MQUICK Upwind Scheme for the Incompressible Navier-Stokes Equations

B. R. Shin*1 and T. Ikohagi*2

非圧縮性 Navier-Stokes 方程式의 解析을 위한 MQUICK 上流解法

申 炳 録, 井小萩 利明

이 논문 에서는, QUICK 해법의 불안정성을 개량하므로써, 수치계산에 있어서 수렴이 빠르고, 수치적으로 안정한 계산을 할 수 있는 새로운 MQUICK 상류해법을 제안하고, 이를 비압축성 충류유동의 계산에 적용하였다. 또한, 해법의 정확성, 안정성, 수렴속도에 대한 검토를 통하여 본 MQUICK 상류해법의 유효성과 타당성이 평가되었다. 이 해법에서는 人工散逸의 加減을 조절하기 위하여 加重係數 α를 써서 정식화 하였고, 위의 검토를 통하여 α의 최적값을 조사하였다. 이 해법을 SMAC 음해법에 적용하여 2 차원 空洞유동, 3 차원 덕트유동과 같은 몇몇 표준문제를 계산하고, 계산된 결과를 실험값 또는, 3 차 정확도의 상류해법 및 QUICK 해법에 의한 결과 들과 비교 하므로써, 본 MQUICK 상류해법이 위의 다른 해법에 비하여 안정하고, 유효성이 높은 해법임을 확인 하였다.

Key Words: QUICK Scheme (QUICK 해법), Upwind Scheme (상류해법), Implicit SMAC Scheme (SMAC 음해법), Finite-Difference Method (유한차분법), Incompressible Navier-Stokes Equation (비압축성 Navier-Stokes 방정식), Curvilinear Coordinate (곡선좌표), Duct Flow (덕트유동)

1. Introduction

Recently, thanks to the rapid progress of the available computer ability, the computational fluid dynamics (CFD) has become vigorously used not only as a design tool for modern flow devices, but as a solver to analyze accurately the practical engineering flow problems. However, most flow devices and their flow fields have geometrically complex computational domains. Furthermore, the majority of such flow consists of high Reynolds number flow. To solve these flows by numerical approach, it is desirable that the available CFD code has a good efficiency, accuracy and numerical stability, and is suitable for supercomputing.

From these points of view, some efforts to improve the numerical methods for the Navier-Stokes equations have been made. For example, on improvements of finite-difference scheme associated with fast convergence as well as high stability for incompressible viscous flows, a series of papers have appeared in some articles addressing the reduction of the computing cost and stable computation at the large Courant (CFL) number[1-3], and improving the convergence rate of iterative method by using the multigrid technique[4].

On the other hand, in order to suppress and overcome the numerical instability come from non linearity of the convective term in the Navier-Stokes equation, the upwinding method for the convection term is widely used. In the sense that they guarantee the high accuracy of the solution from the effect of numerical diffusion, higher-order upwinding schemes such as QUICK[5] and the third-order upwind

*1. Member, Institute of Fluid Science, Tohoku University, Sendai 980-8577, Japan

*2. Institute of Fluid Science, Tohoku University, Sendai 980-8577, Japan

scheme are commonly used nowadays in the CFD community for the incompressible flow.

The QUICK (Quadratic Upstream Interpolations for the Convective Kinematics) scheme given by Leonard is a second-order accurate upwind scheme but is very close to the third-order accurate scheme[6]. That is, because QUICK scheme employs a three-point upwind weighted quadratic interpolation technique taking the upstream-shifted point with the sign of the transport speed of the convection term, it provides accurate results without extensive grid refinement and retains the basic stable convective sensitivity property. After the presence of this QUICK scheme, therefore, a number of articles[7,8] related to the implementation and evaluation of the scheme are appeared, and the application of this scheme for solving incompressible complex flows has brought great success in the past decades.

However, in spite of higher-order upwind scheme in accuracy we often experience that QUICK scheme leads to unstable solutions for the practical computation of convection-dominated problems. As an effective way to overcome this instability, therefore, some SIMPLE users[9-11] have been proposed some new formulations of this scheme. In their literatures, the weighting coefficients of QUICK scheme are partly redistributed on both left- and right-hand sides of systems of linear equation in the implementation of the SIMPLE algorithm, and they demonstrated the improvement of their stability, although each value of the coefficients is determined by trial and error[10,11] with the behavior of the solution during the iteration procedure.

In this paper, a new MQUICK (modified QUICK) upwind scheme is proposed. This schem is developed by the modification of QUICK scheme in order to improve the stability and convergence rate, and is formulated not by the rearrangement of weighting coefficients for just SIMPLE algorithm, but by adjusting the weight of the artificial dissipation with the general selection parameter α when QUICK based scheme is given with the fourth-order artificial dissipation. Also, its validity and effectiveness are examined by the evaluation of the accuracy, stability and convergence rate, and the optimal value for the parameter α is investigated through the error and the stability analysis by using linear equations.

The benchmark evaluation of the present MQUICK scheme are performed through some calculations for incompressible flows such as two-dimensional (2-D) steady flows in the square cavity[12] and three-dimensional (3-D) developing entry flows through a square duct with 90-degree bend[13-15]. The numerical method used in this study is an efficient implicit SMAC scheme[15] which has been developed by authors to solve incompressible Navier-Stokes equations. In order to satisfy the continuity condition and to avoid the occurrence of spurious errors, a staggered grid is applied, and the Poisson equation of pressure is solved by using the Tschebyscheff SLOR method which is suitable in supercomputing.

The comparisons of predicted results from the MQUICK and other high-order upwind schemes, such as QUICK and the third-order upwind scheme, with experimental data are provided. In consequence, it is shown that the MQUICK scheme is the most stable and its efficiency is high compared with the other two upwind schemes.

2. Numerical Method

2.1. Governing Equations

The governing equations of incompressible viscous flows are the Navier-Stokes equations and the continuity equation of volume fluxes JU_ℓ in curvilinear coordinates. They can be written in the conservative forms as[15]

$$\frac{\partial}{\partial t}(JU_\ell) + L(JU_\ell, p) = 0 \quad (\ell = 1,2,3) \tag{1}$$

$$D \equiv \frac{\partial}{\partial \xi_i}(JU_i) = 0 \tag{2}$$

where,

$$L(JU_\ell, p) \equiv \frac{\partial}{\partial \xi_i}(JU_iU_\ell) - JU_i\boldsymbol{u}\cdot\frac{\partial}{\partial \xi_i}\nabla\xi_\ell + \tilde{g}_{\ell i}\frac{\partial p}{\partial \xi_i} + \nu\epsilon_{\ell i j}\frac{\partial}{\partial \xi_i}h_{jk}Z_k, \tag{3}$$

$\tilde{g}_{ij} = Jg_{ij}$, and $\epsilon_{\ell ij}$ is the permutation tensor. The Jacobian J and the metrics g_{ij} and h_{ij} of the transformation from Cartesian coordinates to general curvilinear coordinates are $J = \partial(x,y,z)/\partial(\xi,\eta,\zeta)$, $g_{ij} = \nabla\xi_i \cdot \nabla\xi_j$

and $h_{ij} = \partial x_k/\partial\xi_i \cdot \partial x_k/\partial\xi_j$, respectively. And the relation between the physical velocity u_i and the contravariant velocity U_i is $U_i = (\partial\xi_i/\partial x_j)u_j$. Similarly the contravariant vorticity Z_i is defined by $Z_i = (\partial\xi_i/\partial x_j)\zeta_j$ with physical vorticity ζ_i.

2.2. Delta Formed Implicit SMAC Scheme

Now, Eq.(1) is extend to the implicit SMAC scheme by applying the delta form approximate - factorization method[16] and partially including the viscous term in the left-hand side. Therefore, the momentum equations of the present implicit SMAC scheme are written as follows, for instance, in regard of JU in ξ-direction[15]:

$$\{1+\Delta t(\frac{\partial}{\partial\xi}U^n - \nu\frac{\partial}{\partial\xi}\tilde{h}_{22}\tilde{h}_{33}\frac{\partial}{\partial\xi})\}\cdot$$
$$\{1+\Delta t(\frac{\partial}{\partial\eta}V^n - \nu\frac{\partial}{\partial\eta}\tilde{h}_{33}\frac{\partial}{\partial\eta}\tilde{h}_{11})\}\cdot$$
$$\{1+\Delta t(\frac{\partial}{\partial\zeta}W^n - \nu\frac{\partial}{\partial\zeta}\tilde{h}_{22}\frac{\partial}{\partial\zeta}\tilde{h}_{11})\}\Delta JU^*$$
$$= RHS_1^n \tag{4}$$

where,

$$RHS_1 = -\Delta t\{\frac{\partial}{\partial\xi_i}(JU_iU_1) - JU_i\boldsymbol{u}\cdot\frac{\partial}{\partial\xi_i}\nabla\xi$$
$$+\tilde{g}_{1i}\frac{\partial p}{\partial\xi_i} + \nu\epsilon_{1ij}\frac{\partial}{\partial\xi_i}(\tilde{h}_{jk}JZ_k)\}, \tag{5}$$
$$JU_1^* = JU_1^n + \Delta JU_1^*,$$
$$JZ_\ell = \epsilon_{\ell ij}\frac{\partial}{\partial\xi_i}(\tilde{h}_{jk}JU_k) \qquad (\ell = 1,2,3),$$

and $\tilde{h}_{ij}$=h_{ij}/J. Also, by satisfying the continuity condition D^{n+1}=0 with Eq.(2), we have the following Poisson equation of pressure ϕ.

$$\frac{\partial}{\partial\xi_\ell}(\tilde{g}_{\ell i}\frac{\partial\phi}{\partial\xi_i}) = \frac{1}{\Delta t}\frac{\partial}{\partial\xi_\ell}(JU_\ell^*) \tag{6}$$
$$p^{n+1} = p^n + \phi \tag{7}$$

where the asterisk * denotes the intermediate time level between time step n and $n+1$, and ϕ is the pressure increment.

The present implicit SMAC scheme satisfies a diagonally dominant condition with the first-order upwind scheme in left-hand side convection terms and, is in the TVD stable[3]. Also, this implicit SMAC scheme is suitable for vector or vector parallel computers as compared with HSMAC[17] and SIMPLE scheme[18] developed toward scalar machines.

The momentum equation of Eq.(4) can be solved by dividing them into three steps, and each step is the problem solving simultaneous linear equations with tri-diagonal matrix by the Gaussian elimination.

3. MQUICK Upwind Scheme

The second-order central-difference is basically used for the space derivatives. However, for convection term on the left-hand side of Eq.(4), the first-order upwind scheme is used to reduce the computational efforts and to accelerate the convergence, while the higher-order upwind scheme is applied to the right-hand side to get accurate and stable solution for high Reynolds number flows. Here, a linear hyperbolic equation is considered to explain a MQUICK scheme as

$$\frac{\partial u}{\partial t} + \frac{\partial f(au)}{\partial x} = \frac{\partial u}{\partial t} + a\frac{\partial u}{\partial x} = 0 \tag{8}$$

where $a(u) = \partial f/\partial u$ is the propagating speed of waves or the transport speed of the convection term. The finite-difference equation of Eq.(8) can be written for the advective form

$$u_i^{n+1} + \frac{\Delta t}{\Delta x}a_i\Theta(\hat{f}_{i+1/2} - \hat{f}_{i-1/2})^{n+1} =$$
$$u_i^n - \frac{\Delta t}{\Delta x}a_i(1-\Theta)(\hat{f}_{i+1/2} - \hat{f}_{i-1/2})^n \tag{9}$$

and for the conservative form

$$u_i^{n+1} + \frac{\Delta t}{\Delta x}\Theta(\hat{f}_{i+1/2} - \hat{f}_{i-1/2})^{n+1} =$$
$$u_i^n - \frac{\Delta t}{\Delta x}(1-\Theta)(\hat{f}_{i+1/2} - \hat{f}_{i-1/2})^n \tag{10}$$

where $0 \le \Theta \le 1$ and, $\hat{f}$ is the numerical flux function. In the case of the second-order central-difference scheme, the numerical flux function is taken $\hat{f}_{i+1/2} = (u_i + u_{i+1})/2$ for Eq.(9) and $\hat{f}_{i+1/2} = a_{i+1/2}(u_i + u_{i+1})/2$ for the conservative form.

As mentioned above, to overcome the numerical instability in convection-dominated flow, the upwinding of the convection term is considered. In general, upwinding is obtained by adding the artificial dissipation to the central-difference scheme. Therefore, when the fourth-order artificial dissipation is considered, upwind finite-difference schemes for Eqs.(9) and (10) can be derived as followings for the numerical flux in Eq.(9)

$$\hat{f}_{i+1/2} = \{-(1+\alpha)u_{i-1} + (c_1+3\alpha)u_i + (c_1-3\alpha)u_{i+1} - (1-\alpha)u_{i+2}\}/c_2 \quad (11)$$

and in Eq.(10)

$$\hat{f}_{i+1/2} = a_{i+1/2}\{-(1+\alpha)u_{i-1} + (c_1+3\alpha)u_i + (c_1-3\alpha)u_{i+1} - (1-\alpha)u_{i+2}\}/c_2 \quad (12)$$

with parameter $\alpha = \alpha_0 \mathrm{sign}(a_i)$ in Eq.(11) and $\alpha = \alpha_0 \mathrm{sign}(a_{i+1/2})$ in Eq.(12). This α represents a weight of the artificial dissipation. In QUICK scheme[5], the value c_1=9, c_2=16 and α_0=1 in Eqs.(11) and (12) are chosen. On the other hand, the third-order upwind scheme takes c_1=7, c_2=12 and $\alpha_0 = 1$.

3.1. MQUICK Schemes and Their Stability

As previously stated, QUICK scheme provides accurate results and stable computations for the moderate-to-high Reynolds number flows. However, we often experience that this scheme leads to unstable solutions for complicated and 3-D unsteady flows. This instability depends on the selection of the modulus of α in Eqs.(11) and (12).

To investigate the stability for the previous upwind scheme, von Neumann method for stability analysis is used in this paper. According to the von Neumann, the stability condition requires the modulus of the amplification factor G to be lower or equal to one, that is, $|G|^2 \le 1$ when the amplification factor defined by $G(\Delta t, \Delta x, \theta) \equiv u_i^{n+1}/u_i^n$ as the ratio of the amplitudes u^n. In the case of the third-order upwind scheme, G can be derived as, for advective formed explicit scheme with $a > 0$ and $\Theta = 0$ in Eq.(9)

$$G = 1 - \frac{C}{6}(\cos 2\theta - 4\cos\theta + 3) - i\frac{C}{6}(8\sin\theta - \sin 2\theta) \quad (13)$$

where, C represents CFL number defined by $C = a\Delta t/\Delta x$, $i = \sqrt{-1}$ and θ denotes the phase angle. Thus we know that in order to be stable, this scheme has to take CFL of $C < 0.176$.

In the same manner, for QUICK-type schemes with arbitrary values of weighting parameter α, we obtain G as

$$G = 1 - \frac{\alpha C}{8}(\cos 2\theta - 4\cos\theta + 3) - i\frac{C}{8}(10\sin\theta - \sin 2\theta) \quad (14)$$

For $\alpha = 1$, Eq.(14) corresponds to the G of QUICK scheme. And its CFL range is $0 < C < 0.161$. On the other hand, for MQUICK schemes with $\alpha \neq 1$, we know that the CFL range is increased with α, that is, $C < 0.206$ at $\alpha = 2$, $C < 0.262$ at $\alpha = 4$, $C < 0.28$ at $\alpha = 5$, and so on.

Figure 1 shows characteristics of both the amplitude $|G|$ and the phase Φ for the equations (13) and (14) at several CFL numbers. Particularly in these figures, $|G|$ means an error in the amplitude because this error, usually called the diffusion or dissipation error, is defined as the ratio of the computed amplitude to the exact one. At $|G| < 1$ the waves of u in the numerical simulation is damped. Similarly, Φ/ϕ means the error on the phase of the solution, called the dispersion error, where ϕ means the exact value of the phase for Eq.(8).

As shown in this figure, MQUICK scheme with α of 4 is very stable at most CFL number in comparison with QUICK scheme not shown here. The computed waves are gradually damped up to critical CFL and the phase angle θ, indicating that MQUICK scheme is generating relatively strong numerical dissipation at available large CFL. This dissipation helps the increment of the numerical stability. On the other hand, dispersion errors at large CFL are bigger than one called the leading error which will become a factor to lead an oscillation of the waves, while it was confirmed that this error was everywhere smaller than one in QUICK scheme, that is, showed a lagging error which means the numerical computed waves propagate slower than the physical waves.

Figure 2 shows an investigation of the nature and frequency spectrum of the amplitude and a phase for all QUICK based schemes at CFL=0.5. At this high CFL number, the be-

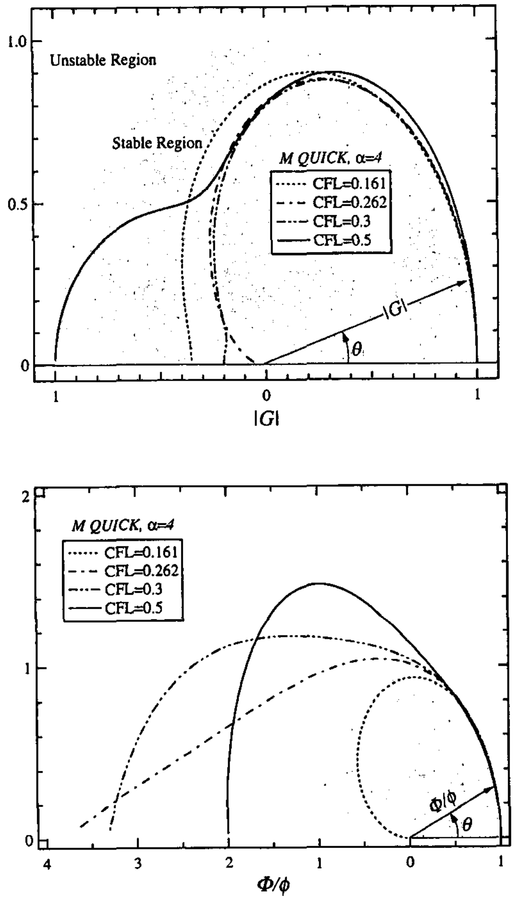

Fig.1 Amplitude and phase errors for MQUICK scheme with $\alpha = 4$.

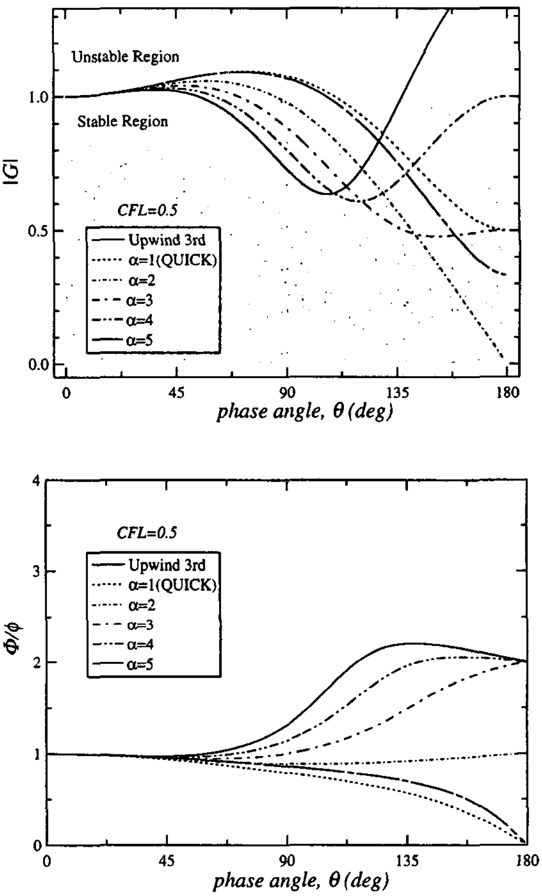

Fig.2 Dissipation and dispersion errors for CFL=0.5.

havior of $|G|$ for large α is more dynamic than small ones. The more we take large α over 4 in MQUICK scheme, the more we have the unstable range converted from damping to amplification between near $\pi/2$ and π in θ. This example helps determine the optimal value of α, although the value of CFL in this figure is beyond the analytical range of the stability condition for all schemes. And, we can infer that the optimal value of α for stable computation will exist between 3 and 5. From this figure, it shows that QUICK scheme is always in the most unstable state.

3.2. Accuracy of MQUICK schemes

According to the Taylor expansion, the truncation error of Eq.(9) which indicates the accuracy of the resulting spatial discretization is obtained as following Table 1 at $a > 0$ for above upwind schemes. Here we know that all QUICK based schemes produce the second-order truncation error but it is clearly much smaller than that of the second-order central-difference approximation with the truncation error leading term of $-(a/6)\Delta x^2 u'''$. These schemes are rather close to the third-order scheme[6]. It is also known that instead of improvement of the stability for the MQUICK schemes as seen in Figs.1 and 2, the truncation error is increased by each $-(a/16)\Delta x^3 u''''$ with the increment of α.

4. Numerical Results

4.1. 1-D Problem of Discontinuity

Table 1 Truncation error

Scheme	α	c_1	c_2	Truncation error
Upwind 3rd	1	7	12	$-(a/12)\Delta x^3 u''''$
QUICK	1	9	16	$-(a/24)\Delta x^2 u'''$ $-(a/16)\Delta x^3 u''''$
MQUICK	2 ~5	9	16	$-(a/24)\Delta x^2 u'''$ $-(\alpha a/16)\Delta x^3 u''''$

In order to investigate an aspect related to the numerical stability for above upwind schemes, the inviscid Burgers' equation (8) in conservative form, that is,

$$\frac{\partial u}{\partial t} + \frac{\partial}{\partial x}(\frac{u^2}{2}) = 0 \qquad (15)$$

for an initial linear distribution is computed numerically with $\hat{f}$ in Eq.(10) and their predictions are compared with exact solution in Figs.3 and 4. In this study, the initial condition of 1-D discontinuity is given by

$$u(x,0) = \begin{cases} 1 & (1.5 \geq x) \\ 2.5 - x & (2.5 \geq x > 1.5) \\ 0 & (x > 2.5) \end{cases} \qquad (16)$$

The discontinuity to be similar to the shock in the compressible flow is firstly occurred at $t = 1.0$ and $x = 2.5$, and the exact solution for this problem is obtained by the following equation for $t < 1.0$ and $t > 1.0$, respectively.

$$u(x,t) = \begin{cases} 1 & (1.5 + t \geq x) \\ \dfrac{2.5 - x}{1 - t} & (2.5 \geq x > 1.5 + t) \\ 0 & (x > 2.5) \end{cases} \qquad (17)$$

$$u(x,t) = \begin{cases} 1 & (2 + 0.5t \geq x) \\ 0 & (x > 2 + 0.5t) \end{cases} \qquad (18)$$

Figure 3 shows a comparison of solutions for Burgers' equation (15) at CFL=0.262 with mesh size of $\Delta x = 0.005$. Here the solid line represents the exact solution. At $t = 0.5$ the solution is still continuous, although velocity steepening has strongly deformed the initial linear distribution. At $t = 1.0$ the discontinuity is present at $x = 2.5$, so that we can check for the correct convection velocity. The existence of oscillations of the wave can be seen near the velocity discontinuity. And at $t = 1.5$ the discontinuity has reached to $x = 2.75$ as Eq.(18). However, in the case of both the third-order upwind and QUICK schemes, the amplitude of the errors appeared before the discontinuity increases continuously with time as compared with MQUICK schemes. The velocity distributions obtained by MQUICK schemes show almost the same pattern in this CFL number.

Another comparison at high CFL number of 0.5 is shown in Fig.4. In this case, the solution obtained by QUICK scheme in Fig.4(b) is near the diverging state at $t = 1.0$, and after $t = 1.5$ this scheme is completely destroyed. Also, at large α of 5 in Fig.4(d), the computation becomes unstable from the beginning of the computation, and then, the solution has been diverged before $t = 1.0$. However, it is shown that the MQUICK scheme with $\alpha = 4$ is still solvable this discontinuity problem. According to the investigation in the present work, MQUICK scheme at $\alpha = 4$ is the most stable. As can be seen in Fig.4(c) the MQUICK scheme resolves the discontinuity with no more than three spatial points and it has been convected correctly.

4.2. Flow in a Square Cavity

Numerical simulations for a 2-D steady incompressible flow in the square cavity is performed by using the present implicit SMAC scheme with QUICK, MQUICK and the first-order upwind schemes. And then, the validity of MQUICK scheme in convergence, efficiency and stability is discussed through some sets of numerical results.

The computational geometry of the cavity flow is the same as that of Ghia, et al.[12]. No-slip boundary condition together with the Neumann condition for the pressure was imposed on the solid wall boundary. Also, the driven velocity on the driven wall of the cavity was prescribed. The weighting parameter α_0 of 4 in Eq.(12) was used for an application of MQUICK scheme. Computations are started with both velocities and pressure free condition except the lid driven velocity at Reynolds number (Re) of 3200, where Re is based on the cavity height and driven velocity with 81×81 grid points.

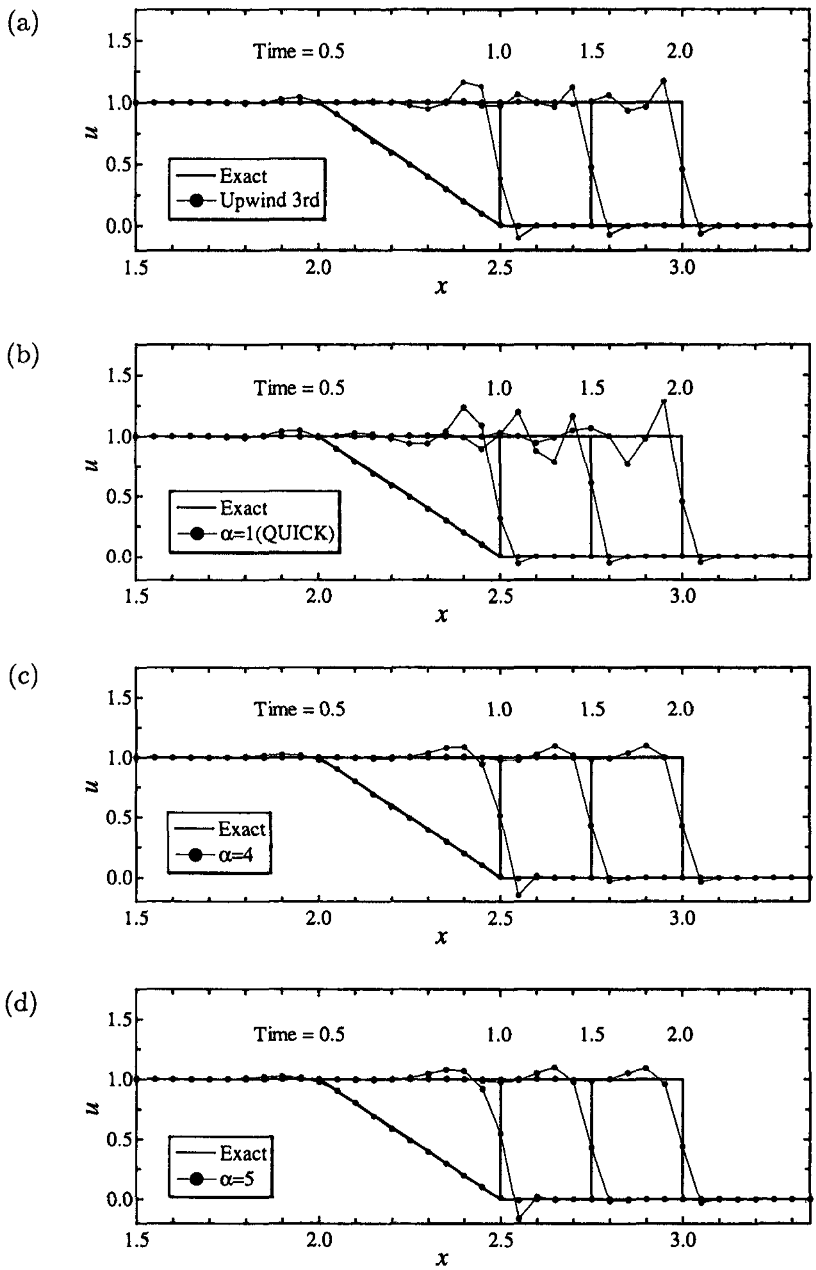

Fig.3 Solutions of Burgers' equation for an initial linear distribution at CFL=0.262: (a) third-order upwind scheme, (b) QUICK scheme, (c) MQUICK with $\alpha = 4$ and (d) MQUICK with $\alpha = 5$.

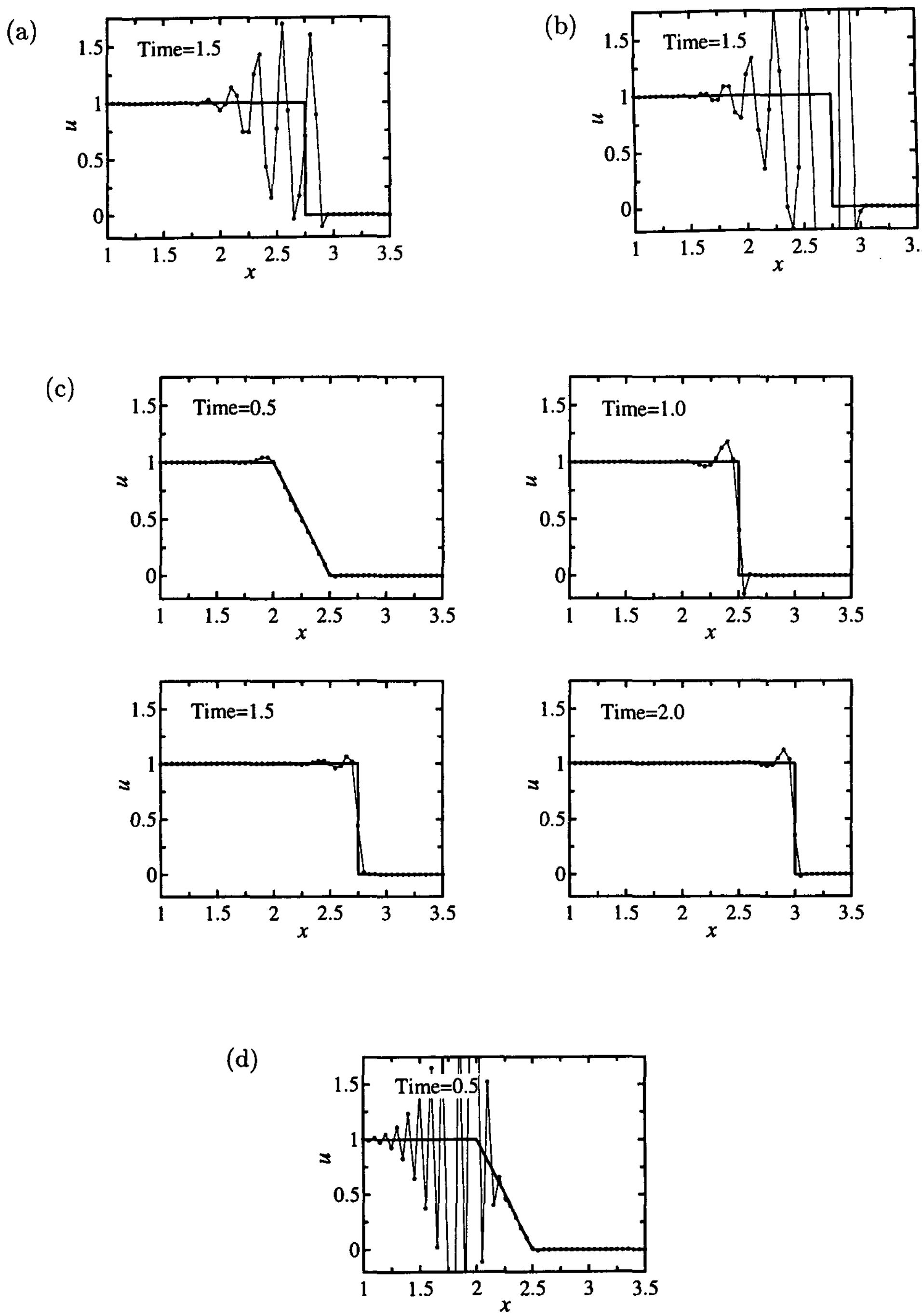

Fig.4 Solutions of Burgers' equation for an initial linear distribution at CFL=0.5: (a) third-order upwind scheme, (b) QUICK scheme, (c) MQUICK with $\alpha = 4$ and (d) MQUICK with $\alpha = 5$.

Firstly, to investigate the numerical stability and convergence rate, Fig.5 is shown iteration histories of the norm of divergence and pressure residual defined respectively as, (Norm of Divergence)^2 = $(1/N)\sum_{ijk}(D^n)^2$ and (Norm of p Residual)^2 = $(1/N)\sum_{ijk}(p^{n+1}-p^n)^2$, where N is the total number of grid points, and i, j and k represent the index for each direction of the ξ, η and ζ, respectively.

According to the present investigation, at the relatively low CFL number, for example, around 1, it showed that the convergence rate of all upwind schemes was in almost same degree. In the case of high CFL number in Fig.5, however, the MQUICK scheme was more predominant than that of QUICK and the third-order upwind scheme at CFL=40 which is the maximum CFL number of QUICK scheme for the present test case. It endorses the fact investigated previously in the sections 3.1 and 4.1. Particularly around 3000 iterations in Fig.5, the curve of the norm of divergence for MQUICK scheme is almost reached to the steady state with high accuracy. To obtain such accuracy with QUICK and the third-order upwind schemes, it requires about 6000 iterations. In the comparison of velocity profiles with those of Ghia et al.[12] which are well-resolved predictions by using a refine mesh with 129×129 grid points, they were in accord quite well with each other.

4.3. Flow through a Square Duct with 90-degree Bend

Another numerical example considered in this study is a developing entry flow through a 3-D square duct with 90-degree bend. The computational geometry is the same as that of Taylor et al.[13]. $107\times41\times41$ grid points are used and Re of 790. Detailed computational conditions are referred to Ref.[15].

The effectiveness for applying the MQUICK scheme is more remarkable for the 3-D computational case given in Fig.6. With the third-order upwind and QUICK scheme the norms are still in the range of order of 10^{-5} after 6000 iterations and they are somewhat unstable. With MQUICK scheme with $\alpha = 4$, however, rapid convergence rate is obtained, indicating the effectiveness for applying the MQUICK scheme for this 3-D case.

Figure 7 shows some comparisons of streamwise velocity (U) profiles along the spanwise (Y) and radial (Z) direction with experiments[13] at two streamwise locations. Here, θ denotes the degree from starting point of the bend and X_H means the downstream distance from the end of the bend. U_0 represents the inlet mean velocity. The agreement between predictions and measurements is quite good at most locations. The discrepancy between results obtained by MQUICK scheme and the others is very small, that is, their accuracy is in the almost same state in this numerical example. Consequently, it is shown that MQUICK scheme is more efficient than the others without the loss of accuracy due to the additional damping.

5. Conclusions

A new MQUICK upwind scheme was developed by the modification of QUICK scheme in order to improve the stability and convergence rate, and is formulated by using the selection parameter α which adjusts the weight of the artificial dissipation in QUICK based schemes. The verification in validity and effectiveness for MQUICK schemes was made by the stability and the error analysis for linear equations with exact solutions. And, they were confirmed by the computation of the inviscid Burgers' equation for an 1-D problem with an initial linear distribution, and the incompressible Navier-Stokes equations for a laminar 2-D lid driven square cavity flow and a 3-D 90-degree bend flow with square cross-section.

Accurate predictions of complex flow characteristics were successfully shown by using the implicit SMAC scheme and the MQUICK scheme with α of 4. A good agreement of the present results with exact solutions and experiments was obtained. In the comparison for the accuracy, stability and convergence rate with QUICK scheme and the third-order upwind scheme, it was shown that the MQUICK scheme is the most efficient one. And it was investigated through the analysis mentioned above that the optimal value of the selection

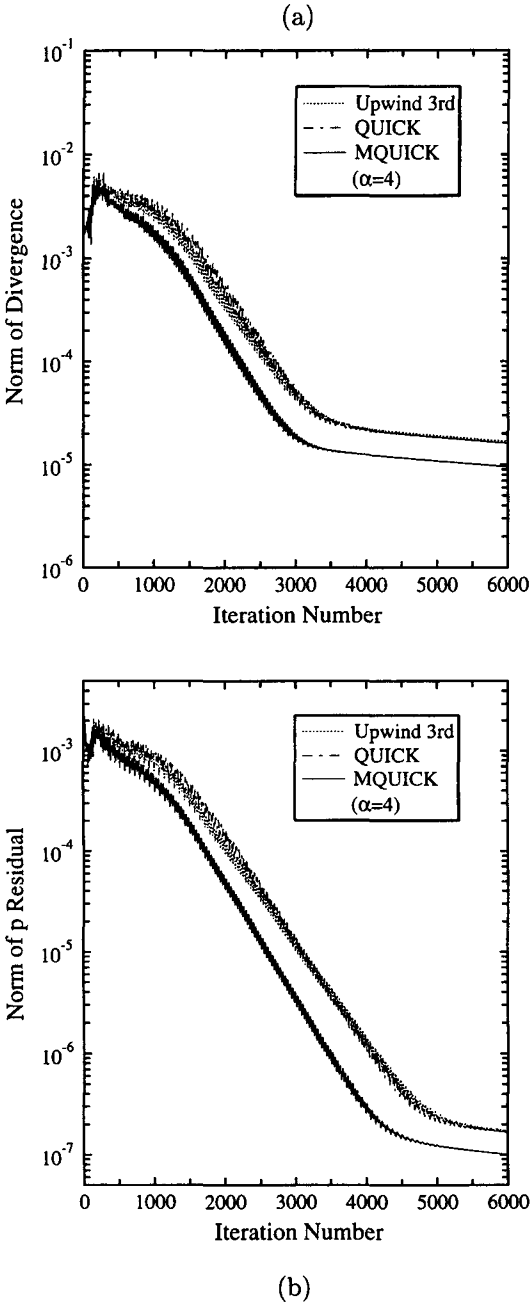

Fig.5 Iteration history of (a) norm of divergence and (b) pressure residual for 2-D cavity flow computation at CFL=40.

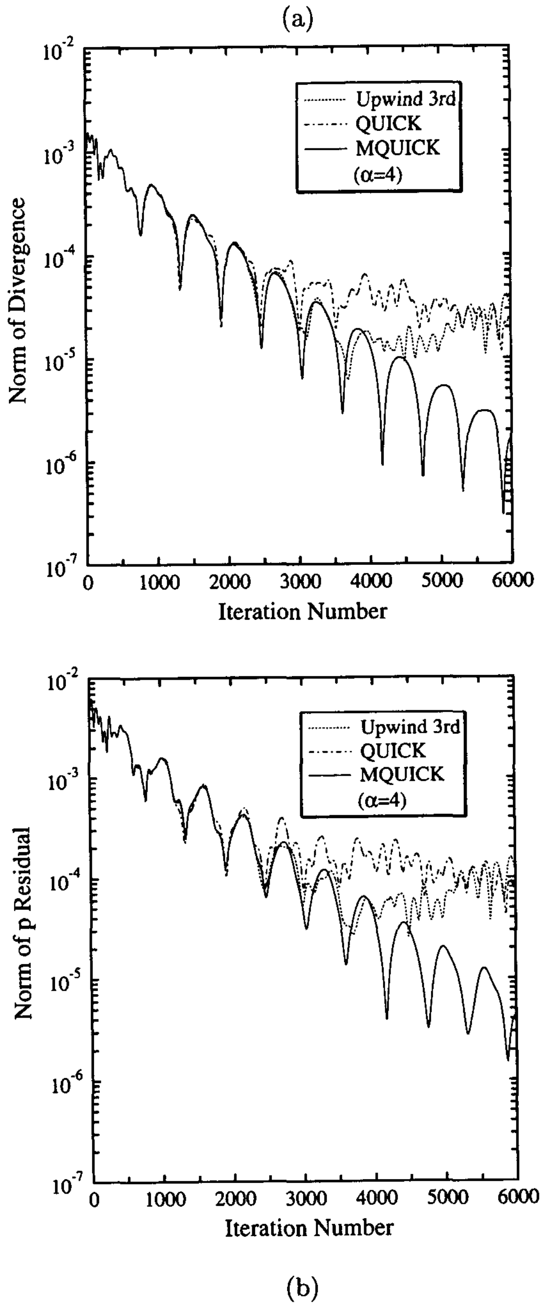

Fig.6 Iteration history of (a) norm of divergence and (b) pressure residual for 3-D curved square duct flow computation.

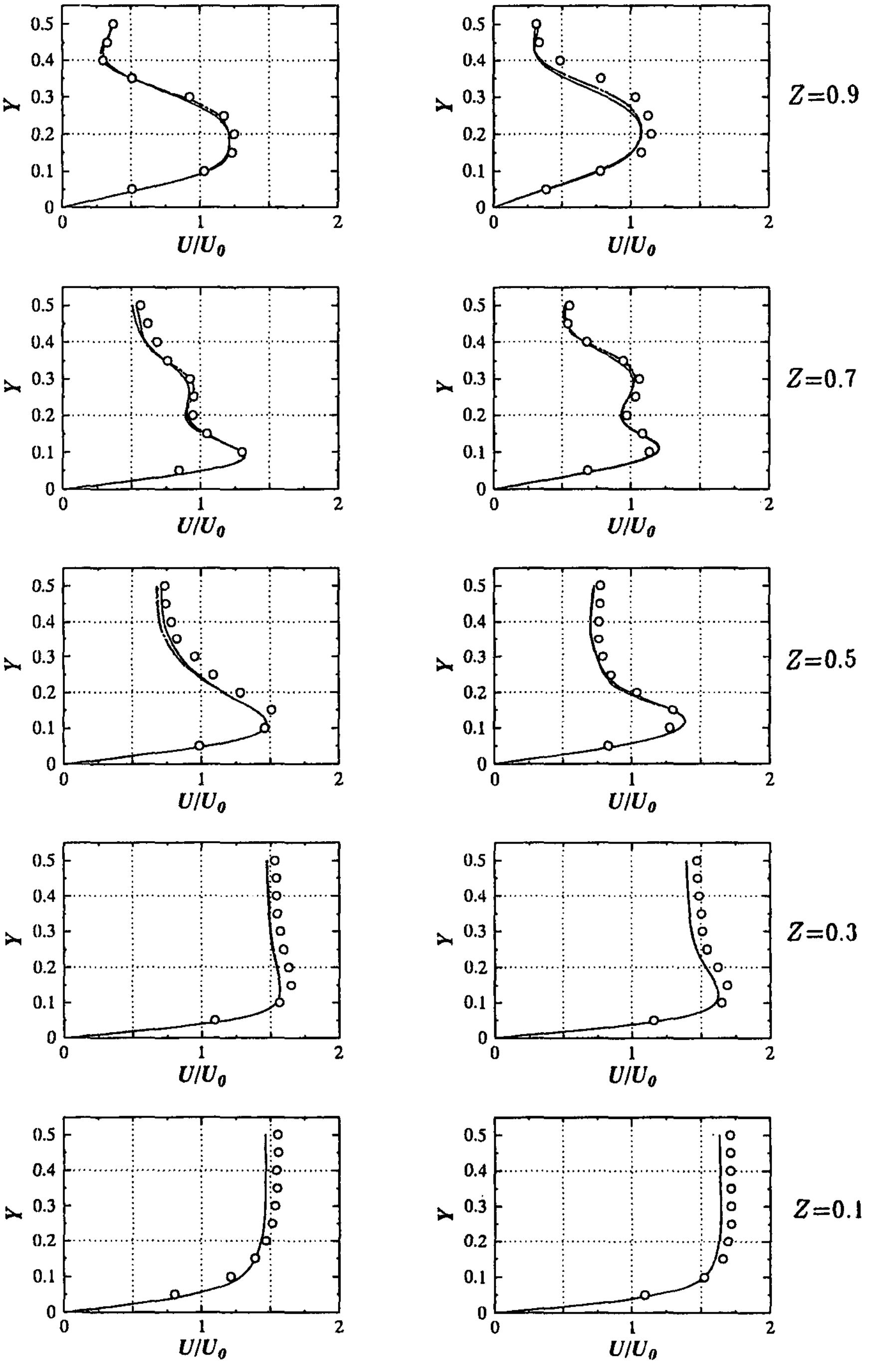

Fig.7 Comparison of streamwise velocity profiles: - - - -, 3rd upwind; – – – , QUICK; —— , MQUICK; ○, measurements.

parameter α exists between 3 and 4 in the MQUICK scheme.

References

[1] Le, H. and Moin, P., "An Improvement of Fractional Step Methods for the Incompressible Navier-Stokes Equations," *J. Comput. Phys.*, **92** (1991), p.369.

[2] Jordan, S.A., "An Iterative Scheme for Numerical Solution of Steady Incompressible Viscous Flows," *Computers & Fluids*, **21** (1992), p.503.

[3] Shin, B.R., et al., "An Implicit Finite Difference Scheme for the Incompressible Navier-Stokes Equations using an Improved Factored Scheme," *Comput. Fluid Dyn. J.*, 4-2 (1995), p.191.

[4] Shyy, W. and Sun, C.H., "Development of a Pressure-Correction / Staggered-Grid Based Multigrid Solver for Incompressible Recirculating Flows," *Computers & Fluids*, **22** (1993), p.71.

[5] Leonard, B.P., "A Stable and Accurate Convective Meodelling Procedure Based on Quadratic Upstream Interpolation," *Computer Methods Appl. Mech. Eng.*, **19** (1979), p.59.

[6] Yeo, R.W., et al., "A Numerical Study of Laminar 90-degree Bend Duct Flow with Different Discretization Scheme," *ASME J. Fluids Eng.*, **113**-12 (1991), p.563.

[7] Pouagare, M. and Delaney, R.A., "Study of Three-Dimensional Viscous Flows in an Axial Compressor Cascade Including Tip Leakage Effects using a SIMPLE-based Algorithm," *Trans. ASME, J. Turbomachinery*, **108** (1986), p.51.

[8] Tamamidis, P., et al., "Comparison of Pressure-Based and Artificial Compressibility Methods for Solving 3D Steady Incompressible Viscous Flows," *J. Comput. Phys.* **124** (1996), p.1.

[9] Hayase, T., et al., "A Consistently Formulated QUICK Scheme for Fast and Stable Convergence using Finite-Volume Iterative Calculation Procedures, *J. Comput. Phys.*, **98** (1992), p.108.

[10] Zhu, J. and Leschziner, M.A., "A Local Oscillation-Damping Algorithm for High-order Convection Scheme," *Computer Methods in Appl. Mech. Eng.*, **67** (1988), p.355.

[11] Freitas, C.J., et al., "Numerical Simulation of Three-Dimensional Flow in a Cavity," *Int. J. Numer. Methods in Eng.*, **5** (1985), p.561.

[12] Ghia, U., et al., "High-Re solutions for incompressible flow using the Navier-Stokes equations and a multigrid method," *J. Comput. Phys.*, **48** (1982), p.387.

[13] Taylor, A.M.K., et al., "Curved Ducts with Strong Secondary Motion: Velocity Measurements of Developing Laminar and Turbulent Flow," *ASME J. Fluid Eng.*, **104**-9 (1982), p.350.

[14] Cabuk, H., et al., "Explicit Runge-Kutta Method for Three-Dimensional Internal Incompressible Flows," *AIAA J.*, **30**-8 (1992), p.2024.

[15] Shin, B.R. and Ikohagi, T., "Incompressible Laminar Entry Flows in a Square Duct of Strong Curvature Using an Implicit SMAC Scheme," *J. Comput. Fluids Eng.*, **1**-1 (1996), p.35.

[16] Beam, R.M. and Warming, R.F., " An Implicit Factored Scheme for the Compressible Navier-Stokes Equations," *AIAA J.*, **16** (1978), p.393.

[17] Hirt, C.W., et al., "SOLA- A Numerical Solution Algorithm for Transient Fluid Flows,"*Los Alamos Scientific Lab.* Rep. LA-5852 (1975).

[18] Patankar, S.V., *Numerical Heat Transfer and Fluid Flow* ,Hemisphere, Washington, DC/New York (1980).