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# Derivation of the Extended Elastic Stiffness Formula of the Holddown Spring Assembly Comprised of Several Leaves

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## Abstract

Based on the Euler beam theory and the elastic strain energy method, the elastic stiffness formula of the holddown spring assembly consisting of several leaves was previously derived. Even though the previous formula was known to be useful to estimate the elastic stiffness of the holddown spring assembly, recently it was reported that the elastic stiffness from the previous formula deviated greatly from the test results as the number of leaves was increased.

The objective of this study is to extend the previous formula in order to resolve such an increasing deviation when increasing the number of leaves. Additionally, considering the friction forces acting on the interfaces between the leaves, we obtained an extended elastic stiffness formula.

The characteristic test and the elastic stiffness analysis on the various kinds of specimens of the holddown spring assembly have been carried out; the validity of the extended formula has been verified by the comparison of their results. As a result of comparisons, it is found that the extended formula is able to evaluate the elastic stiffness of the holddown spring assembly within the maximum error range of +12%, irrespective of the number of the leaves.

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**Key Words** : elastic stiffness formula, holddown spring assembly, leaf spring, HDS

## 1. Introduction

The HoldDown Spring (HDS) assembly, which is attached at the uppermost part of the fuel assembly in pressurized water reactors, has two main functions [1]. The first is to keep the fuel assembly firmly seated on the lower core plate during normal plant operation. To accomplish this function, the springs, in conjunction with the fuel assembly weight, apply a holddown force in excess of the buoyancy forces and the upward hydraulic flow forces that act on the fuel assemblies due to the normal reactor coolant flow. The second is to allow changes to occur in the length of the fuel assembly relative to the space between the upper and lower core plates, while still providing an acceptable holddown force. These changes in relative length can occur due to differential thermal expansion between the fuel assembly structure made of Zircaloy-4 and the core support structure made of stainless steel, and due to the neutron irradiation-induced growth of the fuel assembly [2].

The elastic stiffness of the HDS assembly consisting of several leaf springs that are bent into design shapes and machined to have a uniformly tapered thickness or width along the leaf length is one of the most fundamental parameters in evaluating the holddown force of the fuel assembly. Then, it is known to be difficult to reliably estimate the elastic stiffness of the HDS assembly because (1) the springs are not simple cantilevers; (2) friction occurs between leaves; and (3) the spring screw and end plate may add some flexibility. Two kinds of HDS assemblies are currently prevailing in the fuel assembly. They are called as the tapered-thickness HDS assembly named TT-HDS and the tapered-width HDS assembly named TW-HDS, respectively. Up to now, most of the foreign nuclear fuel vendors determines the elastic stiffness from characteristic test results or from empirical formulas based on the test results. However these methods are limited in evaluation of the elastic stiffness of the TT-HDS in such cases that the shape of the leaf is changed or modified. Therefore, it is very useful for the design of the HDS assembly to develop a methodology to be able to reliably evaluate the elastic stiffness of the HDS assembly with the geometric data and the material properties of the leaf.

Based on the Euler beam theory and the elastic strain energy method, the elastic stiffness formula of the HDS assembly was previously derived [4]. Even though the previous formula was known to be useful to estimate the elastic stiffness of the HDS assembly, recently it was reported that the elastic stiffness from the previous formula deviated greatly from the test results, as the number of leaves was increased [5]. In this study, in order to resolve such an increasing deviation as the number of leaves increased, we extended the previous formula in consideration of the friction forces acting on the interfaces between the leaves. In addition, to verify the extended formula, the characteristic test and the elastic stiffness analysis on the various kinds of specimens of the HDS assembly has been carried out.

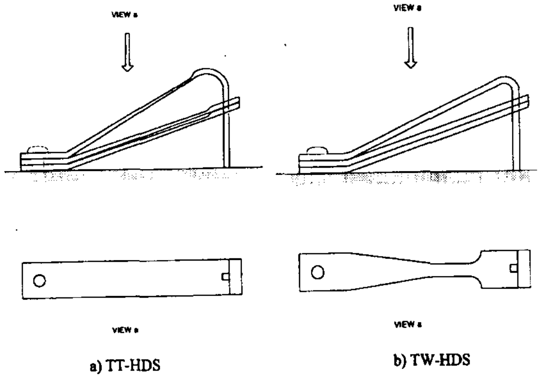
## 2. Derivation of the Extended Elastic Stiffness Formula

In order to analytically derive the extended elastic stiffness formula of the HDS assembly as shown in Fig. 1, each leaf spring is divided into regions, as designated in Figs. 2 and 3, for the TT-HDS and the TW-HDS, respectively. When a leaf spring is deformed, normal reaction forces and friction forces are acting on the interfaces between the leaves. The bending moments, axial and shear forces are obtained from the equilibrium conditions of a free-body diagram in each region of the leaf spring. The procedure to derive the extended elastic stiffness formula is summarized in the following three stages. First, the bending moments, axial and shear forces in each region are put to use to calculate the total strain energy in each leaf. Second, in-line deflections at loading and reaction points are obtained by applying Castigliano's theorem. Third, the extended elastic stiffness formula is obtained by imposing constraint conditions on the in-line deflections of each leaf. In the following subsections, these procedures are described in detail.

### 2.1. Total Strain Energy in Each Leaf

When a leaf spring is deformed, the total strain energy ($U_n$) in the *n-th* leaf is expressed as [6]:

$$U_n = \int \frac{M_i^2}{2E_iI_i} ds + \int \frac{P_i^2}{2A_iE_i} ds + \int \frac{\tau^2}{2G_i} dv \qquad (1)$$



a) TT-HDS b) TW-HDS

**Fig. 1. Leaf Type Holddown Spring Assembly.**

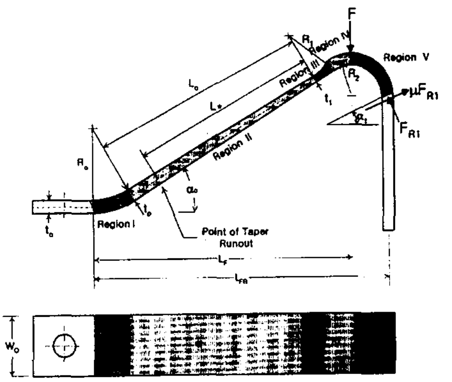
*where,*
$dv$ : Element of volume (or Differential volume)
$ds$ : Differential length
$U_n$ : Total strain energy in *n-th* leaf
$M_i$ : Bending moment in the *i-th* region
$E_i$ : Elastic modulus in the *i-th* region
$A_i$ : Cross-sectional area in the *i-th* region
$P_i$ : Axial force in the *i-th* region
$G_i$ : Shear modulus in the *i-th* region
$I_i$ : Second moment of the beam cross-sectional area in the *i-th* region
$\tau$: Shear stress
$i$ = I, II, III, IV, V: Region number of the leaf shown in Figs. 2, 3, and 4

Assuming that the shear stresses are distributed uniformly across the width and by solving the equilibrium equations for the plane stress condition, we can obtain the shear-stress distribution in a beam of rectangular cross-section as follows:

$$\tau \equiv \frac{V_i}{2I_i}\left[\left(\frac{t_x}{2}\right)^2 - y_1^2\right] \tag{2}$$

where,
$V_i$ : Shear force on the beam cross-section in the *i-th* region

a) For the uppermost leaf



b) For the lower(n≥2) leaf

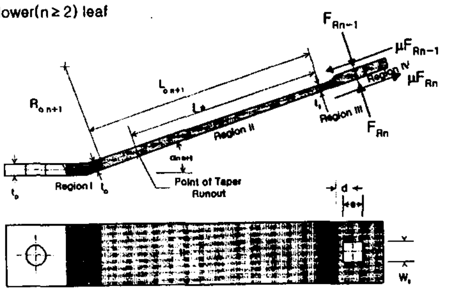


**Fig. 2. Design Variables for Each Leaf of TT-HDS.**

$t_x$ : Full thickness of the beam
$y_1$ : Distance from the neutral axis on the beam cross-section

### 2.2. In-line Deflections at Loading (F) and Reaction (F$_R$) Points

In-line deflections ($\delta$) at loading and reaction points are obtained by differentiating the total strain energy with respect to the load at that point (Castigliano's theorem).

#### 2.2.1. For the Top Leaf

$$\delta_{1F} = \frac{\partial U_1}{\partial F} = AA_1F - AB_1F_{R1} \tag{3}$$

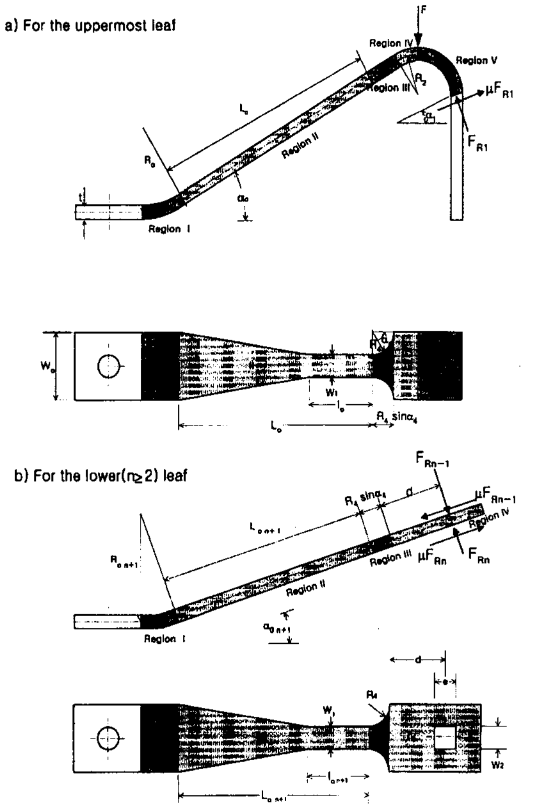


Fig. 3. Design Variables for Each Leaf of TW-HDS.

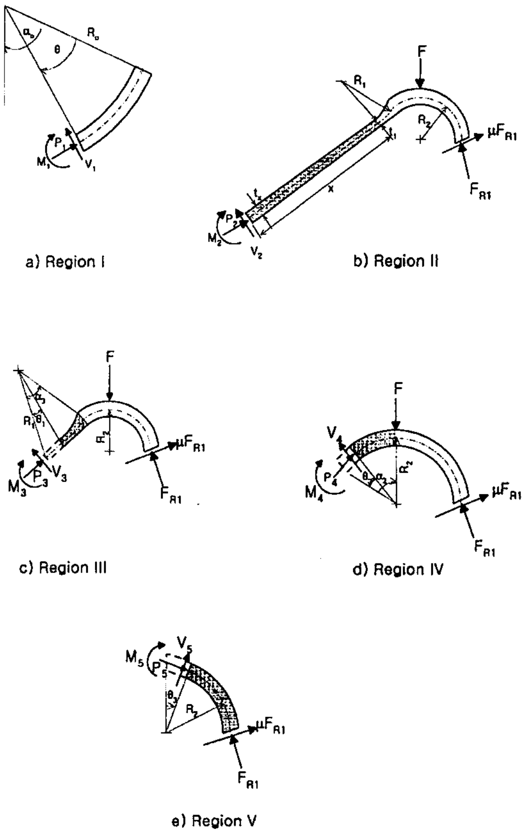
$$\delta_{1F_{R1}} = \frac{\partial U_1}{\partial F_{R1}} = -AB_1F + BB_1F_{R1} \tag{4}$$

### 2.2.2. For the Lower (n≥2) Leaf

$$\delta_{2F_{R1}} = \frac{\partial U_2}{\partial F_{R1}} = BB_2(F_{R1} - F_{R2}), \text{ for the 2nd leaf} \tag{5a}$$

$$\delta_{3F_{R2}} = \frac{\partial U_3}{\partial F_{R2}} = BB_3\, F_{R2}, \text{ for the 3nd leaf} \tag{5b}$$

$AA_1$, $AB_1$, $BB_1$, $BB_2$, and $BB_3$ are the coefficients [7] expressed as a function of the design variables and the coefficient of friction ($\mu$) between leaves. And $F_{R1}$ and $F_{R2}$ are the reactions at the reaction points of each leaf, as shown in Figs. 2 and 3.



a) Region I b) Region II c) Region III d) Region IV e) Region V

Fig. 4. Free Body Deagram in Each Region for Uppermost Leaf of TT-HDS.

### 2.3. Constraint Conditions on the In-line Deflections at Reaction Points

Assuming that the in-line deflections at reaction points between leaves are equal, the constraint conditions are as follows:

$$\delta_{1F_{R1}} = -\delta_{2F_{R1}}, \text{ for the top and 2nd leaf} \tag{6a}$$

$$\delta_{2F_{R1}} = \delta_{3F_{R2}}, \text{ for the top and 2nd leaf} \tag{6b}$$

### 2.4. Extended Elastic Stiffness Formula

From the in-line deflections of Eq. (3), Eq. (4), and Eq. (5a, b) and the constraint conditions of Eq. (6a, b), we can obtain the extended elastic stiffness formula of the HDS assembly as follows:

$$K_{ass} = \frac{F}{\delta_{1F}} = \frac{1}{AA_1 - \dfrac{AB_1^2}{BB_1 + \dfrac{1}{\sum_{i=2}^{3} \dfrac{1}{BB_i}}}} \qquad (7)$$

The coefficients in Eq. (7) for the TT-HDS are differently expressed from those for the TW-HDS because of the different dimensions and shape of the HDS assemblies.

## 3. Characteristic Test on the HDS Assemblies

A characteristic test has been carried out on the TW-HDS and the TT-HDS. Three sets of holddown springs, which are composed of one leaf, two leaves, and three leaves, are prepared for the TW-HDS and the TT-HDS; for each set of holddown springs, five test specimens are prepared. The test specimens composed of one leaf are designed to investigate the elastic stiffness for the case that no friction forces are acting on the leaf spring. And the test specimens composed of two or three leaves are designed to investigate the effect of friction forces on the elastic stiffness.

For the determination of the elastic stiffness of the HDS assemblies, the test specimens were mounted in the same manner as in the fuel assembly, i.e.; it was fastened to the end piece using the original screws at the fixed end of the spring sets. The screw was tightened in accordance with installation requirements. The end piece with the test specimens attached was mounted on the working table of a MTS (Material Testing System) to obtain the force-deflection curve. Compressive tests on the test specimens are performed until the test specimens are deflected vertically up to 46mm and the test results are input to the on-line delayed time personal computer in which the force-deflection curves are generated by ORIGIN program [8]. From the force-deflection curves, the elastic stiffness of the test specimens is obtained by curve-fitting the data within an elastic range.

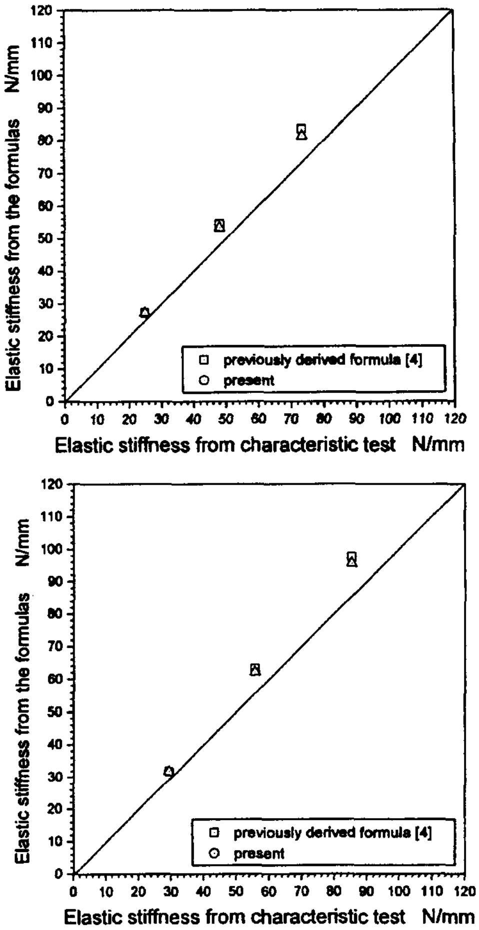


Fig. 5. Comparison of Elastic Stiffness Derived by the Formula and Test Results.

## 4. Verification of the Extended Elastic Stiffness Formula

In order to check the validity of the extended

**Table. 1. Ratio of the Elastic Stiffness from the Formulas to the Characteristic Test Result**

| | Number of leaves | Ratio of the elastic stiffness | |
|---|---|---|---|
| | | Previously derived formula [4] | Present |
| | 1 | 1.098 | |
| TT-HDS | 2 | 1.137 | 1.112 |
| | 3 | 1.141 | 1.110 |
| | 1 | 1.087 | |
| TW-HDS | 2 | 1.137 | 1.117 |
| | 3 | 1.145 | 1.121 |

elastic stiffness formula, the elastic stiffness from the extended formula is compared with those from the characteristic tests. In the elastic stiffness analysis, the coefficient of sliding friction for steel on steel (=0.20) [9] is used. The comparisons for the HDS specimens are shown in Fig. 5 and Table 1.

For the TT-HDS specimen composed of only the top leaf, Table 1 shows that the ratio of the elastic stiffness from the formulas to the characteristic test result is 1.098. In that case, the elastic stiffness from the extended formula ("present" in Fig. 5 and Table 1) is the same as that from the previously derived formula [4] because no friction forces are acting on the top leaf. Such an over-estimation of the elastic stiffness from the formulas is assumed to the imposition of different boundary conditions at the fixed end of the spring sets. For example, actually the fixed end of the spring sets is held in place by the screw, which allows the spring set to rotate and lead to more deflections, while in the analytical method all displacements at that part of the leaf are constrained as an ideal clamped support. In addition, Table 1 shows that as the number of leaves is increased, the ratio of elastic stiffness derived from the previously derived formula to the test result deviates greatly while the ratio for the extended elastic stiffness formula is maintained around 1.11. This fact denotes that the extended elastic stiffness formula can properly consider the friction forces acting on the interfaces between the leaves, which were not properly considered in the previously derived formula.

For the TW-HDS specimen composed of only the top leaf, Table 1 shows that the ratio of the elastic stiffness derived from the formulas to the characteristic test result is 1.087. Such an over-estimation of the elastic stiffness for the formulas is assumed to the imposition of different boundary conditions at the fixed end of the spring sets. In addition, Table 1 shows that as the number of leaves is increased, the ratio for the previously derived formula deviates greatly from 1.0 while the ratio for the extended formula is maintained around 1.12. This fact also denotes that the extended elastic stiffness formula properly considers the friction forces acting on the interfaces between the leaves, which were not properly considered in the previously derived formula

## 5. Conclusions

The elastic stiffness formula of the HDS assembly, which was previously derived, has been extended to additionally consider the friction forces acting on the interfaces between the leaves. And the extended elastic stiffness formula is verified by comparing the values of the elastic stiffness from the extended formula with the characteristic test results. The results from this study are as follows:

1. For the HDS specimen composed of only the top leaf, the ratios of the elastic stiffness derived from the extended elastic stiffness formula to the characteristic test result is around 1.08. And this deviation from the test results is

attributed to the presumptions in derivation of the extended elastic stiffness formula.

2. As the number of leaves is increased, the ratio of the elastic stiffness from the extended formula to the test result is maintained around 1.10~1.12 while the ratio for the previously derived formula deviates greatly. This fact denotes that the extended elastic stiffness formula properly considers the friction forces acting on the interfaces between the leaves, which were not properly considered in the previously derived formula.

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