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A note on the Geostrophically Controlled Volume Transport of the Tsushima Current

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A simple analytic model for estimating the volume transport of the Tsushima Current is considered by assuming that the transports through the connecting channels, the Korea and Tsugaru Straits, are geostrophically controled. The model gives a much simpler form of solution than that by Minato and Kimura (1980). It depends no longer on the geometry of the marginal sea and on the thickness of western boundary layer relative to the dimension of the ocean, but considers the geometry of the connecting channels ignored by Nof (1993). The external parameters turn out to be the oceanic meridional sea level difference between the two channels, the depth of the channels and the meridional position of the marginal sea. For typical value of the depth ratio of the channels to the ocean, the model gives an estimate of the Tsushima Current transport of acceptable magnitude.

INTRODUCTION

The Tsushima Current is a major warm current passing through the East (Japan) Sea basin. It originates from the Kuroshio by leaking onto the East China Sea shelf, flows into the East Sea through the Korea Strait and flows out mostly through the Tsugaru Strait to join the Pacific (Fig. 1). The volume transport of the Tsushima Current is not quite clearly defined because is not yet accurately measured and varies from time to time. Considering various estimates made up to present (*e.g.*, Moriyasu, 1971; Yi, 1966; Sugimoto, 1990), 2 Sv is widely accepted as transport. Concerning the location of the branching of the Tsushima Current from the Kuroshio, there are two different views: One shows it at the southwestern corner of the continental shelf break (*e.g.*, Fang *et al.*, 1991) and the other shows it at the northeastern corner (*e.g.*, Lie and Cho, 1994). Finding out the mechanism of the formation of the Tsushima Current and the external parameters determining the volume transport of the Tsushima Current have long been one of the major subjects for local oceanographers. The first attempt to solve this problem is that by Minato and Kimura (1980; hereafter MK). They considered a linear dissipative wind-driven ocean connected on the west to a shallow marginal sea through narrow inflowing and outflowing channels. The resulting volume transport depends on various parameters such as the oceanic meridional sea level difference between the two channels, meridional distance between the two channels, depth ratio of the marginal sea to the ocean, thickness of the western boundary layer relative to the scale of the ocean and geometries of the marginal sea and channels.

On the other hand, Nof (1993) considers the problem as a geostrophic adjustment of warm Kuroshio water injecting through a narrow gap into the marginal sea filled with deep motionless water, where the whole domain is assumed to be infinitely deep and the gap exerts no effect on the flow. In this study, the major driving force of the Tsushima Current is the pressure gradient developing from the ocean toward the marginal sea across the gap. This pressure difference (or the thickness of upper Kuroshio water at the gap) is equivalent to that between the gap and the separation point of the Kuroshio from the western boundary where, as in the marginal sea, the deep motionless water outcrops. This meridional pressure difference is related in a simple manner to the meridional distance of the gap from the separation position. On the other hand, the marginal sea can be considered to be connected to the ocean north of the separation point. Therefore, his model is substantially analogous to the inflow-outflow system with wide outflowing channel north of the separation point, where the volume transport is solely determined by the oceanic meridional pressure difference between the inflowing and outflowing channels.

The common external parameter in the above two models is the oceanic meridional pressure difference between the two connecting channels. Indeed, both

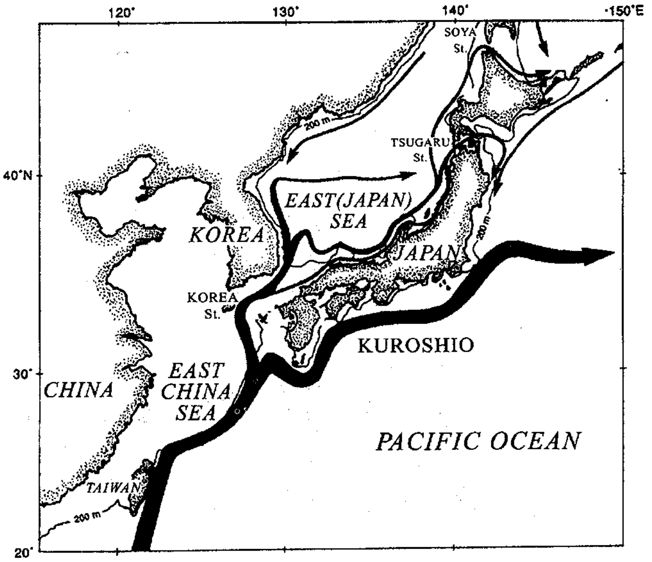

Fig. 1. Study area with schematic presentation of the branching of the Tsushima Current from the Kuroshio.

Toba *et al.* (1982) and Ohshima (1994) show that there really exist sea level differences between inside and outside of the East Sea. Although these models successfully shed light on the physics of the formation of the Tsushima Current, some comments should be made on them. The model by MK relies much on the geometry of the marginal sea. It is relatively sensitive to α, the magnitude of the friction coefficient (or the thickness of the western boundary layer relative to the dimension of the ocean). Furthermore, it is not quite convincing that the volume transport is inversely proportional to the meridional distance between the two channels independently of the oceanic meridional pressure difference between them, and that in the limit of vanishingly narrow western boundary layer (or in the limit of large friction), the transport becomes independent of r, the depth ratio of the marginal sea to the ocean. For typical parameter values of $\alpha=40$ and $r=0.2$, it gives the Tsushima Current transport of about 0.3 times the Kuroshio transport (see Fig. 6 of MK). Taking the latter as about 30 Sv, the resulting Tsushima Current transport turns out to be about 9 Sv, which is larger than the presently accepted value of about 2 Sv.

The model by Nof (1994) does not consider any geometric effect of the connecting channels although it considers the non-linearity and the beta effect. Consequently, it misses the depth ratio of the marginal sea (or gap) to the ocean as the external parameter. It gives the Tsushima Current transport of about 3–4 Sv for the meridional distance of 300 Km between the gap and the separation point. This distance is, however, minimal and the transport will increase further by taking larger distance.

In present study, we propose another model which has simpler result than that by MK and considers the geometric effect of the connecting channel ignored by Nof (1994). It owes the simplicity to the assumption that the flow through the Korea and Tsugaru Straits are geostrophically controled (Garrett, 1983; Toulany and Garrett, 1984). Major comparisons are made with the model by MK because this is similar to the present one in that both models are barotropic, linear and dissipative. This paper is organized as follows: The linear barotropic wind-driven ocean model is briefly presented in the second section. In the third section, the theory of geostrophic control is applied and the results are discussed. Finally, in the fourth section, concluding remarks are made.

WIND-DRIVEN OCEAN

Take a barotropic rectangular ocean with dimension a in zonal (x) direction and b in meridional (y) direction (Fig. 2); the origin of the coordinate (x, y) is at the southwestern corner of the ocean. The depth of the oecan is D. A zonal wind with stress $\tau=-\tau_0$ cos $(\pi y/b)$ blows over the ocean. The model ocean thus corresponds to the subtropical gyre but can be extended northward to include the subpolar gyre. To the western side of the ocean, a relatively small marginal sea is connected through two narrow channels. The southern channel lies in the subtropical gyre but the northern channel lies either in the subtropical or

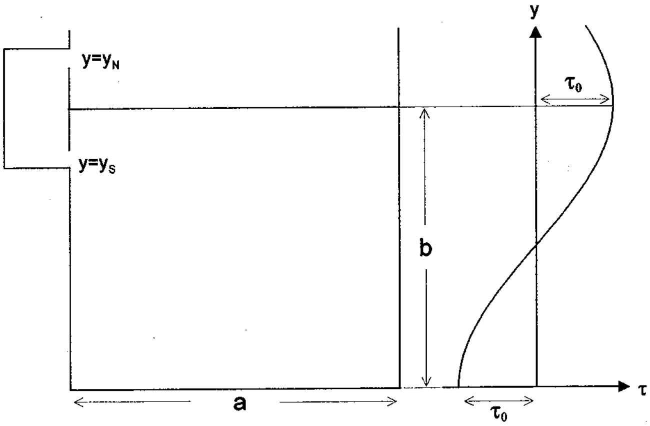

Fig. 2. Model ocean with zonal and meridional scales, respectively, a and b. A marginal sea on the west is connected to the ocean through narrow channels at $y=y_S$ and $y=y_N$. Meridional distribution of the zonal wind stress with amplitude τ_0 is shown on the right.

in the subpolar region near the boundary between the two gyres. The channels are so narrower and shallower than inside the marginal sea that the current speed in the latter is negligibly small compared to those in the formers. Note that the depth of the marginal sea here is deeper than that in MK as it really is. Assuming that the disturbance due to the presence of marginal sea is negligibly small, the Stommel's solution (Stommel, 1948) is valid in the ocean. For simplicity, while retaining the sufficient accuracy, we express the Stommel's solution by using the boundary layer approximation. We present here only the major results and detailed procedures are referred to Pedlosky (1989). For the ocean interior, it is just the Sverdrup solution:

$$\Psi_I = \frac{\tau_0 \pi}{\beta b}(a-x)\sin\left(\frac{\pi y}{b}\right) \tag{1}$$

where Ψ_I is the transport streamfunction with subscript I denoting the interior and β is the meridional gradient of the Coriolis parameter f. For the western boundary layer, it is

$$\Psi_w = \frac{\tau_0 \pi a}{\beta b}\left(1-e^{-\frac{\beta D}{k}x}\right)\sin\left(\frac{\pi y}{b}\right) \tag{2}$$

where subscript W denotes the western boundary and k is the friction coefficient. The physical quantity $\beta D/k$ is the inverse of the western boundary layer thickness which is negligibly small compared to the dimension of the ocean, *i.e.*, $a\beta D/k \gg 1$. The composite solution is

$$\Psi = \frac{\tau_0 \pi}{\beta b}\left(a-x-ae^{-\frac{\beta D}{k}x}\right)\sin\left(\frac{\pi y}{b}\right) \tag{3}$$

Maximum transport of the subtropical gyre, $\Psi_{\max}$, is equivalent to $\Psi_I(0, b/2)$, *i.e.*,

$$\Psi_{\max} = \frac{\tau_0 \pi a}{\beta b} \tag{4}$$

The surface elevation $\eta(0,y)$ along the western boundary is obtained using the fact that the meridional momentum balance is between the pressure gradient and friction, *i.e.*,

$$\frac{\partial \eta(0,y)}{\partial y} = -\frac{k}{gD^2}\left.\frac{\partial \Psi}{\partial x}\right|_{x=0} \tag{5}$$

where g is the gravity constant. Application of (2) [or (3) with approximation $a\beta D/k \gg 1$] to (5) leads to

$$\eta(0,y) = \frac{\tau_0 a}{gD}\cos\left(\frac{\pi y}{b}\right) \tag{6}$$

where the integration constant $\eta(0, 0)$ is taken arbitrarily to be $\tau_0 a/gD$. In terms of $\Psi_{\max}$, it becomes

$$\eta(0,y) = \frac{\beta b}{\pi g D}\Psi_{max}\cos\left(\frac{\pi y}{b}\right) \tag{7}$$

GEOSTROPHIC CONTROL

We assume that the transports through the narrow channels are geostrophically controled. For steady state and for frictional decay time much longer than the inertial period, the geostrophically controled transport (Garrett, 1983) at each channel is given by

$$\Psi_T = \left(\frac{gd}{f}\Delta\eta\right)_S = \left(\frac{gd}{f}\Delta\eta\right)_N \tag{8}$$

where Ψ_T is the magnitude of transport with subscript T denoting the Tsushima Current, d is the channel depth, $\Delta\eta$ is the along-channel sea level difference and subscripts S and N mean that the quantities in parentheses are evaluated, respectively, at the southern and orthern channels. Note that the flow, when geostrophically controlled, is dependent on the along-channel sea level difference rather than the cross-channel difference. Inside the marginal sea, the horizontal gradient of the sea level is presumed much smaller than that along the channel because, as noted earlier, the current speed becomes vanishingly small inside the marginal sea; major sea level difference occurs along each channel. Note therefore that the sum of the two along-channel sea level differences is equivalent to the oceanic meridional sea level difference between the two channels, *i.e.*,

$$(\Delta\eta)_S + (\Delta\eta)_N = \eta(0,y_S) - \eta(0,y_N) \tag{9}$$

Assuming that the geometries are not so different between the two channels, we combine (8) and (9) to give

$$\Psi_T = \frac{g\bar{d}}{2f}[\eta(0,y_S) - \eta(0,y_N)] \tag{10}$$

where the overbar means the average between the two channels. Using (7), the Tshushima Current transport can also be expressed in terms of the maximum transport of the Kuroshio as follows:

$$\frac{\Psi_T}{\Psi_{\max}} = \frac{\beta b \bar{d}}{2\pi f D}\left[\cos\left(\frac{\pi}{b}y_S\right) - \cos\left(\frac{\pi}{b}y_N\right)\right] \tag{11}$$

Equation (10) shows that the Tsushima Current transport is a function of the oceanic meridional pressure difference between the two channels as already shown in previous studies. Compared to the model by MK [their Equation (47)], it is much simpler and does not show any dependence on various geometries of the marginal sea and channels except the depth of the channel. Furthermore, it is independent of the meridional distance between the two channels, $y_N - y_S$, and the magnitude of friction coefficient. The dependence on depth, $\bar{d}$, is not related to the western boundary layer thickness whereas in MK, the dependence on $\bar{d}$/D disappears in the limit of large α [see their Equation (52)]. Eq. (11) gives an estimate of the Tsushima Current transport relative to the maximum Kuroshio transport. It depends on the difference of wind stress between the two channels, the depth ratio $\bar{d}$/D and the meridional scale of the gyre relative to the meridional position of the marginal sea, $\beta b/\bar{f}$, which is of O(1) in this case. Compared to Nof's result, it depends on the meridional distance only through the wind stress rather than the stress itself. As expected, the depth ratio $\bar{d}$/D appears to be an important parameter, which is ignored by Nof. Since the cosine terms in (11) give the value of O(1) for reallistic geographic positions of the channels (Fig. 1), the relative magnitude of the Tsushima Current transport becomes $\bar{d}/2\pi D$ which for typical value of $\bar{d}/D=0.2$, as in MK, is about 0.03. Taking Ψ_{max} =30 Sv, it becomes 0.9 Sv which is on the acceptable order of magnitude.

CONCLUDING REMARKS

The result of this study confirms the previous model results that the major external parameter of the Tsushima Current transport is the meridional sea level difference measured on the ocean side of the two channels connecting the ocean to the marginal sea. Owing to the geostrophic control of the transport through the channels, the geometry of the marginal sea does not appear to be external parameters. Consequently, the transport of the Tsushima Current is determined in much simpler form than that by MK. Note, however, that the geometric effect of the connecting channels is an essential ingredient of this model contrary to the model by Nof where it is ignored. The relative magnitude of the Tsushima Current transport to the Kuroshio transport depends on the depth ratio of the channels to the ocean. The meridional distance between the two channels interferes only through the wind stress in determining the meridional sea level difference whereas the distance itself is important in Nof's model. Note that in Nof (1993), the relative transport depends only on the so called "beta control" parameter which is the ratio of the meridional distance between the gap and separation position to the meridional position (from the equator) of the gap. According to the model result, the transport becomes maximum when the separation point coincides with the outflowing channel and decreases as the former occurs farther away from the latter. However, the order of magnitude will remain unchanged because that distance is not expected to be large. For typical value of depth ratio between the channels and the ocean, the Tsushima Current transport becomes about 0.9 Sv which, being ten times smaller than that by MK, is about one half of the presently known transport value.

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