Prediction of Critical Reynolds Number in Stability Curve of Liquid Jet (I)

S. Y. No, K. Y. Ryu, J. H. Rhim and S. B. Lim
Dept. of Agricultural Machinery Engineering,
Chungbuk National University, Cheongju, 361-763, Korea

ABSTRACT

The first maximum point in the stability curve of liquid jet, i.e., the critical point is associated with the critical Reynolds number. This critical Reynolds number should be predicted by simple means. In this work, the critical Reynolds number in the stability curve of liquid jet are predicted using the empirical correlations and the experimental data reported in the literatures. The critical Reynolds number was found to be a function of the Ohnesorge number, nozzle lengh-to-diameter ratio, ambient Weber number and nozzle inlet type. An empirical correlation for the critical Reynolds number as a function of the Ohnesorge number and nozzle length-to-diameter ratio is newly proposed here. Although an empirical correlation proposed in this work may not be universal because of excluding the effects of ambient pressure and nozzle inlet type, it has reasonably agrees with the measured critical Reynolds number.

Keywords : jet stability curve, critical Reynolds number, Ohnesorge number, nozzle length-to-diameter ratio

INTRODUCTION

The phenomenon of disintegration of liquid jet emerging from a nozzle into drops has been studied both theoretically and experimentally for a long time. The liquid jet instability requires to be clearly understood to allow good control of spray fonnation. This is the reason why the stability analysis of liquid jet has received interests from engineers for a long time, on either an experimental or a theoretical basis.

The jet behaviour have characterized by determining exprimentally the relationship between the jet velocity and the breakup length. The change of breakup length of a jet versus its average exit velocity is called the jet stability curve. The breakup length is defined as the length of continuous liquid column of the jet, measured from the nozzle exit to the first breakup point where the drop formation occurs. It is well known that as the jet velocity is increased, the breakup length increases linearly. If the jet velocity is further increased, a point is reached where the breakup length no longer increases, but reaches a maximum and then decreases. This first maximum point in the stability curve is called the critical point and is associated to the critical velocity and the critical Reynolds number. The prediction of this critical velocity or the critical Reynolds number is, of course, of much practical interest. According to Weber, the critical point in the stability curve occurs due to the effect of aerodynamic forces, which increase with the relative velocity between the liquid and the surrounding gaseous medium, i.e. jet velocity. Other investigators reported that the occurance of this critical point in the stability curve is attributed to the initial disturbance level, i.e. the onset of turbulence in the jet. In addition, the others explain that both mechanisms contribute simultaneously to promote jet disintegration ; however, the predominancy between the two mechanisms has yet to be resolved[1].

Main concern in this paper is the analysis of the critical point, the first maximum in the stability curve. Thus, for the details on the transition, turbulent flow, and spray regions in the stability curve, the readers should consult the books[1,2] and papers[3,4,5].

It has been known for a long time that the Reynolds number based on the nozzle diameter is the parameter that describes the disturbance level of a jet flow. According to the literature, for each stability curve, there is a critical Reynolds number above which the breakup length decreases rapidly[6,7,8]. Therefore the critical Reynolds number should be predicted clearly with the proper physical interpretation. The purpose of this work is to review the critical Reynolds number in the stability curve previously reported for the liquid jet in air and to suggest a general empirical correlation.

REVIEW OF CORRELATIONS ON CRITICAL REYNOLDS NUMBER

The critical Reynolds number can be predicted by theoretical consideration or empirical correlations. The critical Reynolds number cannot be simply obtained from the Weber's theory and the modified Weber's theories such as by Grant and Middleman[9], Sterling and Sleicher[4] and Leroux et al.[3,10] Rather, for given values of the parameters, numerical methods must be used to obtain the critical Reynolds number. If the numerical results were plotted and curve-fitted with simple expressions, the repetition of numerical calculations for each case would be unnecessary. Thus a brief review of empirical correlations on the critical Reynolds number is presented.

Tanasawa and Toyota[11] observed the modes of dripping, laminar flow, turbulent flow and spray of different kinds of liquid jets. They suggested the empirical formula for the transition velocities from laminar to turbulent and from turbulent to spray regions respectively. The expression for the critical Reynolds number at which the jet changes from laminar to turbulent flow was given by

$$St^{0.12}\, Re_c^{0.39} = 10 \qquad (1)$$

which may be rewritten as

$$Re_c = 367\, Oh^{-0.31} \qquad (2)$$

where St, Re_c and Oh are the stability number, the critical Reynolds number and the Ohnesorge number, respectively.

For the jets with fully developed parabolic velocity profiles for a wide range of nozzles and liquids, Grant and Middleman[9] developed the following empirical correlation for the critical Reynolds number.

$$Re_c = 325\ Oh^{-0.28} \quad (3)$$

They argued that the critical Reynolds number should be correlated with the Ohenesorge number and the density ratio between liquid and the ambient medium. They, however, suggested the empirical correlation as Eq. (3) with considering the Ohnesorge number only. In addition, they compared their empirical correlation with the following theoretical equation from Weber's theory under their experimental conditions.

$$Re_c = 40\ Oh^{-1.0} \quad (4)$$

They concluded that Eq.(4) couldn't predict correctly the position of the observed maximum in their experimental results.

Phinney[12] claimed that the Ohnesorge number is not the only parameter to express the critical Reynolds number, and particlular study is required to relate the Ohnesorge number and the critical Reynolds number.

On the other hand, McCarthy and Molloy[5] in their review paper pointed out the importance of nozzle geometry in determining the critical point in the stability curve. They tried to form a qualitative correlation between the nozzle shape(i.e., the contraction ratio, the nozzle length-to-diameter ratio, the contraction angle and the smoothness of nozzle interior) and issuing jet shape. They concluded that the nozzle length-to-diameter ratio(L/d) has a highly significant effect on the initial jet velocity profile and subsequent jet surface shape up to L/d values where the shear stress and static pressure gradient are fully developed.

Van de Sande and Smith[13] proposed the following expression for the critical Reynolds number as only a function of nozzle length-to-diameter ratio.

$$Re_c = 12000\ (L/d)^{-0.3} \quad (5)$$

where L/d is the nozzle length-to-diameter ratio. They reported that for convergent 3mm nozzles, the critical Reynolds number for the laminar-turbulent transition can be reasonably predicted by Eq. (5).

From the experimental data of Arai et al.[14] on the disintegration of a liquid jet in the atmospheric environment, the following empirical correlations are derived here for the different types of nozzle hole entrances.

For water and for sharp-edged inlet nozzle,

$$Re_c = 1997\ (L/d)^{0.087} \quad (6)$$

For water and for round-edged inlet nozzle,

$$Re_c = 7325\ (L/d)^{-0.448} \quad (7)$$

and for viscous liquids and for sharp-edged inlet nozzle,

$$Re_c = 542\ (L/d)^{0.095} \quad (8)$$

for viscous liquids and for round-edged inlet nozzle,

$$Re_c = 1546\ (L/d)^{-0.248} \quad (9)$$

It is evident from these equations that the effect of the nozzle length-to-diameter ratio on the critical Reynolds number shows the opposite tendency between the sharp-edged and the round-edged inlet types of nozzles for both liquids.

To clarify this result more precisely, the effect of nozzle length-to-diameter ratio on the critical Reynolds number for the different shapes of nozzle inlet is shown in Fig. 1. It is clear from the figure that the critical Reynolds number is increased with the nozzle length-to-diameter ratio for both liquids and the sharp-edged nozzle. On the other hand, for round-edged nozzle, the critical Reynolds number is decreased with the nozzle length-to-diameter ratio. This means that the nozzle-inlet type is one of the important parameters affecting the disintegration of liquid jet, and hence, the critical Reynolds number.

From the above discussion, it can be summarized that the critical Reynolds number may be expressed as a function of the Ohnesorge number, nozzle length-to-diameter ratio, the density ratio between liquid and ambient medium, or the ambient Weber number. In additon, the effects of nozzel inlet type such as sharp-edged, round-edged, re-entrant and convergent nozzles on the critical Reynolds number may be explicitly included in the

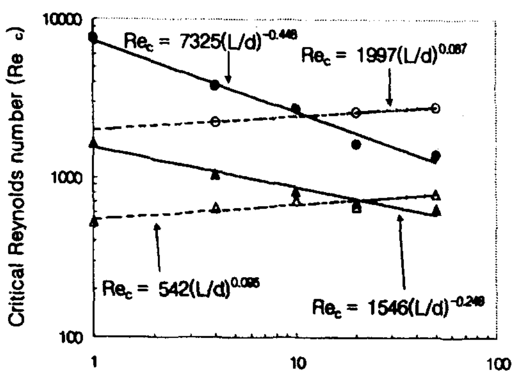

Fig. 1 Effect of L/d on Re_c for the different shape of nozzle : ----- Sharp-edged inlet ; ○ $\nu = 0.66\times10^{-6}m^2/s$, △ $\nu = 20\times10^{-6}m^2/s$, —— Round-edged inlet ; ● $\nu = 0.66\times10^{-6}m^2/s$, ▲ $\nu = 20\times10^{-6}m^2/s$

empirical correlation in any form. However, the experimental data employed to establish the emprical correlation in this study are limited to data for the liquid jet through atmospheric pressure and for the nozzles with re-entrant, convergent and sharp-edged inlet types.

ANALYSIS OF CRITICAL REYNOLDS NUMBER

Fig.2 compares the predicted results from Eqs. (2), (3) and (4) discussed ealier along with the various sources of experimental data. It is clear from the figure that the Weber's theory does not represent the experimental data correctly. It can be found that an increase in the Ohnesorge number, (i.e. in the liquid viscosity) generally, induces a decrease of the critical Reynolds number. In addition, Fig. 2 suggests that the critical Reynolds number cannot be expressed as a function of Ohnesorge number only. However, Eq. (2) by Tanasawa and Toyota[11] correlates the experimental data relatively well as pointed out by No et al.[15]

The variation of the critical Reynolds number with the nozzle length-to-diameter ratio is shown in Fig. 3 along with the correlations Eqs.(5~9). It is obvious that the empirical correlations deviate largely from the experimental data. It is well known that the nozzle length-to-diameter ratio has a remarkable effect on the initial jet velocity profile of the issuing liquid jet, i.e., subsquently the development of the critical Reynolds number. Fig.3 reveals that the liquid jet may have either a flat initial velocity profile or viscosity high enough to rapidly relax the issuing velocity profile. Hence the critical Reynolds number is likely inflenced slightly by the initial velocity profile,i.e., by the nozzle length-to-diameter ratio.

Therefore, it is required to establish the empirical correlation for the critical Reynolds number including both the effects of Ohnesorge number and nozzle length-to-diameter ratio simultaneouly. It was not possible to investigate the effects of ambient pressure and nozzle inlet type on the critical Reynolds number in this work due to the lack of the experimental data to compare. In obtaining an empirical correlation for the critical Reynolds number, a power equation was assumed as follows.

$$Re_c = C(Oh)^m (L/d)^n. \tag{10}$$

The multiple linear regression and Gauss elimination method was employed to obtain the constant coefficients C, m and n in Eq.(10). The results of these calculations give

$$Re_c = 218 (Oh)^{-0.338} (L/d)^{0.086} \tag{11}$$

It is clear from Eq.(11) that the relative contribution of length-to-diameter nozzle ratio to the critical Reynolds number is not remarkable. The reason can be inferred from the distribution of the critical Reynolds number with length-to-diameter nozzle ratio of Fig. 3.

The agreement with the predicted critical Reynolds number by Eq. (11) and the experimental data from the literature is shown in Fig. 4. There were some difficulties to determine the critical Reynolds number in the data selection of the critical velocity. In addition, there must be error due to data reading from graphs of the various literatures.

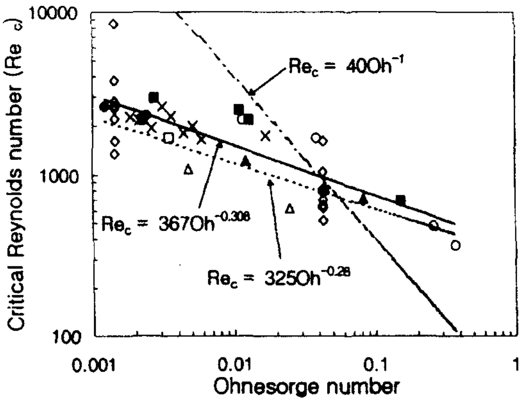

Fig. 2 Variation of critical Reynolds number with Ohnesorge number : —— Tanasawa & Toyota, ----Grant & Middleman, - - - Weber ; ◇ Arai el al.(1991), ● Arai et al.(1997), ▲ Grant & Middleman(1966), ■ Sterling & Sleicher(1975), ○ Fenn & Middleman(1969), △Leroux et al.(1997), □ Mansour & Chigier(1994), × No et al.(1998)

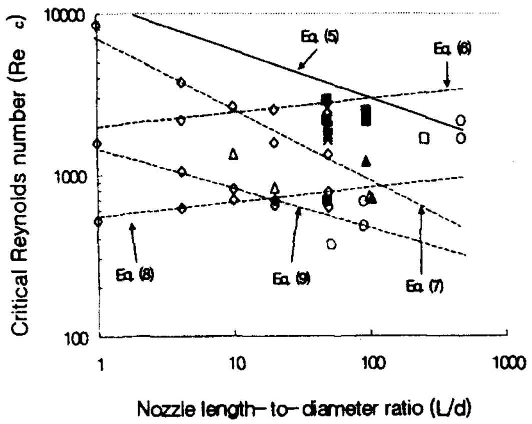

Fig. 3 Variation of critical Reynolds number with length-to-diameter nozzle ratio : —— Van de Sande & Smith(1976), ----- Derived eqs. from data of Arai et al.(1991) ; ◇ Arai el al.(1991), ▲Grant & Middleman(1966), ■ Sterling & Sleicher(1975), ○ Fenn & Middleman(1969), △Leroux et al.(1997), □ Mansour & Chigier(1994), × No et al.(1998)

The Eq.(11) with obtained constants here provides relatively a good correlation with appropriate experimental data. To find out the reliablity of Eq.(11), the test of correlation significance level was conducted.[16] It was assumed that the two variables i.e. the measured and the predicted critical Reynolds number are random. The correlation coefficient r of 0.89 between the two variables for Eq.(11) was obtained. The correlation between two variables are significant with 33 degrees of freedom for Eq.(11) at the significant level(α) of 1 %(i.e., the critical value is around 0.4304).

The data in this figure, reported by the different researchers, are for jets through the atmospheric environment and for nozzles with re-entrant, convergent and sharp-edged inlet type. Thus the result in this work is not valid for jet formed by round-edged nozzle inlet type and formed in the environment with higher or lower than atmospheric pressure. The tendency of the data for the round-edged inlet type of nozzle appears to be largely different from that of other types. This is clear from the comparisons between Eq.(6) and Eq.(7) for water, and between Eq.(8) and Eq.(9) for viscous liquids.

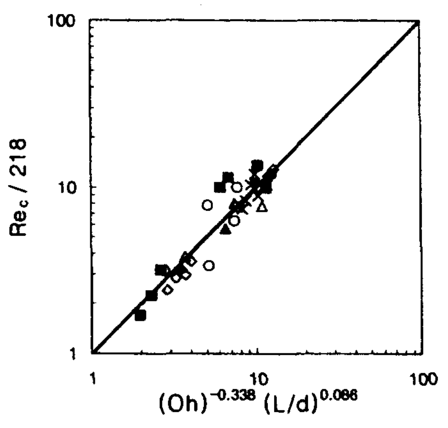

Fig. 4 Comparison of measured critical Reynolds number with predicted one by Eq.(10) : ◇ Arai el al.(1991), ● Arai et al.(1997), ▲Grant & Middleman(1966), ■ Sterling & Sleicher(1975), ○ Fenn & Middleman(1969), △Leroux et al.(1997), □ Mansour & Chigier(1994), × No et al.(1998)

CONCLUSIONS

The critical Reynolds number in the stability curve of liquid jet in the atmospheric condition are predicted from the empirical correlations and the experimental data reported previously. The critical Reynolds number is found to be described by a function of the Ohnesorge number, nozzle lengh-to-diameter ratio, ambient conditions and nozzle inlet type. An empirical correlation for the critical Reynolds number as a function of the Ohnesorge number and the nozzle length-to-diameter ratio was newly derived. The proposed correlation in this study can not claim to be universal, due to lack of the experimental data and failure to explicitly include the effects due to ambient pressure and nozzle entrance type etc. However, it reasonably agrees with the experimental data within a limited range.

For more precise correlation to predict the critical Reynolds number, a systematic evaluation of the role of the various factors such as ambient pressure and nozzle inlet type in the destabilization of liqid jet should be conducted.

ACKNOWLEDGEMENT

The authors wish to express their appreciation to Dr. C. Dumouchel at University of Rouen, France and Prof. M. Shimizu at Kinki University for supplying

their experimental data.

REFERENCES

1. A. H. Lefebvre, *Atomization and Sprays*, Hemisphere Pub. Co., 1989.
2. L. Bayvel and Z. Orzechowski, *Liquid Atomization*, Taylor & Francis, 1993
3. S. Leroux, C. Dumouchel and M. Ledoux, The Stability Curve of Newtonian Liquid Jets, *Atomization and Sprays*, Vol. 16, pp. 623-647, 1996.
4. A. M. Sterling and C. A. Sleicher, The Instability of Capillary Jets, *J. Fluid Mech.*, Vol. 68, pp. 477-495, 1975.
5. M. J. McCarthy and N. A. Molloy, Review of Stability of Liquid Jets and the Influence of Nozzle Design, *The Chemical Engineering Journal*, Vol. 7, pp. 1-20, 1974.
6. R. W. Fenn and S. Middleman, Newtonian Jet Stability: The Role of Air Resistance, *AIChE J.*, Vol. 15, No. 3, pp. 379-383, 1969.
7. A. Mansour and N. Chigier, Effect of Turbulence on the Stability of Liquid Jets and the Resulting Droplet Size Distributions, *Atomization and Sprays*, Vol. 14, pp. 583-604, 1994.
8. M. Arai, K. Amagai and S. Y. No, Disintegration of Liquid Jets from a Coaxial Dual Nozzle, Part I: Observation of Breakup Phenomena, *Atomization and Sprays*, Vol. 7, pp. 507-517, 1997.
9. R. P. Grant and S. Middleman, Newtonian Jet Stability, *AIChE J.*, Vol. 12, No. 4, pp. 669-678, 1966.
10. S. Leroux, C. Dumouchel and M. Ledoux, The Break-up Length of Laminar Cylindrical Liquid Jets: Modification of Webers Theory, *Proc. of ICLASS-97*, Vol. 1, pp. 353-360, 1997.
11. Y. Tanasawa and S. Toyoda, A Study on the Atomization of High Speed Liquid Jets(2nd Report), *J. of JSME*, Vol. 20, No. 92, pp. 306-309, 1954.(in Japanese)
12. R. E. Phinney, Stability of a Laminar Viscous Jet - The Influence of the Initial Disturbance Level, *AIChE J.*,Vol. 18, No. 2, pp. 432-434, 1972.
13. E. Van de Sande and J. M. Smith, Jet Break-up and Air Entrainment by Low Velocity Turbulent Water Jets, *Chem. Eng. Sci.*, Vol. 31, pp. 219-224, 1976.
14. M. Arai, M. Shimuzu and H. Hiroyasu, Disintergrating Process of a High Viscous Liquid Jet, *Journal of Japan Institute of Fuel*, Vol. 70, No. 11, pp. 1032-1037, 1991.(in Japanese)
15. S. Y. No, G. P. Ko and S. B. Lim, Measurement of Breakup Length of Viscous Liquid Jet in Stagnant Air, *Proc. of ILASS-Korea*, Oct. 17, 1998.(in Korean)
16. R. P. Sokal and F. J. Rohlf, *Introduction to Biostatistics*, W. H. Freeman and Company, New York, 1987.