement age. First, a system of partial differential equations based on enumerating the various probabilistic events, is derived. Then, solutions of such system of equations for a model considered in the steady-state are obtained. Finally, an economic analysis is performed to determine the optimal replacement ages of both equipment.

1. Introduction

Replacement models of equipment correspond to situations in which equipment deteriorates with age. That is, the longer it is retained, the higher the cost of operating it. Thus, as an alternatives, it may be profitable to acquire a new equipment that is more economical to operate. The fundament al problem that one is faced with is to make an appropriate balance between the cost of increased upkeep of the old equipment and acquisition cost and reduced upkeep of a new equipment.

Consider a system consisting of two continuously (continuous state space, continuous parameter set in a stochastic process) and independently operating equipment subject to breakdown and repair. Repair is caused by equipment breakdown, and the length of repair time is the time necessary to set it back into an operating state. It is necessary for such a system segregate between the two distinct phases in equipment state, the operating phase and the repair phase. In general, an element of uncertainty is present both in the frequency of breakdown and the length of time to repair. The followings are assumed:

(i) Both equipment, denoted by equipment 1 and equipment 2, age only in operating. Such aging will be termed as service aging to distinguish it from chronological aging. The state of the system is specified by their ages only.

(ii) Service ages of both equipment are not affected by breakdowns and repairs.

(iii) A group replacement policy is in effect which takes the form; replace both equipment simultaneously by new equipment whenever equipment 1 reaches age X_1 or equipment 2 reaches age X_2, whichever comes first; otherwise do not replace. The desirability of a group replacement policy over an individual replacement policy is dictated by economic consideration. For example, very often the purchase of two

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or more equipment at the same time may lead to quantity discounts which may be substantial. The replacement ages X_1 and X_2 could be used as decision variables whose values may be determined by the selection of an appropriate objective function.

The group replacement strategy is peculiar only to systems involving several equipment items. This should be distinguished from the individual replacement strategies in which each equipment is treated independently(see, e.g. Cox[3], Jorgensen, McCall and Rander[6] and Barlow and Prochan[1], Grant, Ireson and Leavenworth[5], Taylor and Karlin[10], Martino[7], Blanchard[2]). Although the concept of group replace- ment is an old issue dating back to the 1950's, little has appeared in the literature(see Sivazlian and Mahoney[8]).

2. Objectives of Research

In order to characterize the model under consideration, define the followings for $0 \leq x_1 \leq X_1$ and $0 \leq x_2 \leq X_2$:

$P_{11}(t, x_1, x_2)dx_1dx_2$: the probability that at time t equipment 1 and 2 are both operating and their ages lie between (x_1, x_1+dx_1) and (x_2, x_2+dx_2), respectively.

$P_{10}(t, x_1, x_2)dx_1dx_2$: the probability that at time t equipment 1 is operating and equipment 2 is not operating and their ages lie between (x_1, x_1+dx_1) and (x_2, x_2+dx_2), respectively.

$P_{01}(t, x_1, x_2)dx_1dx_2$: the probability that at time t equipment 1 is not operating and equipment 2 is operating and their ages lie between (x_1, x_1+dx_1) and (x_2, x_2+dx_2), respectively.

$P_{00}(t, x_1, x_2)dx_1dx_2$: the probability that at time t equipment 1 and 2 are both not operating and their ages lie between (x_1, x_1+dx_1) and (x_2, x_2+dx_2), respectively.

It is assumed that breakdown rate depends only on the state of the system determined by the service ages x_1 and x_2 of equipment 1 and 2, and do not depend on time. However, without loss of generality, repair rate for each equipment is assumed to be a constant. Let $\lambda_i(x_i)dx_i$ where $i=1,2$: the probability that equipment i will breakdown between age x_i and x_i+dx_i given that equipment i is operating at its age x_i. and $\mu_i dt$ where $i=1,2$: the probability that equipment i, in a state of repair at time t, will be repaired between time t and $t+dt$.

Using the above system parameters, the system of equations in $P_{ij}(t, x_1, x_2)$ where $i,j=0,1$ will be derived. Then, under the stated replacement policy, an appropriate non-trivial function which satisfies (i) the system of equations, (ii) the normalizing condition and (iii) the boundary condition which is dictated by the replacement policy, will be obtained. Finally, an economic model for the system considering the various cost entities such as the operating cost per unit time for each equipment, the repair cost per unit time and the group replacement cost of both equipment, will be constructed to determine the optimal replacement ages of both equipment. First, the system of equations in $P_{ij}(t, x_1, x_2)$ where $i,j=0,1$ is derived.

3. Derivation of the System of Equations

3.1 The System of Equations

Upon enumerating the various probabilistic events and definitions of $\lambda_i(x_i)dx_i$ where $i=1,2$ and $\mu_i dt$ where $i=1,2$, the following expression for $P_{11}(t+dt, x_1+dx_1, x_2+dx_2)$ is stated:

$$\begin{aligned} &P_{11}(t+dt, x_1+dx_1, x_2+dx_2) \\ &= P_{11}(t, x_1, x_2)[1-\lambda_1(x_1)dx_1][1-\lambda_2(x_2)dx_2] \\ &\quad + P_{10}(t, x_1, x_2+dx_2)[1-\lambda_1(x_1)dx_1]\mu_2 dt \\ &\quad + P_{01}(t, x_1+dx_1, x_2)\mu_1 dt[1-\lambda_2(x_2)dx_2] \\ &\quad + P_{00}(t, x_1+dx_1, x_2+dx_2)\mu_1 dt\ \mu_2 dt \end{aligned}$$

For convenience, let $P_{ij}(t) = P_{ij}(t, x_1, x_2)$ for $i,j=0, 1$. Expanding as Taylor's series yields

$$\begin{aligned} &\frac{\partial P_{11}(t)}{\partial t} + \frac{\partial P_{11}(t)}{\partial x_1}dx_1 + \frac{\partial P_{11}(t)}{\partial x_2}dx_2 \\ &= -[\lambda_1(x_1)+\lambda_2(x_2)]\ P_{11}(t) \end{aligned}$$

$$+ \ [P_{10}(t) + \frac{\partial P_{10}(t)}{\partial x_2} dx_2] \ \mu_2 dt$$
$$+ \ [P_{01}(t) + \frac{\partial P_{01}(t)}{\partial x_1} dx_1] \ \mu_1 dt \ + o(dt)$$

where $\lim_{dt \to 0} \frac{o(dt)}{dt} = 0$. Dividing both sides by dt and letting $dt \to 0$ yield

$$\frac{\partial P_{11}(t)}{\partial t} + \frac{\partial P_{11}(t)}{\partial x_1} + \frac{\partial P_{11}(t)}{\partial x_2}$$
$$= -[\lambda_1(x_1) + \lambda_2(x_2)] P_{11}(t) + \mu_2 P_{10}(t) + \mu_2 P_{01}(t) \quad (1)$$

Using similar procedures, expressions for $P_{10}(t, x_1, x_2)$, $P_{01}(t, x_1, x_2)$ and $P_{00}(t, x_1, x_2)$ are obtained as

$$\frac{\partial P_{10}(t)}{\partial t} + \frac{\partial P_{10}(t)}{\partial x_1}$$
$$= \lambda_2(x_2) P_{11}(t) - [\lambda_1(x_1) + \mu_2] P_{10}(t) + \mu_1 P_{00}(t) \quad (2)$$
$$\frac{\partial P_{01}(t)}{\partial t} + \frac{\partial P_{01}(t)}{\partial x_2}$$
$$= \lambda_1(x_1) P_{11}(t) - [\lambda_2(x_2) + \mu_1] P_{01}(t) + \mu_2 P_{00}(t) \quad (3)$$
$$\frac{\partial P_{00}(t)}{\partial t}$$
$$= -(\mu_1 + \mu_2) P_{00}(t) + \lambda_2(x_2) P_{01}(t) + \lambda_1(x_1) P_{10}(t) \quad (4)$$

Let $P_{ij} = P_{ij}(x_1, x_2) = \lim_{t \to \infty} P_{ij}(t, x_1, x_1)$. Under the stated group replacement policy, assuming steady-state conditions, the system of equations in (1) to (4) becomes

$$\frac{\partial P_{11}}{\partial x_1} + \frac{\partial P_{11}}{\partial x_2} = -[\lambda_1(x_1) + \lambda_2(x_2)] \ P_{11} + \mu_2 P_{10} + \mu_2 P_{01} \quad (5)$$
$$\frac{\partial P_{10}}{\partial x_1} = \lambda_2(x_2) \ P_{11} - [\lambda_1(x_1) + \mu_2] \ P_{10} + \mu_1 P_{00} \quad (6)$$
$$\frac{\partial P_{01}}{\partial x_2} = \lambda_1(x_1) \ P_{11} - [\lambda_2(x_2) + \mu_1] \ P_{01} + \mu_2 P_{00} \quad (7)$$
$$0 = -(\mu_1 + \mu_2) P_{00} + \lambda_2(x_2) P_{01} + \lambda_1(x_1) P_{10} \quad (8)$$

3.2 The Normalizing Condition

Define the followings for $0 \le x_1 \le X_1$ and $0 \le x_2 \le X_2$:

(i) $f(x_1, x_2)$: the joint probability density function of ages for both equipment.

(ii) $f_1(x_1)$ and $f_2(x_2)$: the marginal probability density functions of ages of equipment 1 and 2, respectively.

Then, the followings hold:

$$f(x_1, x_2) = P_{11}(x_1, x_2) + P_{10}(x_1, x_2) + P_{01}(x_1, x_2) + P_{00}(x_1, x_2) \quad (9)$$

$$f(x_1, x_2) = f_1(x_1) f_2(x_2) \quad (10)$$

$$\int_0^{X_1} \int_0^{X_2} f(x_1, x_2) \ dx_1 dx_2 = 1 \quad (11)$$

3.3 The Boundary Conditions

Since both equipment are replaced simultaneously as soon as equipment 1 reaches the replacement age X_1 or equipment 2 reaches the replacement age X_2, whichever comes first; otherwise do not replace, the following boundary conditions may be stated for any $0 \le x_1 \le X_1$ and $0 \le x_2 \le X_2$:

$$P_{11}(X_1, x_2) = P_{11}(x_1, x_2) = P_{10}(X_1, x_2)$$
$$= P_{01}(x_1, X_2) = P_{11}(X_1, x_2) \quad (12)$$

However, $\lambda_1(X_1)$ and $\lambda_2(X_2)$ will be determined based on the boundary conditions in (12) in the sequel.

Note that the system of equations of partial differential equations in (5) to (8) should be solved subject to the normalizing condition in (11) and the boundary conditions in (12). Also note that

$$\frac{\partial P_{11}(x_1, x_2)}{\partial x_1} + \frac{\partial P_{11}(x_1, x_2)}{\partial x_2} + \frac{\partial P_{10}(x_1, x_2)}{\partial x_1} + \frac{\partial P_{01}(x_1, x_2)}{\partial x_2} = 0 \quad (13)$$

4. Solution of the System of Equations

In order to determine an appropriate non-trivial function of $P_{ij}(x_1, x_2)$ where $i, j=0,1$ explicitly which satisfies relations (5) to (8) subject to (10) to (12), solve relation (8) with respect to P_{00}, i.e.,

$$P_{00}=\frac{\lambda_2(x_2)}{\mu_1+\mu_2}P_{01}+\frac{\lambda_1(x_1)}{\mu_1+\mu_2}P_{10} \quad (14)$$

Substituting (14) into relations (5) to (7), and then, solving them with respect to P_{11} yield the 3rd order linear partial differential equation. The same equation would be obtained for P_{10} and P_{01}. Since these equations are not regular, difficulties are encountered to solve them: for example, the method of separation of variables fails in this situation. Hence, the following procedures are strongly recommended. Using the fact that both equipment are operating independently, define the followings:

(i) $P_1(x_1)dx_1$: the probability that equipment 1 is operating and its age lies between (x_1, x_1+dx_1).

(ii) $P_0(x_1)dx_1$: the probability that equipment 1 is not operating and its age lies between (x_1, x_1+dx_1).

(iii) $P_1'(x_2)dx_2$: the probability that equipment 2 is operating and its age lies between (x_2, x_2+dx_2).

(iv) $P_0'(x_2)dx_2$: the probability that equipment 2 is not operating and its age lies between (x_2, x_2+dx_2). Then, $P_{ij}(x_1, x_2)$ can be rewritten as

$$P_{11}(x_1, x_2) = P_1(x_1)P_1'(x_2) \quad (15)$$

$$P_{10}(x_1, x_2) = P_1(x_1)P_0'(x_2) \quad (16)$$

$$P_{01}(x_1, x_2) = P_0(x_1)P_1'(x_2) \quad (17)$$

$$P_{00}(x_1, x_2) = P_0(x_1)P_0'(x_2) \quad (18)$$

It is clear that $P_1(x_1)$ and $P_0(x_1)$ for equipment 1 satisfy the following system of equations (e.g., see Sivazlian and Stanfel[9]):

$$\frac{dP_1(x_1)}{dx_1} = -\lambda_1(x_1)P_1(x_1)+\mu_1P_0(x_1) \quad (19)$$

$$0 = -\mu_1P_0(x_1)+\lambda_1(x_1)P_1(x_1) \quad (20)$$

Thus, solving the above (19) and (20) yields

$$P_1(x_1) = C \quad (21)$$

$$P_0(x_1) = C\frac{\lambda_1(x_1)}{\mu_1} \quad (22)$$

where C is an arbitrary constant.
Since

$$f_1(x_1) = P_1(x_1)+P_0(x_1) \quad (23)$$

and

$$\int_0^{X_1} f_1(x_1)dx_1 = 1 \quad (24)$$

it can be shown that

$$P_1(x_1) = \frac{1}{\left[X_1+\frac{\alpha(X_1)}{\mu_1}\right]} \quad (25)$$

$$P_0(x_1) = \frac{\frac{\lambda_1(x_1)}{\mu_1}}{\left[X_1+\frac{\alpha(X_1)}{\mu_1}\right]} \quad (26)$$

$$f_1(x_1) = \frac{1+\frac{\lambda_1(x_1)}{\mu_1}}{\left[X_1+\frac{\alpha(X_1)}{\mu_1}\right]} \text{ for } 0\le x_1\le X_1 \quad (27)$$

where $\int_0^{X_1}\lambda_1(x_1)dx_1 = \alpha(X_1)$.

Similarly, it is possible to obtain $P_1'(x_2), P_0'(x_2)$ and $f_2(x_2)$ for equipment 2 as

$$P_1'(x_2) = \frac{1}{\left[X_2+\frac{\beta(X_2)}{\mu_2}\right]} \quad (28)$$

$$P_0'(x_2) = \frac{\frac{\lambda_2(x_2)}{\mu_2}}{\left[X_2+\frac{\beta(X_2)}{\mu_2}\right]} \quad (29)$$

$$f_2(x_2) = \frac{1+\frac{\lambda_2(x_2)}{\mu_2}}{\left[X_2+\frac{\beta(X_2)}{\mu_2}\right]} \text{ for } 0\le x_2\le X_2 \quad (30)$$

where $\int_0^{X_2}\lambda_2(x_2)dx_2 = \beta(X_2)$.

Thus, substituting $P_i(x_1)$ and $P_i'(x_2)$ for $i=0,1$ in (25), (26), (28) and (29) into relations in (15) to (18) yields $P_{ij}(x_1, x_2)$ for $i, j=0,1$. It is easy to verify that the obtained $P_{ij}(x_1, x_2)$ for $i, j=0, 1$ satisfies the system of equations in (5) to (8) and the normalizing conditions in (9) to (11). Thus, from (10), (27) and (30), the joint probability density function of service ages for both equipment, $f(x_1, x_2)$ is obtained easily. These results will be used to develop an economic model for determination of the optimal replacement ages which minimize the total cost function associated with the various cost entities including replacement, operation and repair.

5. Economic Model for the System

A new equipment will usually lose value in a continuous fashion as it ages. Simultaneously, the cost of operation and repair will, in general, be non-decreasing function of equipment age. Hence, the higher value and lower operating costs associated with the acquisition of a new equipment could be weighted with the contribution of loss in value and increased costs of keeping the old equipment. Let new equipment installed in the future have identical procurement cost and benefit characteristics as the existing equipment for which replacement is being contemplated. The problem then consists in determining the optimal replacement age so as to minimize the total expected cost per unit time which are represented by the sum of the various cost entities such as the expected group replacement cost per unit time, the expected cost of operation per unit time and the expected cost of repair per unit time. In order to construct a function of the total expected cost per unit time, define the followings:

(i) K: the cost of group replacement for both equipment

(ii) T: the random variable denotes the time interval between two successive replacements. $E[T]$ is the expected value of T.

(iii) $O_i(x_i)dx_i$ where $i=1, 2$: the cost of operation per unit time for equipment i whose service age lies between x_i and x_i+dx_i given that equipment i is in an operating state. In general, $O_i(x_i)$ is a nondecreasing function of x_i.

(iv) $R_i(x_i)$ where $i=1, 2$: the cost associated with one unit time of repair for equipment i whose service age is x_i. In general, $R_i(x_i)$ is a non-decreasing function of x_i.

(v) $E[G]$: the expected cost of group replacement per unit time for both equipment.

(vi) $E[O]$: the expected cost of operation per unit time for both equipment.

(vii) $E[R]$: the expected cost of repair per unit time for both equipment.

(viii) $TC(X_1, X_2)$: the total expected cost per unit time associated with the group replacement, operation and repair of both equipment where X_1 and X_2 are replacement ages of equipment 1 and 2, respectively.

Then, $TC(X_1, X_2)$ is given by

$$TC(X_1, X_2) = E[G] + E[O] + E[R] \tag{31}$$

Note that $E[G]$, $E[O]$ and $E[R]$ are obtained from

$$E[G] = \frac{K}{E[T]} \tag{32}$$

$$\begin{aligned} E[O] = &\int_0^{X_1}\int_0^{X_2} O_1(x_1)[P_{11}(x_1, x_2) \\ &+ P_{10}(x_1, x_2)]dx_2dx_1 \\ &+ \int_0^{X_1}\int_0^{X_2} O_2(x_2)[P_{11}(x_1, x_2) \\ &+ P_{01}(x_1, x_2)]dx_2dx_1 \end{aligned} \tag{33}$$

$$\begin{aligned} E[R] = &\int_0^{X_1}\int_0^{X_2} R_1(x_1)[P_{01}(x_1, x_2) \\ &+ P_{00}(x_1, x_2)]dx_2dx_1 \\ &+ \int_0^{X_1}\int_0^{X_2} R_2(x_2)[P_{01}(x_1, x_2) \\ &+ P_{00}(x_1, x_2)]dx_2dx_1 \end{aligned} \tag{34}$$

5.1 Heuristic Approach to find $E[T]$

Let T_i for $i=1, 2$ be random variables represent the total repair time of equipment i when equipment i reaches its service age X_i. Then, $E[T]$ is given by

$$E[T] = E[Min(X_1+T_1, X_2+T_2)] \tag{35}$$

where $E[X_1+T_1] = X_1 + \frac{\alpha(X_1)}{\mu_1}$ and $E[X_2+T_2] = X_2 + \frac{\beta(X_2)}{\mu_2}$. However, since T_i is characterized by nonhomogeneous Poisson distributions, difficulties are encountered to find the exact expression of $E[T]$. Furthermore, even though exact $E[T]$ is obtained, the explicit expression of the optimal replacement ages, denoted by, X_1^* and X_2^*, may not be derived because of form of $E[T]$. Hence, heuristic approaches to find $E[T]$ are suggested like the following method. It is clear that $E[T] \le X_1 + \frac{\alpha(X_1)}{\mu_1}$ and $E[T] \le X_2 + \frac{\beta(X_2)}{\mu_2}$. Let $P(1,0)$ be the probability that the age of equipment 1 reaches X_1 while equipment 2 is in the breakdown state and $P(0,1)$ be the probability that the age of equipment 2 reaches X_2 while equipment 1 is in the breakdown state. Then, $P(1,0)$ and $P(0,1)$ are obtained from

$$P(1,0) = \int_0^{X_1}\int_0^{X_2} P_{10}(x_1,x_2)dx_1dx_2$$

$$P(0,1) = \int_0^{X_1}\int_0^{X_2} P_{01}(x_1,x_2)dx_1dx_2$$

where $P_{10}(x_1,x_2)$ and $P_{10}(x_1,x_2)$ are shown in (16) and (17). Then,

$$E[T] = \frac{1}{2}\left\{E[X_1+T_1]\frac{P(1,0)}{P(1,0)+P(0,1)}\right. \quad (36)$$
$$\left. +E[X_2+T_2]\frac{P(1,0)}{P(1,0)+P(0,1)}\right\}$$

5.2 Numerical Examples for the Heuristic Approach

Example 1: Suppose that $\lambda_1(x_1)=0.01$, $\lambda_2(x_2)=0.005$, $\mu_1=5$ and $\mu_2=10$. Also suppose that $K=$ \$50,000, $O_1(x_1)=0.1x_1^2$, $O_2(x_2)=5x_2$, $R_1(x_1)=2x_1$ and $R_2(x_2)=x_2$. Then, $\alpha(X_1)=0.01X_1$ and $\beta(x_2)=0.05x_2$. Hence, from (36), $E[T]=1.431X_1+0.3589X_2$, $E[O]=0.0333X_1^2+2.4876X_2$ and $E[R]=E[R]=0.02X_1+0.025X_2$. Thus, $F(X_1,X_2)$ is given by

$$TC(X_1,X_2) = \frac{50000}{0.143X_1+0.359X_2}$$
$$+0.033X_1^2+2.388X_2+0.02X_1$$
$$+0.003X_2$$

Setting $\frac{\partial TC(X_1,X_2)}{\partial X_1}=0$ and $\frac{\partial TC(X_1,X_2)}{\partial X_2}=0$ yields $X_1^*=14.6$ unit time and $X_2^*=230.7$ unit time.

Example 2: Suppose that the values of all input parameters are given by the example 1 except $O_i(X_i)$ and $R_i(x_i)$ for $i=1,2$. Suppose that $O_i(x_i)=A_i$ and $R_i(x_i)=B_i$ for $i=1,2$ where A_i and B_i are constants. Then, $X_1^*=\infty$ unit time and $X_2^*=\infty$ unit time. These results does make sense because it is economical to keep both equipment as long as possible due to the fact that the operating cost and the repair cost are independent of service ages of both equipment.

6. Summary

A group replacement model of two equipment system has been analyzed under assumptions that each equipment is operating continuously and independently, and that each of them is subject to random breakdown and repair. A system of four linear partial differential equations which lead to derive the various probabilities to characterize the system states, is obtained. An economical model is developed using the results obtained and the various cost entities such as the cost of group replacement, the cost of operation and the cost of repair to obtained the optimal replacement ages for both equipment.

Although the replacement policy is governed by two decision variables X_1 and X_2 in this paper, nevertheless, in general, such policy is dictated by selection of a curve to determine the operating and repair costs: this is a problem in variational calculus whose solution is not obvious. From an economic standpoint, an analysis is not complete unless a group replacement policy is compared on a cost basis with an individual replacement policy. This type of analysis may be performed in a straightforward fashion since the analytic results for an individual replacement policy are known (see e.g., Sivazlian and Stenfel[8]). A comparison of the results between these two policies will determine whether a group replacement policy is preferred to an individual replacement police or vice versa.

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