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센서/액츄에이터 콜로케이션을 이용한 트러스 구조물에 대한 구조계와 제어계의 동시 최적설계

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Simultaneous Optimum Design of Structural and Control Systems for Truss Structure with Collocated Sensors and Actuators

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ABSTRACT

3차원 트러스 구조물을 설계대상으로, 구조계와 제어계의 동시최적설계문제에 대하여 고찰하였다. 구조 설계에대한 최소중량설계와 제어 설계에대한 외란 억제문제를 설계목적으로 고려하였다. 그리고, 본연구의 유용성을 입증하기위한 수치 시뮬레이션의 결과를 기술하였다.

Key Words : Optimum Structural Design (최적 구조 설계), Structural Control (구조 제어), H_∞ Controller (H_∞ 제어), 3-D Truss Structure (3차원 트러스 구조물), Sensor/Actuator Collocation (센서/액츄에이터 동위치/동방향 배치)

1. Introduction

In the field of designing flexible structures such as large space structures, they are required to have lighter weight in consideration of transportation cost. However, when the structures are made lighter, their stiffness becomes small and even a little disturbance causes big vibrations. Besides, generally, the inner damping of space structures is so small that once vibrations are caused, it is not easy to suppress them. A simultaneous optimum design of structural and control systems is considered as one of powerful methods for the cases like these[1][2].

In this paper, we consider a simultaneous optimum design of structural and control systems for 3-dimensional truss structure as design object. The structural objective function is the structural weight and the control objective function is H_∞ norm of the transfer function from the disturbance input to the controlled output in closed-loop system. The objective function of simultaneous optimum design problem is the linear sum of the normalized structural objective function and control objective function which are on competitive terms. By minimizing this objective function, it is possible to make optimum design by which the balance of structural weight and control performance is taken. We propose an optimum design method for truss structure with control system to find cross sectional areas of the truss members which minimize the objective function. We also consider the validity of sensor/actuator

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collocation for control system design.

2.System Formulation and H$_\infty$ Control Problem

Generally, flexible structures are expressed by the following equation of motion.

$$M_s \ddot{q} + D_s \dot{q} + K_s q = L_1 w + L_2 u \qquad (1)$$

where M_s, D_s, and K_s are the mass, the damping, and the stiffness matrices. q, w and u are the displacement, the disturbance input and the control input and L_1 and L_2 are the disturbance and control input matrices. The descriptor system of the model becomes Eqs. (2), (3) and (4), where x, z and y are the descriptor variable, the controlled output and the measured output, C_1 and C_2 are the controlled and measured output matrices. D_{12} and D_{21} are the matrices which satisfy $D_{12}^T D_{12} > 0$ and $D_{21}D_{21}^T > 0$ where super script T means its transposed matrix.

$$E\dot{x} = Ax + B_1 w + B_2 u \qquad (2)$$

$$z = C_1 x + D_{12} u \qquad (3)$$

$$y = C_2 x + D_{21} w \qquad (4)$$

$$E = \begin{bmatrix} I & 0 \\ 0 & M \end{bmatrix}, \quad A = \begin{bmatrix} 0 & I \\ -K_s & -D_s \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ L_1 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0 \\ L_2 \end{bmatrix}, \quad x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

In this paper, the control system is designed with the H_∞ control to suppress the effect of the disturbance. In Fig.1, H_∞ control problem is to find such a controller $K(s)$ that the closed-loop system is internally stable and the following H_∞ norm condition is satisfied

$$N = \|T_{zw}(s)\|_\infty < \gamma \qquad (5)$$

$$\|T_{zw}(s)\|_\infty = \sup_w \sigma_{\max}(T_{zw}(jw))$$

where $T_{zw}(s)$ is the transfer function from the disturbance input w to the controlled output z in the closed-loop system. γ is a prescribed positive number and $\sigma_{\max}(T_{zw})$ is the maximum singular value of T_{zw}. Eq.(5) which is expressed in frequency domain is equivalent to the following equation in time domain.

$$\int_0^\infty z^T(t)z(t)dt < \gamma^2 \int_0^\infty w^T(t)w(t)dt \qquad (6)$$

It is considered from Eqs.(5) and (6) that H_∞ norm, N, denotes the degree of disturbance suppression because the right-hand side of Eq.(6) denotes the effect of disturbances.

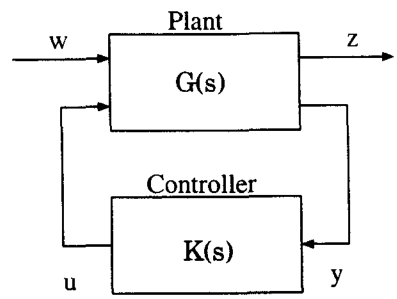


Fig. 1 H$_\infty$ Control System

The necessary and sufficient conditions for the existence of H_∞ controller are that there exist X and Y which satisfy the following symmetric Ricatti equations[3]

$$(A - B_2 D_{12}^{\#} C_1)^T X + X^T (A - B_2 D_{12}^{\#} C_1)$$
$$- X^T (B_2 R_{12}^{-1} B_2^T - \frac{1}{r^2} B_1 B_1^T) X \qquad (7)$$
$$+ (D_{12}^{\perp} C_1)^T (D_{12}^{\perp} C_1) = 0$$

$$E^T X = X^T E \geq 0 \qquad (8)$$

$$(A - B_1 D_{21}^{\#} C_2) Y + Y^T (A - B_1 D_{21}^{\#} C_2)^T$$
$$- Y^T (C_2 R_{21}^T C_2^T - \frac{1}{r^2} C_1 C_1^T) Y \qquad (9)$$
$$+ B_1 C_{21}^{\perp} (B_1 D_{21}^{\perp})^T = 0$$

$$EY = Y^T E^T \geq 0 \tag{10}$$

$$\det(I_n - \frac{1}{\gamma^2} YX) \neq 0 \tag{11}$$

$$Z = (I_n - \frac{1}{\gamma^2} YX)^{-1} \geq 0, \tag{12}$$

where R_{12}, R_{21}, $D_{12}^{\#}$, $D_{12}^{\perp}$, $D_{21}^{\#}$ and $D_{21}^{\perp}$ are

$$R_{12} = D_{12}^T D_{12} > 0, \quad R_{21} = D_{21} D_{21}^T > 0$$

$$D_{12}^{\#} = R_{12}^{-1} D_{12}^T, \quad D_{12}^{\perp} = I - D_{12} D_{21}^{\#}$$

$$D_{21}^{\#} = D_{21}^T R_{21}^{-1}, \quad D_{21}^{\perp} = I - D_{21}^{\#} D_{21}$$

Now, we have the following H_∞ controller

$$E x_k = A_k x_k + B_k y \tag{13}$$

$$u = C_k x_k + D_k y \tag{14}$$

where A_k, B_k, C_k, and D_k are

$$A_k = A - B_k C_2 + \gamma^{-2} Y^T C_1^T C_1 + (B_2 + \gamma^{-2} Y^T C_1^T D_{12}) C_k$$

$$B_k = (Y^T C_2^T + B_1 D_{21}^T) R_{21}^{-1}$$

$$C_k = R_{12}^{-1} \{ D_{12}^T C_1 + (B_2 + \frac{1}{\gamma^2} Y^T C_1^T D_{12})^T Z \}$$

$$D_k = 0$$

3. Optimum Design Problem

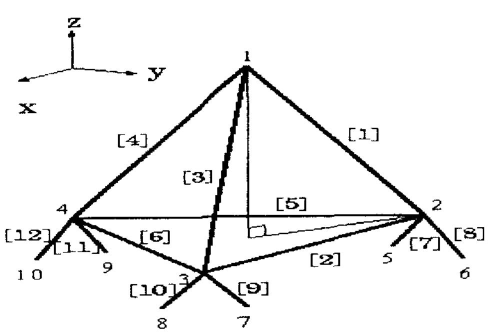


Fig.2 3-D Truss Structure

In this paper, we consider a minimum weight design problem for structural system and suppression problem of the effect of disturbances for control system as the purpose of the design. Taking a 3-dimensional truss structure as an object, the mass, damping, and stiffness matrices of the system can be modeled as the function of the cross sectional areas of the truss members from FEM formulation[4]. The structural objective function is the structural weight W and the control objective function is N, that is, H_∞ norm of the transfer function from the disturbance input to the controlled output in closed-loop system.

We take a 3-D truss structure shown in Fig.2 as design object. $1, \cdots, 10$ are nodes and $[1], \cdots, [12]$ are truss members. Considering non-dimensional form, the length of long members is 10, short members $2\sqrt{2}$, density 1.0, and Young's modulus 10^4. The nodes from 5 to 10 are fixed. The damping matrix is assumed by

$$D_s = 0.001 M_s + 0.001 K_s \quad .$$

The structural objective function W is calculated by

$$W = \sum_{i=1}^{12} \rho_i l_i a_i \tag{15}$$

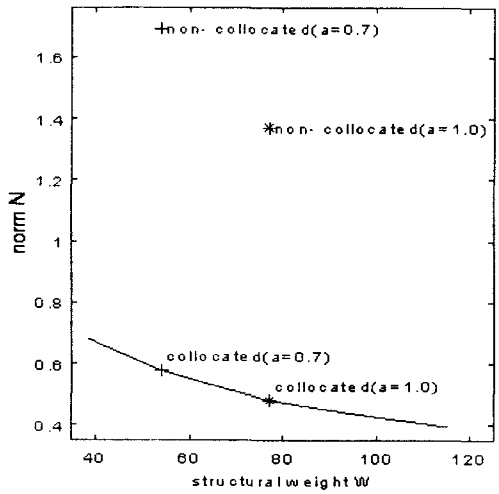
which ρ_i, l_i, and a_i are density, length and cross sectional area of the $i-th$ truss member. Control objective function N is the minimum value of γ in Eq.(5) which can be calculated by iteration method[5] (γ iteration) in the following interval.

$$0 < \gamma \leq 10 \tag{16}$$

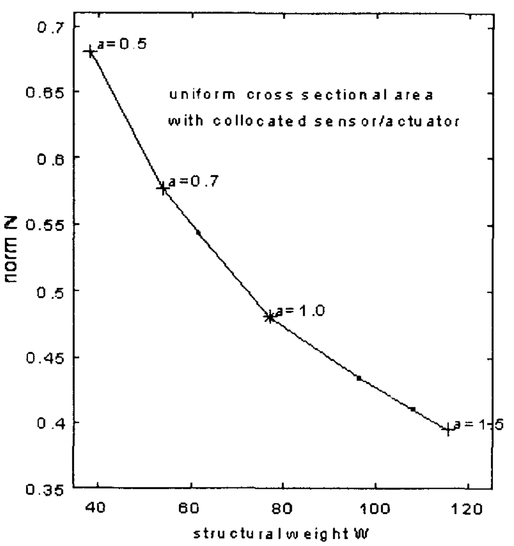
3.1 Sensor/Actuator Collocation

The structures in which their sensors and actuators are located in the same directions and at the same positions (called collocation) are to be minimum phase system, i.e. real parts of invariant zero points are all negative. Minimum phase system is known as better than non minimum phase system for control system design[6]. In this section, we perform the control system design to minimize only the control objective function N

under the condition that the structural system is given for several cases that all members of truss structure have uniform cross sectional areas. And we consider the positioning problem of sensors and actuators, which is important for the control system design. We compare the cases that sensors and actuators are collocated with non-collocated cases.



(a) comparison between collocated case and non-collocated case



(b) collocated case

Fig. 3 Relation between weight W and norm N (uniform cross sectional area)

These results show that the structures with collocated sensors and actuators are better for control system design than those with which they are not collocated as shown in Fig.3 (a). Therefore, from now in this paper, we consider the optimum design problem for truss structure as shown in Fig.2 on the assumption that both the sensors and actuators are located at the node 1 in x, y and z directions. In Fig.3 (b), the relation between the structural weight, W, and H_∞ norm, N, is shown for several cases that all members of truss structure have uniform cross sectional areas. We recognize that two objective functions are on competitive terms.

3.2 Simultaneous Optimum Design

In this section, we consider a simultaneous optimum design problem of structural and control systems for flexible structure. The objective function in this approach is the linear sum of the normalized structural objective function and control objective function as follows

$$J(a) = w_W \frac{W(a)}{W_0} + w_N \frac{N(a)}{N_0} \quad (17)$$

where w_W and w_N are the weightings for structural weight, W, and H_∞ norm, N, and $W_0(=76.971)$ and $N_0(=0.485)$ are the values of the structural weight and of H_∞ norm for the initial structure in which all members have uniformcross sectional are as $a_i = 1 (i = 1, \cdots, 12)$.

We formulate the simultaneous optimum design problem such as to find cross sectional areas of truss members for the minimization of the objective function, J, as follows :

$$\min_a J(a) = w_W \frac{W(a)}{W_0} + w_N \frac{N(a)}{N_0} \quad (17)$$

$$w_W + w_N = 1 \quad (18)$$

$$
\text{subject to}
\begin{cases}
\textit{There exists } X \textit{ in Eq.}(7). \\
\textit{There exists } Y \textit{ in Eq.}(9). \\
\textit{Eqs.}(8),(10),(11) \textit{ and } (12) \\
\textit{are satisfied} \\
a^{\min} \le a \le a^{\max}
\end{cases}
$$

where a is the set of cross sectional areas of the truss members, $a^{\min}$ and $a^{\max}$ are lower and upper limits of the cross sectional area.

By minimizing the objective function, J, it is possible to make optimum design by which the balance of structural weight and control performance is taken. The simplex method is used to solve the optimization problem above. We take into consideration the following constraint for cross sectional areas as design variables.

$$0.5 \le a_i \le 1.5 \qquad (i = 1,\cdots,12) \tag{19}$$

First, we perform the simultaneous optimum design in the case of the set of weightings for the structural and control objective functions $(w_W, w_N) = (0.5,0.5)$. In this case (case1), from the result by the minimization of J, the structural weight, W, is 71.383 and the H_∞ norm, N, is 0.473. We get 7.3% lighter structural weight, W, and 9.9% smaller value of H_∞ norm, N, than their initial values, W_0 and N_0.

Then, in the case of the set of weightings for $(w_W, w_N) = (0.6,0.4)$, we perform the simultaneous optimization. In this case (case2), W is 64.945 and N is 0.460. We get 15.6% lighter weight and 5.2% smaller value of the norm than those of initial structure. In both cases, we get lighter structural weight, W, and smaller H_∞ norm, N, than the initial structure, i.e.the reduction of the cost for structural system designand the improvement of the suppression for the effect of disturbances for control system by the simultaneous optimum design are obtained.

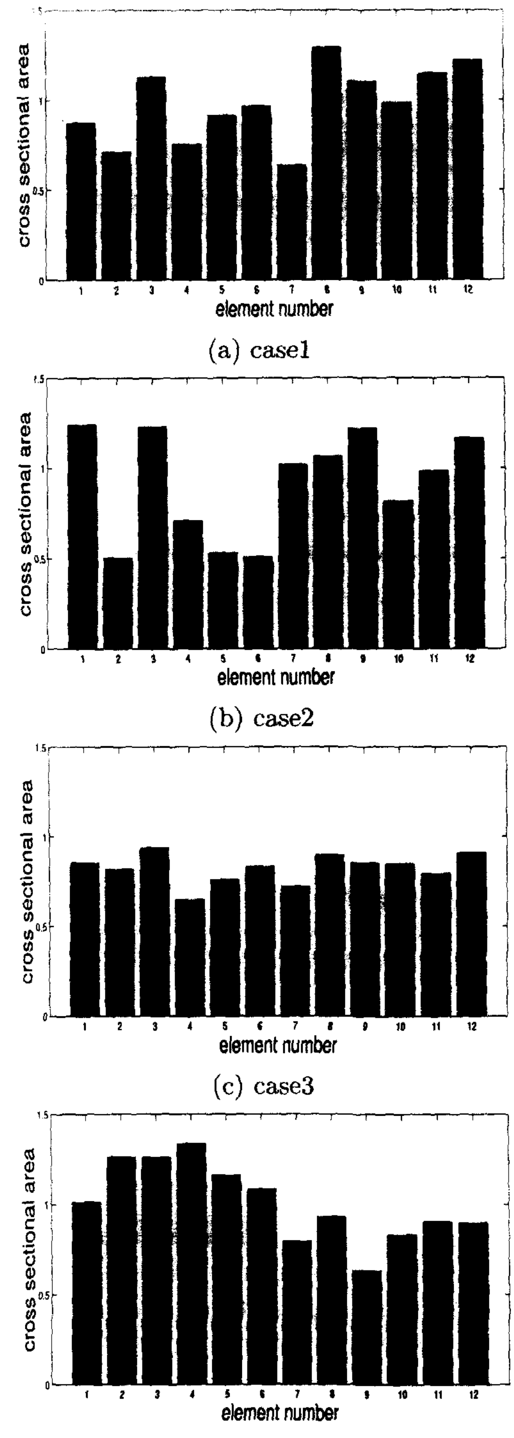


Fig. 4 Distribution of Cross Sectional Area

Next, we perform the simultaneous optimum design in the case of the set of weightings for $(w_W, w_N) = (0.7, 0.3)$. In this case (case3), the structural weight, W, is 54.693 and the H_∞ norm, N, is 0.519. We get 28.9% lighter structural weight, W, than the initial weight W_0. But, the value of H_∞ norm, N, is 7.0% increased than N_0, that is, the suppression problem of the effect of disturbances got worse than the initial structure. And in the case of $(w_W, w_N) = (0.3, 0.7)$, W is 85.308 and N is 0.419. In this case (case4), we get 13.6% smaller norm, N, than N_0, but the structural weight, W, is 10.8% increased than W_0. These results show the effect of weightings for structural and control objective functions in our optimum design method. Fig.4 shows the distribution of the optimum cross sectional areas for these cases.

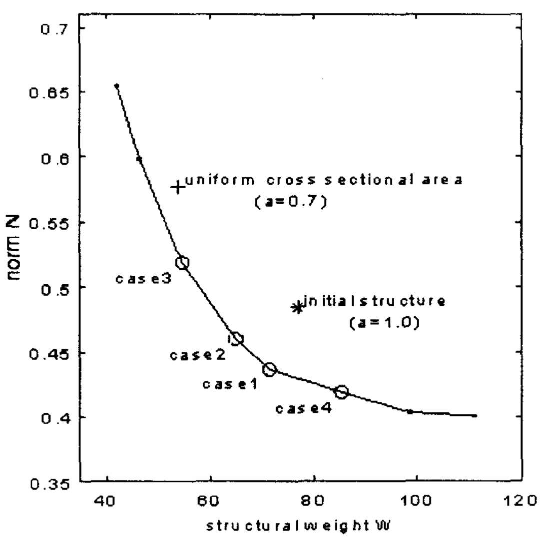


Fig. 5 Pareto Optimality for Weight and Norm

In Fig.5, the set of structural weight and H_∞ norm, (W, N), corresponding to the minimum state of J in Eq.(17) is shown for several sets of (w_W, w_N).

4. Conclusion

In this paper, we formulated a simultaneous optimum design problem of structural and control systems and suggested a design method for 3-dimensional truss structure as the design object.

By our simultaneous design method, we obtained the reduction of the cost for structural system design and the improvement of the suppression for the effect of disturbances for control system compared with the design which considers only the control system. We also showed the validity of sensor/actuator collocation for control system design and that of weighting method for simultaneous optimization of structural and control objective functions.

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