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Spatial Distribution of Mobiles in Cellular Communication Network

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이동 통신망에서의 셀 내 가입자 분포 분석

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We present a simulation model to generate the spatial distribution of mobiles in cellular communication network. Three types of spatial distributions are considered; biased, random, and ratio-based distributions. This study also points out and corrects the critical errors performed by Das and Morgera(1997) in getting random location of mobiles. By applying a simple path loss model, the effects of our correction on the signal-to-interference (SIR) ratio are discussed. The numerical results indicate that the variation of SIR in the Das's biased distribution is larger than that of other distributions. As compared with the random distribution, the average SIR error of the biased distribution is 91.1%.

1. Introduction

The spatial distribution of mobiles in a cell area is used to simulate the wireless circumstances in the mobile cellular communication network. Using the spatial distribution of mobiles, the multiple-access interference in an uplink (mobile-to-base station) or downlink (base station-to-mobile) can be characterized by considering the moving direction and speed of the mobiles. Furthermore, interference in the area with the non-uniform distributions (for example, hot spot or sparse area) is different from that of the random spatial area.

Das and Morgera(1997) presented the simulation model in a DS-CDMA (direct sequence-code division multiple access) integrated voice/data wireless network under the principle of random spatial distribution of the mobiles. However, they used the biased distribution to get the random location of the mobiles. In this paper, we correct their critical errors and present the right spatial random distribution of mobiles. The numerical results for the multiple-access interference are also discussed by using the simple path loss model. Three types (biased, random, and ratio-based) of spatial distributions are analyzed. For the cell area with the hot spot or sparse distribution of mobiles, the ratio-based simulation model can be used.

2. Spatial Distribution of Mobiles

2.1 Biased distribution

Das and Morgera performed a simulation to characterize the multiple-access interference in a DS-CDMA integrated voice/data wireless network. In the simulation, they tried to generate all the mobiles under the principle of random spatial distribution as indicated in the following expression; '*A fixed number of mobiles is randomly distributed within each cell*' in the Section III. A of the paper (Das and Morgera, 1997). For the purpose, they used the spatial distribution of the mobiles as follows:

The polar coordinates of the mobiles are denoted by (r, θ), where r and θ represent the radial and angular distance of a mobile from the center of its cell. The probability density function (pdf) of the polar coordinates is

$$p(r, \theta) = \frac{1}{2\pi R}, \quad 0 < r < R, \quad 0 < \theta < 2\pi \quad (1)$$

where R is the radius of the circle that circumscribes a hexagonal cell. We have found that the distribution is not appropriate to generate the mobiles randomly within each cell. Making a change of variables, $x = r\cos(\theta)$, $y = r\sin(\theta)$, we have

$$p(x, y) = \frac{1}{2\pi R\sqrt{x^2+y^2}}, \quad 0 < x^2+y^2 < R^2, -R < x, y < R \quad (2)$$

The pdf implies that the possibility of generating the mobile of a location is inversely proportional to the square-rooted radial distance of the location. Therefore, the mobiles will be more concentrated near center rather than the outer area of the cell. In the following sections, we will derive the right spatial distribution substituting for Eq. (1) and discuss the effects of our correction on the interference by using a numerical study. Moreover, a simulation model for the ratio-based spatial distribution is proposed to evaluate the interference in the hot spot or sparse cell area.

2.2 Random distribution

When the mobiles are distributed uniformly over the cell area, the cumulative distribution function (cdf) of (r, θ) should be

$$P(r, \theta) = \frac{r^2\theta}{2\pi R^2}, \quad 0 < r < R, \quad 0 < \theta < 2\pi \quad (3)$$

Thus, the pdf of (r, θ) is

$$P(r, \theta) = \frac{r}{\pi R^2}, \quad 0 < r < R, \quad 0 < \theta < 2\pi \quad (4)$$

Similarly to Eq. (2), we have the corresponding pdf of (x, y)

$$P(x, y) = \frac{1}{\pi R^2}, \quad 0 < x^2+y^2 < R^2, \quad -R < \theta < R \quad (5)$$

Unlike the Das's biased method, the above pdf indicates that the probability of selecting a location is inversely proportional to the cell area. That is, the mobiles can be randomly generated by using Eq. (4).

2.3 Ratio-based distribution

As shown in <Figure 1>, a cell can be partitioned into N subareas included in the range of radius, $(r_1 - r_0)$, $(r_2 - r_1)$, $\cdots$, and $(r_N - r_{N-1})$, respectively, where $r_0 = 0$, $r_N = R$. The length, $(r_n - r_{n-1})$, of each subarea is all R/N in the case of equal division. Define the weight parameter of the n^{th} length $(r_n - r_{n-1})$ to the divided equal length(R/N), as $\alpha_n = \frac{(r_n - r_{n-1})}{R/N}$ for $n = 1, 2, ..,$ and N.

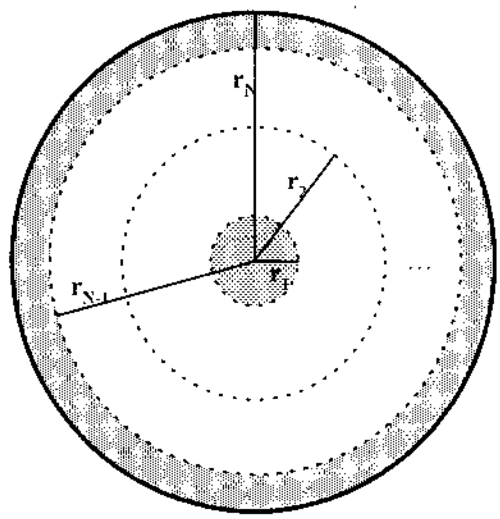


Figure 1. Subareas in a cell.

Then, the size of the subarea can be determined by using the α_n.

$$\begin{aligned} r_1 &= \alpha_1 \frac{R}{N}, \\ r_2 &= r_1 + \alpha_2 \frac{R}{N} = (\alpha_1 + \alpha_2)\frac{R}{N}, \\ &\cdots \\ r_N &= r_{N-1} + \alpha_N \frac{R}{N} = \sum_{n=1}^{N} \alpha_n \frac{R}{N} \end{aligned} \quad (6)$$

where $\sum_{n=1}^{N} \alpha_n = N$.

Denote the ratio of the number of mobiles located in each subarea to the total number of mobiles by p_n, $n = 1, 2, \cdots, N$. Assuming the uniform distribution of mobiles in each subarea, the weight α_n is recursively calculated by using the following equation.

$$\begin{aligned} \alpha_1 &= N\sqrt{p_1}, \\ \alpha_n &= \sqrt{\left(\sum_{i=1}^{n-1} \alpha_i\right)^2 + N^2 p_n} - \sum_{i=1}^{n-1} \alpha_i, \quad n = 2, 3, \cdots, N \end{aligned} \quad (7)$$

3. Simulation

To locate the mobiles in the cell area, the polar coordinates of the mobiles are determined by using the following simulation model. In the biased model, the spatial location of the mobile is calculated based on the uniform distribution.

$r \sim Uniform(0, R),\ \theta \sim Uniform(0, 2\pi)$:

Biased Distribution

Unlike the biased distribution, the random spatial distribution of mobiles is gathered by using the uniform distribution of 0 and 1.

$r \sim R\sqrt{Uniform(0, 1)},\ \theta \sim Uniform(0, 2\pi)$:

Random Distribution

Finally, given the ratio for the number of mobiles in each subarea and the size of the subarea, the ratio-based spatial distribution of mobiles is determined by applying the following routine.

$U \sim Uniform(0, 1)$

if ($U < p_1$) then

$r \sim r_1\sqrt{Uniform(0, 1)}$

elseif ($\sum_{i=1}^{n-1} p_i \le U < \sum_{i=1}^{n} p_i$) then: *Ratio – based Distribution*

$r \sim r_{n-1} + (r_n - r_{n-1})\sqrt{Uniform(0,1)},\ n = 2, 3, \cdots, N$

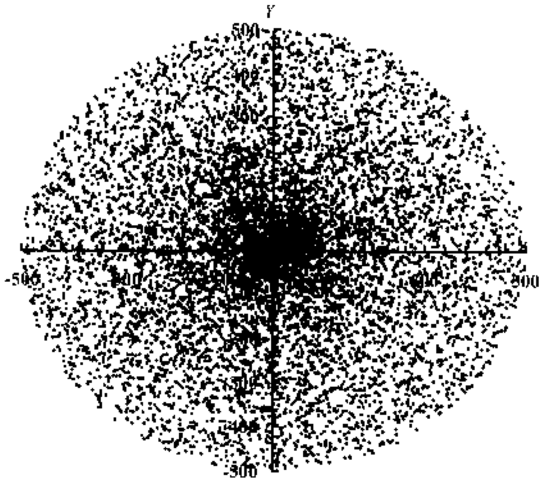
endif

4. Numerical Results

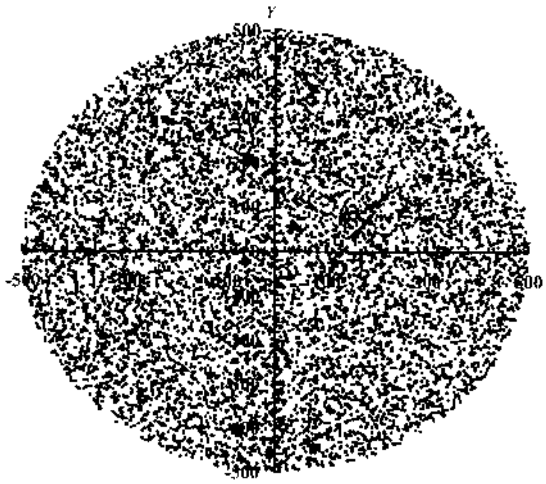
We perform a computer simulation to confirm our correction and analyze the interference for each spatial distribution of mobiles. For a visual validation, <Figure 2> plots the mobile's location by Das's biased distribution (a), random distribution (b), and ratio-based distribution (c, d, and e). We gene- rate 10,000 *mobiles* within the cell of radius $R = 500$ meter. The three types of test example for the ratio- based distribution are summarized as <Table 1>.

Table 1. The types for the ratio-based distribution

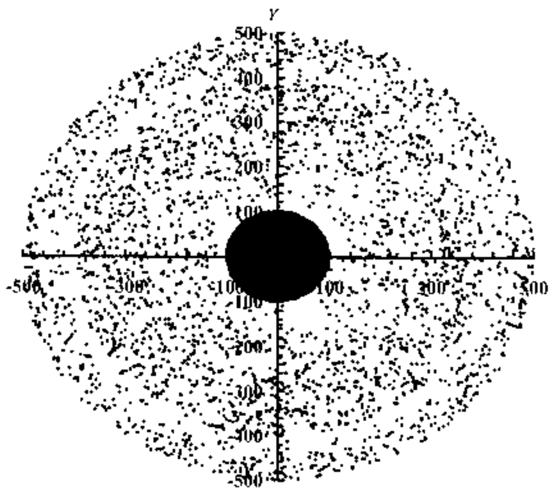
	r_1 100	r_2 400	r_3 500
Ratio1	0.7	0.2	0.1
Ratio2	0.1	0.8	0.1
Ratio3	0.1	0.2	0.7



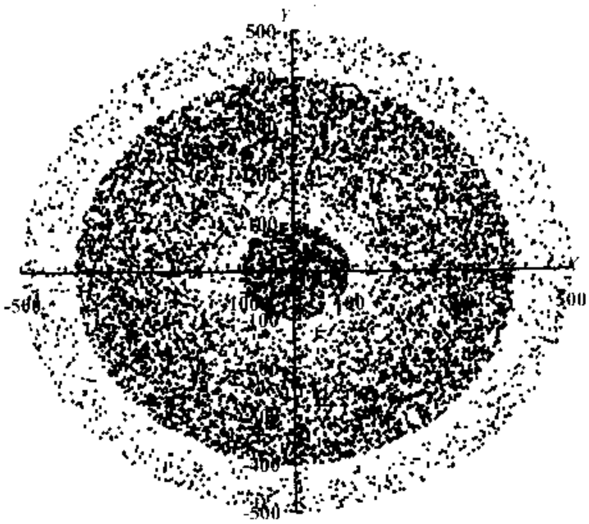
(a) Biased distribution



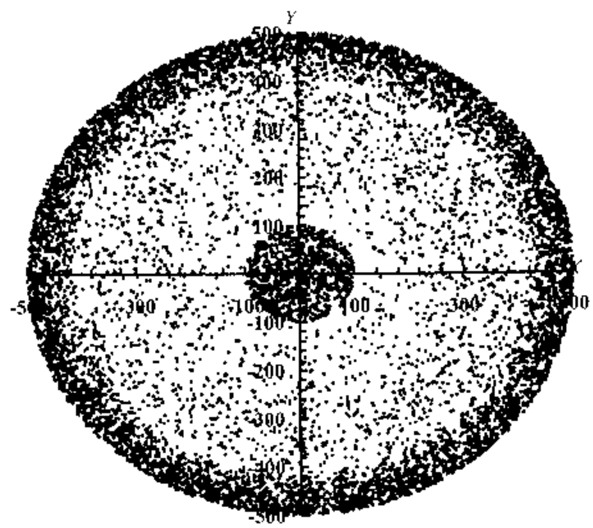
(b) Random distribution



(c) Ratio-based distribution ($p_1 = 0.7, p_2 = 0.2, p_3 = 0.1$)



(d) Ratio-based distribution ($p_1=0.1, p_2=0.8, p_3=0.1$)



(e) Ratio-based distribution ($p_1=0.1, p_2=0.2, p_3=0.7$)

Figure 2. Spatial distribution of mobiles.

The cell is composed of three regions ($r_1=100$ meter, $r_2=400$ meter, and $r_3=500$ meter). The values indicated in the table represent the ratio of the mobiles included in each region to the total number of subscribers.

From the results of the visual configuration, it is observed that the mobiles generated by Das's biased distribution are not uniformly distributed, but are more concentrated near center rather than the outer area of the cell. On the other hand, random method ensures that the mobiles are randomly distributed over the cell area. It is also observed that the range of $r_1=100$ meter near the base station is more dense for all p_1's. That is due to the small area of r_1 as compared with (r_2-r_1), and (r_3-r_2).

<Figure 3> shows the number of mobiles occurred in each radius less than R (see also the <Figure 13 (b)>of Das and Morgera(1997)). The figure indicates that we should gather a lot of mobiles in larger radius to guarantee the randomness and to satisfy the ratio of mobiles in each range of radius.

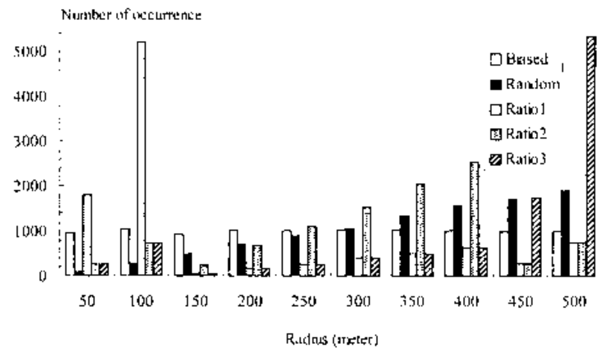


Figure 3. Histogram for simulated radius within a cell.

Finally, we adopt the simple path loss model (William, 1995) to evaluate the signal-to-interference ratio (SIR) for the downlink (base station-to-mobile). The model assumes that i) received power is inversely proportional to d_i^4, where d_i is the distance between mobile i and base station, ii) no power control system, and iii) the interference due to base stations in surrounding cells is not included. By applying the path loss model, the SIR received by a mobile is expressed as

$$\left(\frac{S}{I}\right)_i = 10\log\left[\frac{d_i^{-4}}{\sum_{k=1,k\neq i}^{K} d_k^{-4}}\right] \quad \text{for } mobile \;\; i, \tag{8}$$

where I is the interference from the base station of the cell in which the mobile i is located. The K is the total number of mobiles that are concurrently calling within the same cell as mobile i stays in. <Figure 4>

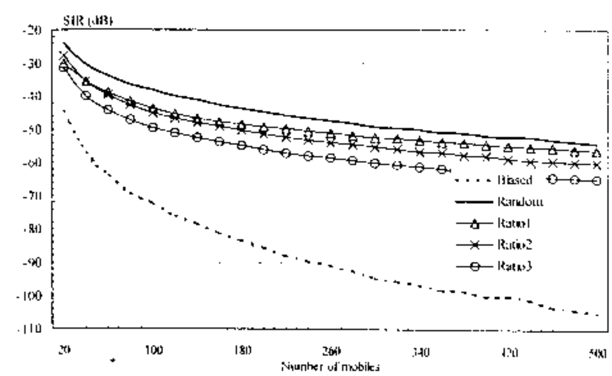


Figure 4. SIR as a function of number of mobiles.

shows the SIR results as a function of K. The SIR values for 1,000 *mobiles* are randomly gathered. As compared with the random distribution, the Das's biased results significantly reduce the SIR. The ratio-based methods also slightly reduce the SIR. The SIR in the hot spot area (Ratio1) near base station is less reduced than that in the examples of Ratio2 and Ratio3. The deviation of SIR between three methods becomes larger for a larger value of K. The figure also indicates that the variation of SIR in the biased distribution is larger than that of other methods. The average errors (computed by (*biased* or *ratio based* - *random*)/*random*) of SIR are evaluated to be 91.1% (biased), 9.5% (Ratio1), 14.2% (Ratio2), and 24.7% (Ratio3), respectively.

5. Conclusions

A simulation method for the spatial distribution of mobiles was presented to characterize the multiple-access interference in the mobile cellular communication network. We corrected the critical errors performed by Das and Morgera(1997) in getting random spatial location of mobiles. The ratio-based spatial distribution of mobiles was also discussed.

Adopting the simple path loss model, the effects of spatial location of mobiles on the SIR were evaluated. From the numerical results, it was observed that the Das's biased distribution significantly reduce the SIR as compared with the random distribution. On the other hand, the ratio-based distribution slightly reduced the SIR. The results also indicated that the variation of SIR in the biased distribution is larger than that of other methods. The average error of SIR between the biased and random distributions was 91.1%. Therefore, the computed interference and SIR statistics in the paper(Das and Morgera, 1997), such as uplink and downlink interference and SIR results, remain to be reevaluated with correct random spatial distribution of mobiles.

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