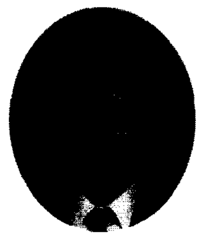
콘크리트 구조물의 합리적인 압축강도 추정기법 연구

Realistic Estimation Method of Compressive Strength in Concrete Structure

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ABSTRACT

실제 구조물의 정확하고 합리적인 압축강도 추정을 위해서는 통계학적으로 많은 실험데이타가 필요하다. 그러나, 실제로 압축강도 자료가 제한되어 있기 때문에 추정에 어려움이 있다. 따라서, 본 연구에서는 적은 자료를 가지고 콘크리트의 실제적인 압축강도 추정을 위해 합리적인 베이시안 기법을 도입하여 콘크리트 강도추정 방법을 제시하였다. 여기서, 콘크리트의 평균 압축강도는 확률변수로 고려한다. 콘크리트 압축강도의 베이시안 업데이팅을 위해 사전확률분포는 기존의 자료를 반영하여 표현하며, 우도함수는 측정치의 특성을 반영하였다. 사후확률분포는 사전확률분포와 우도함수를 조합하여 나타내었다. 콘크리트 교량 현장에서 제작한 실린더 공시체로부터 측정한 자료를 이용하여 수치해석을 수행하였다. 수치해석결과는 상대적으로 적은 개수의 측정자료를 사용하고도 실제에 가까운 사후확률분포를 추정할 수 있는 것을 보여 주고 있다. 또한, 우도함수 분포의 신뢰구간에 대한 사전확률분포의 신뢰구간의 상대적인 크기는 사후확률분포의 결정에 영향을 미치는 것으로 나타났다. 본 논문에서 제시된 방법은 적은 현장측정자료를 가지고도 합리적인 강도추정이 가능함을 보여주고 있으며, 실제에 유용하게 활용될 수 있을 것으로 사료된다.

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1. Introduction

The estimation of concrete strength has been major subject of interest in non-destructive testing[1,2]. A common procedure in the estimation of concrete strength is the analysis of test data from site. Compressive strength of concrete is uncertain input variable for structural analysis or evaluation. The stochastic nature of concrete has prompted the development of mathematical methods for compressive strength.

It is still desirable to have many compressive strength data at a site that will be used in structural evaluation. However, a frequently encountered situation is that of making a preliminary evaluation of the strength in site with limited observational data. Classical statistical techniques are not adequate to characterize the concrete strength at such sites due to very few limited data. In this paper, the Bayesian technique is used to estimate realistically the compressive strength of concrete with limited data.

Bayesian estimation is a powerful tool that can be used to combine information from a expert knowledge with site-specific information. Geyskens et al.[3] have recently used this approach to improve their prediction of elastic modulus of concrete and Kajner et al.[4] to forecast the progression of roughness in hot-mix asphalt concrete overlays using data and expert judgement. The objective of this paper is to propose a realistic method which can be used to estimate the compressive strength of concrete in site with limited data.

2. Mathematical Theory

2.1 Bayesian Theorem

The Bayesian approach has many advantages in the area of engineering planning and design[5]. It systematically combines uncertainties associated with randomness and those arising from error of estimation and prediction. It provides a formal procedure for systematic updating of information and increases the prediction precision.

In the Bayesian approach the underlying probability distribution of a basic random variable, denoted X, is assumed to be known, but the parameters of the distribution are considered random with a distribution of their own, referred to as the prior. All available intuitive and judgmental information, as well as objective assessments, are quantified to choose the prior distribution, $p(\theta)$, in which the unknown parameters have been considered as θ. Subsequently observed values of X can be formally combined with the prior distribution of θ through the use of the Baye's theorem to obtain an updated posterior distribution, $p(\theta \mid x)$, from which decisions and inferences are made.

Let $L(\theta \mid x)$ denote the likelihood function, then Baye's theorem can be expressed as follows:

$$p(\theta \mid x) = c\,L(\theta \mid x)\,p(\theta) \qquad (1)$$

in which c = a normalizing constant; $L(\theta \mid x)$ = likelihood function of the sample given θ; $p(\theta)$ = prior probability of θ, that is, before availability of experimental information and $p(\theta \mid x)$ = posterior probability of θ, that is, probability that has been revised in the light of experimental outcome x.

It is observed from Eq. (1) that both the prior distribution and the likelihood function contribute to the posterior distribution of θ. The prior information enters the posterior probability density function(pdf) via the prior pdf, whereas all of the sample information enters via the likelihood function. In this manner judgmental and observational data are combined properly and systematically.

Given a set of observed values $x_1, x_2, \cdots, x_n$, which represent a random sample from a population of X with underlying density $f_x(X)$, the likelihood function can be written as

$$L(\theta \mid x) = L(\theta \mid x_1, x_2, \cdots, x_n) = \prod_{i=1}^{n} f_x(x_i \mid \theta) \qquad (2)$$

in which $f_x(x_i \mid \theta)$=probability distribution of the basic random variable, X, given θ and n =number of observations of X. The posterior distribution is the combination of the prior information and the likelihood function. Just as the prior distribution reflects beliefs about θ prior to experimentation, so $p(\theta \mid x)$ reflects the updated beliefs about θ after observing the sample x.

2.2 Bayesian Formulation

Suppose that an unbiased method of experimental measurement is available and that an observation x made by this method follows a normal distribution with mean θ and standard deviation σ. If now a single observation x is made, the standardized likelihood function is represented by a normal curve centered at x with standard deviation σ_1. Then Bayesian theorem can be applied to show how knowledge regarding θ is modified by the information coming from measurement data.

Assuming that a prior is distributed normally with its mean θ_0 and standard deviation σ_0, then a prior for parameter θ is expressed as

$$p(\theta) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp\left[-\frac{1}{2}\left(\frac{\theta-\theta_0}{\sigma_0}\right)^2\right] \qquad (3)$$

and the likelihood function of θ is proportional to a normal function[6)]

$$L(\theta \mid x) \propto \exp\left[-\frac{1}{2}\left(\frac{\theta-x}{\sigma_1}\right)^2\right] \qquad (4)$$

Then the posterior distribution of θ given the data x is

$$p(\theta \mid x) = \frac{L(\theta \mid x)\,p(\theta)}{\int_{-\infty}^{\infty} L(\theta \mid x)\,p(\theta)\,d\theta} = \frac{f(\theta \mid x)}{\int_{-\infty}^{\infty} f(\theta \mid x)\,d\theta} \qquad (5)$$

in which,

$$f(\theta \mid x) = \exp\left\{-\frac{1}{2}\left[\left(\frac{\theta-\theta_0}{\sigma_0}\right)^2 + \left(\frac{\theta-x}{\sigma_1}\right)^2\right]\right\}$$

Using the identity

$$A(z-a)^2 + B(z-b)^2 = (A+B)(z-c)^2 + \frac{AB}{A+B}(a-b)^2 \qquad (6)$$

with $c = \frac{1}{(A+B)}(Aa+Bb)$

We can write

$$\left(\frac{\theta-\theta_0}{\sigma_0}\right)^2+\left(\frac{\theta-x}{\sigma_1}\right)^2$$

$$= (\sigma_0^{-2}+\sigma_1^{-2})(\theta-\overline{\theta})^2+d \qquad (7)$$

where $\overline{\theta}=\frac{1}{\sigma_0^{-2}+\sigma_1^{-2}}(\sigma_0^{-2}\theta_0+\sigma_1^{-2}x)$

and d is a constant independent of θ. Thus,

$$f(\theta \mid x)=\exp\left(-\frac{d}{2}\right)\times$$
$$\exp\left[-\frac{1}{2}(\sigma_0^{-2}+\sigma_1^{-2})(\theta-\overline{\theta})^2\right] \quad (8)$$

so that

$$\int_{-\infty}^{\infty} f(\theta \mid x)=\exp\left(-\frac{d}{2}\right)\times$$
$$\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2}(\sigma_0^{2}+\sigma_1^{-2})(\theta-\overline{\theta})^2\right]d\theta$$
$$=\sqrt{2\pi}\,(\sigma_0^{-2}+\sigma_1^{-2})^{-1/2}\exp(-d/2) \quad (9)$$

It follows that

$$p(\theta \mid x)=\frac{(\sigma_0^{-2}+\sigma_1^{-2})^{1/2}}{\sqrt{2\pi}}\times$$
$$\exp\left[-\frac{1}{2}(\sigma_0^{-2}+\sigma_1^{-2})(\theta-\overline{\theta})^2\right] \quad (10)$$

which is normal distribution.

Therefore, the updated statistics or the posterior distribution of θ given x, $p(\theta \mid x)$, can be calculated as the mean and the variance as follows.

$$\overline{\theta} = \frac{1}{w_0+w_1}(w_0\theta_0+w_1x) \qquad (11a)$$

$$\frac{1}{\overline{\sigma}^2} = w_0+w_1 \qquad (11b)$$

where, $w_0=\frac{1}{\sigma_0^2}$ and $w_1=\frac{1}{\sigma_1^2}$

The posterior mean $\overline{\theta}$ is a weighted average of the prior mean θ_0 and the observation x, the weights being proportional to w_0 and w_1 which are, respectively, the reciprocal of the variance of the prior distribution of θ and that of the observation.

Also, the likelihood function of θ given n independent observations from the normal population $N(\theta, \sigma^2)$, is

$$L(\theta \mid x) \propto \exp\left[-\frac{1}{2}\left(\frac{\theta-\overline{x}}{\sigma/\sqrt{n}}\right)^2\right] \qquad (12)$$

The result in Eq. (11) can thus be applied as if $\overline{x}$ were a single observation with variance σ^2/n, that is, with weight n/σ^2. The posterior distribution of θ obtained by combining the likelihood function in Eq. (12) with a normal prior $N(\theta_0, \sigma_0^2)$ is the normal distribution, which can be expressed as follows.

$$\overline{\theta}_n = \frac{1}{w_0+w_n}(w_0\theta_0+w_n\overline{x}) \qquad (13a)$$

$$\frac{1}{\overline{\sigma}_n^2} = w_0+w_n \qquad (13b)$$

where, $w_0=\frac{1}{\sigma_0^2}$ and $w_n=\frac{n}{\sigma^2}$

3. Numerical Application for Estimating Compressive Strength of Concrete

The strength of concrete cylinders which were made by writers in an actual prestressed concrete box girder bridge in Korea[7] is to be evaluated. Concrete cylinders were cast under construction of bridge. The data were taken from results of laboratory test in construction site. From the 53 test strengths measured, samples were randomly selected for this numerical example.

3.1 Sequential Estimation of Concrete Strength

At a site with limited observational data, the choice of the prior distribution is obtained by consideration of all related information such as previous experience at similar sites or previous work. The distribution of concrete strength were proposed by Mirza[8], Bartlett[9] and Tabsh[10]. The standard deviation is assumed to be less than 0.2 times the mean strength.

Consider a case for which all information available prior to sampling leads to estimates of mean of 450 kg/cm^2 and standard deviation of 45 kg/cm^2, respectively. The use of Eq. (3) yields the prior probability density function of the compressive strength of concrete cylinders. Now with n observations of the sample, the prior distribution is updated to obtain the posterior distribution through Eq. (13). The results of Bayesian estimation are shown in Table 1 and Fig. 1 which show the sequential nature of the Bayesian updating process.

Table 1 Posterior distribution parameters after obtaining observational data at site

Number of observations (n)	Posterior distribution parameters	
	Mean (kg/cm^2)	Standard deviation (kg/cm^2)
0	450.0	45.0
3	469.3	20.6
10	479.9	12.2
30	494.1	6.4

The posterior distribution after 3 observation serves as the prior for the next set of 7 observations. The posterior after 10 observations then serves as the prior for the next set, and so on. Theoretically, as the number of observations increases, the variance of compressive strength should approach zero. Fig. 1 shows the tendency for the variance of the Bayesian compressive strength distribution to decrease with increasing number of observations. The emphasis placed on site measurements should increase as the number of site-specific measurements increase. This point is apparent by examining the posterior mean given in Table 1. For the asymptotic limit as $n \rightarrow \infty$, the posterior mean is equal to site mean.

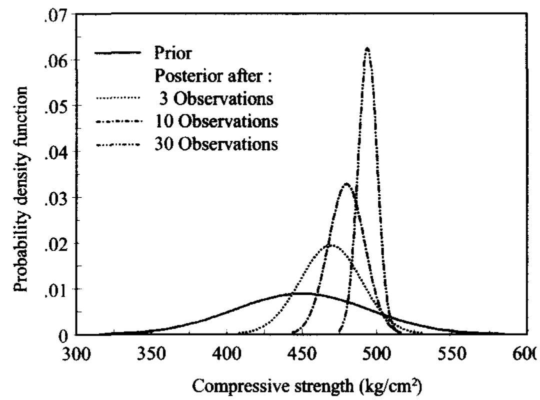


Fig. 1 Prior and posterior distributions of the compressive strength of concrete

3.2 Effect of Sample Size of Site Measurements for Compressive Strength

For Bayesian technique, desirable feature is that the technique provides reasonable results even for small data sets. This point will be examined by considering the spread of pdfs. The effect of small sample size on the spread of the site and posterior distributions will be illustrated.

The prior, site, and posterior distributions using the first three observations and three other observations are shown in Figs. 2(a) and (b), respectively. Consider a case for which prior distribution has mean value of 450 kg/cm^2 and standard deviation of 45 kg/cm^2, respectively. Assume that the experimental design had been changed to collect only three data. If only the first three values is collected, then the resulting analysis is as shown in Fig. 2(a). Here the site standard deviation is 75.4 kg/cm^2. On the other hand, if those three data is collected to be those values with narrow spread distribution, the posterior distributions shown in Fig. 2(b) would be obtained. The site standard deviation is now only 22.1 kg/cm^2.

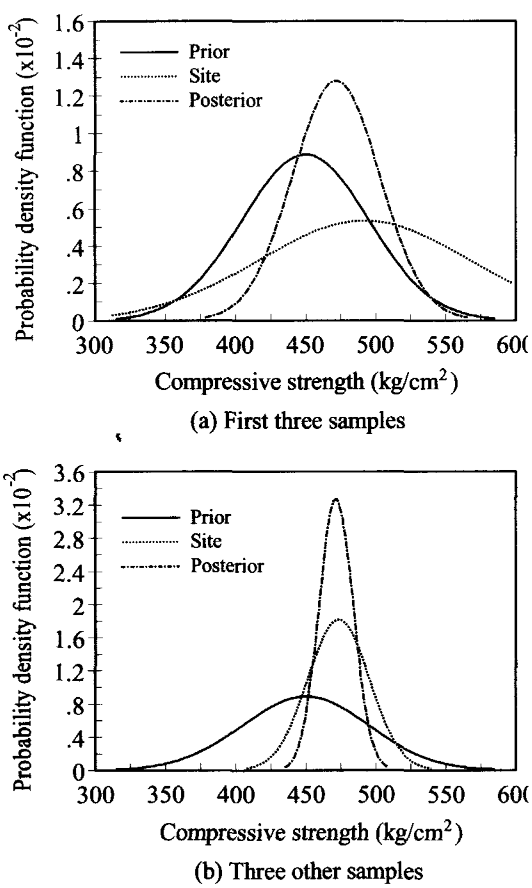


Fig. 2 Bayesian analysis with three sampling

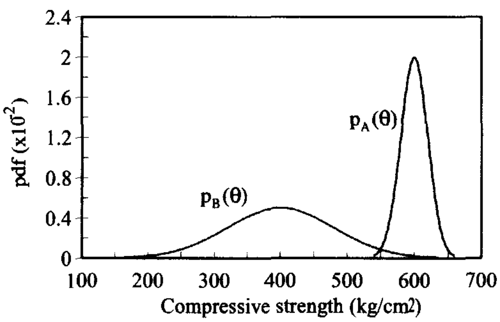
For the data subset using the first three values(Fig. 2(a)) the mean value posterior is 469.9 kg/cm^2, and for the data subset using three values with narrow spread distribution(Fig. 2(b)) the mean value of posterior is 471.7 kg/cm^2, which indicate that respective mean values of two posteriors are almost same. However, for the data subset using the first three values the standard deviation is 31.2 kg/cm^2, and for the data subset using three values with narrow spread distribution the standard deviation is 12.2 kg/cm^2.

The standard deviation of the posterior distribution is smaller than those of prior and site distributions. In comparison to site values, posterior standard deviations are considerably smaller. Considering that the posterior estimates are more reliable and smaller than either of the other two distributions, the present method represents a much improvement in the estimation of compressive strength of concrete.

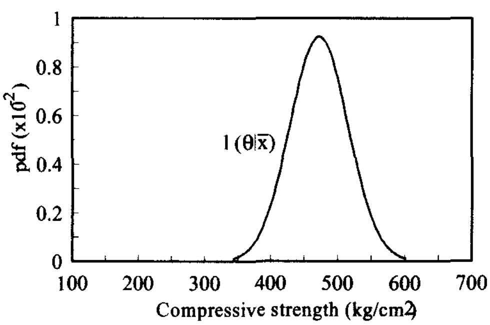
3.3 Effect of the Prior Distribution on the Posterior Distribution of Concrete Strength

Suppose two prior distributions, $p_A(\theta)$ and $p_B(\theta)$, are concerned with obtaining more accurate estimates of concrete strength. Prior distribution $p_A(\theta)$ is approximately represented by a normal distribution centered at 600 kg/cm^2 with a standard deviation of 20 kg/cm^2, $N(600, 20^2)$, while prior distribution $p_B(\theta)$ has little previous information and rather vague prior beliefs are represented by the normal distribution, $N(400, 80^2)$. Considering that the sample mean of three observations is 472.5 kg/cm^2, then the likelihood function is shown in Fig. 3(b). These posterior distributions are shown in Fig. 3(c). It is seen that after three observations posterior distributions are much closer than prior distributions, although they still differ considerably. It is seen that prior $p_A(\theta)$, relatively speaking, did not learn much from the observational data, while prior $p_B(\theta)$ learned a great deal.

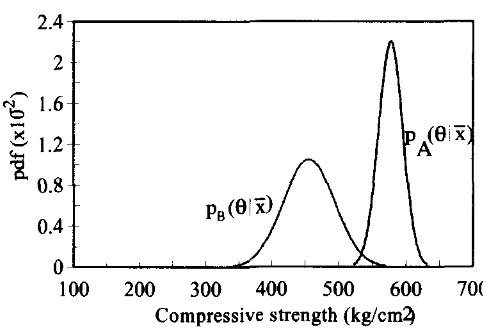
Suppose 47 further independent observations are made and the sample mean of the entire 50 observations is 494.2 kg/cm^2. Now the likelihood is the normal function centered at 494.2 kg/cm^2 with standard deviation of $\sigma/\sqrt{n}$= 5.1 kg/cm^2 as shown in Fig. 3(d).



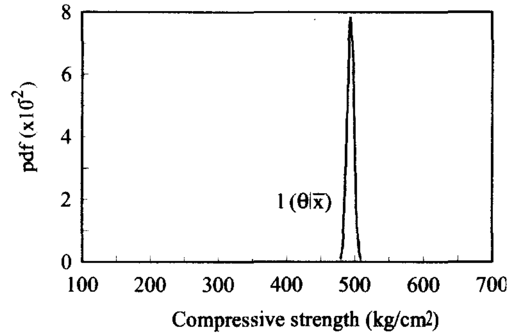
(a) Prior distribution for A and B



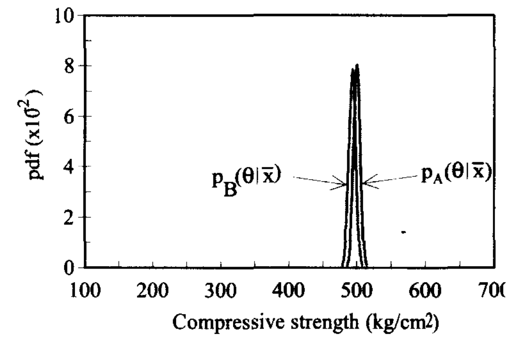
(b) Standardized likelihood for 3 observations



(c) Posterior for A and B after 3 observations



(d) Standardized likelihood for 50 observations



(e) Posterior for A and B after 50 observations

Fig. 3 Posterior distributions for different priors

Thus the posterior distributions of $p_A(\theta \mid \bar{x})$ and $p_B(\theta \mid \bar{x})$ are $N(500.7,\ 4.97^2)$ and $N(493.8,\ 5.08^2)$, respectively. These two distributions which are shown in Fig. 3(e) are, for all practical purpose, the same. After 50 observations, A and B would be in almost complete agreement which indicates that the prediction is almost perfect. This is because the information coming from the data almost completely overrides prior differences. In this numerical example, the contribution of the prior in determining the posterior distribution of parameter θ is seen to depend on its sharpness or flatness in relation to the sharpness or flatness of the likelihood with which it was to be combined.

4. Conclusions

A method of establishing the realistic compressive strength of concrete at a site with limited information has been proposed. The estimation of actual concrete strength is necessary especially when conducting a site assessment. Due to the limited observational data, classical statistical methods are inadequate and thus a Bayesian inference has been applied here to estimate the compressive strength of concrete. The compressive strength of concrete has been modeled with a normal prior distribution, and a closed form has been obtained for the Bayesian distribution for the estimation of compressive strength of concrete. The prior distribution was combined with site-specific information to obtain posterior distribution.

Application to actual compressive strength data from concrete cylinder has demonstrated that by combining prior estimation with information from observation more precise estimation is possible with relatively small sampling. It is also seen that the contribution of the prior in determining the posterior distribution depends on its sharpness or flatness in relation to the sharpness or flatness of the likelihood function. The proposed method allows more realistic determination of concrete strength with limited site data.

Acknowledgement

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ABSTRACT

To estimate the compressive strength of concrete more realistically, relative large number of data are necessary. However, it is very common in practice that only limited data are available. The purpose of the present paper is therefore to propose a realistic method to estimate the compressive strength of concrete with limited data in actual site. The Bayesian method of statistical analysis has been applied to the problem of the estimation of compressive strength of concrete. The mean compressive strength is considered as the random parameter and a prior distribution is selected to enable updating of the Bayesian distribution of compressive strength of concrete reflecting both existing data and sampling observations. The updating of the Bayesian distribution with increasing data is illustrated in numerical application. It is shown that by combining prior estimation with information from site observation, more precise estimation is possible with relatively small sampling. It is also seen that the contribution of the prior in determining the posterior distribution depends on its sharpness or flatness in relation to the sharpness or flatness of the likelihood function. The present paper allows more realistic determination of concrete strength in site with limited data.

Keywords : compressive strength, concrete, Bayesian estimation, prior distribution, posterior distribution, likelihood function, updating, field data

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